# One-Loop Renormalization Group Equations in the Standard Model and Beyond

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*A dissertation submitted for the partial fulfillment of BS-MS dual degree in Science*

**Indian Institute of Science Education and Research Mohali**

**April 2019**

## Certificate of Examination

This is to certify that the dissertation titled **"One-Loop Renormalization Group Equations in the Standard Model and Beyond"** submitted by **Ms. Meenakshi** (Reg. No. MS14077) for the partial fulfillment of BS-MS dual degree programme of the Institute, has been examined by the thesis committee duly appointed by the Institute. The committee finds the work done by the candidate satisfactory and recommends that the report be accepted.

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## Declaration

The work presented in this dissertation has been carried out by me under the guidance of Dr. Ketan Patel at the Indian Institute of Science Education and Research Mohali. Some later part of my thesis was done at Physical Research Laboratory, Ahemdabad.

This work has not been submitted in part or in full for a degree, a diploma, or a fellowship to any other university or institute. Whenever contributions of others are involved, every effort is made to indicate this clearly, with due acknowledgement of collaborative research and discussions. This thesis is a bonafide record of original work done by me and all sources listed within have been detailed in the bibliography.

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In my capacity as the supervisor of the candidate's project work, I certify that the above statements by the candidate are true to the best of my knowledge.

Prof. Charanjit Singh Aulakh
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Dr. Ketan Patel
(Supervisor)

# Acknowledgement

I would take this opportunity to thank my thesis supervisor “Dr Ketan Patel” for allowing me to pursue my master’s thesis project under his guidance. His motivation, knowledge and positively encouraging attitude helped me a lot during my thesis.

I also thank Dr. Kinjalk Lochan and Dr. Ambresh Shivaji for being a part of my thesis committee. I also thanks Prof. Charanjit S. Aulakh for being my local guide at IISER-M for my thesis work. I also thank him for the course on “Gauge theories and the Standard Model-PHY654” whose lecture notes I followed for taking help in some calculations. I want to express my gratitude towards the founding Director Prof. N. Sathyamurthy, former Director Prof. D.P. Sarkar, Contemporary Director Prof. Arvind for their support.

I thank my peer group “Chhatta Floor”(Ritika, Swati, Sanjana, Tejasi, Avneet, Himanshi) for their continuous support and for all the fun we had to make this period a cheerful journey. Especially, I thank my friend, Himanshi Singh for always being there for me in my troubles and comfort in my five years of BS-MS.

I also thank Ravneet S. Bedi for all the valuable discussions which helped in clearing many doubts and giving more insight into the work. Thanks to a lot of my friends to name who directly or indirectly cheered me up. A special mention to Jorawar Singh, Subhashree, and Geeta.

I am grateful to my parents, Mr. Ramkumar Moun and Mrs. Ramrati for believing in me and motivating me throughout this project and my life in general. I thank my siblings, Sheetal di, Sapna and Deepak for their love and support.

Last but not least, I thank the DST, Government of India for the Inspire Fellowship that has helped in covering a large part of my educational expenses during BS-MS. I thank IISER Mohali(IISER-M) for giving me this opportunity to pursue my master’s thesis and explore more in my field of interest.

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# Abstract

This thesis deals with one-loop Renormalization Group Equations(RGEs) for the Standard Model and Beyond the Standard Model. Among the SM couplings, the quartic coupling goes to negative before the Planck scale which unbound the Higgs potential at higher field values. This makes the present Higgs vacuum metastable and gives an indication of Physics beyond the Standard Model.

Thus, we have worked on extending the SM via Clock-Work(CW) fermions and analyzed their effects on the SM RGEs. We found that adding CW fermions can stabilize the Higgs potential by breaking the asymptotic freedom of $g_2$ and $g_3$ couplings and contributing more positively to the $g_1$ coupling, leading to faster decay of Top-Yukawa coupling. Also, we have shown that one can unify the gauge couplings by CW extension keeping the perturbation theory valid till the Planck Scale.

# Chapter 1

# Introduction

This master's thesis investigates the Renormalization Group Equations, i.e., the evolution of the couplings with energy scale, of the Standard Model(SM) and Beyond the SM. The standard model is a gauge quantum field theory containing the internal symmetries of the unitary product group,

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

The group SU(3) corresponds to the three colors, i.e., red, green, blue and only quarks families from the SM are found to transform as a triplet under SU(3). In SU(2) transformations, the particles transform as a doublet under SU(2) generators. Only left-handed particles are found to transform under this group. Each particle in the SM has its hyper-charge(denoted by Y), and it transforms under U(1) group. The hypercharge of the particle is related to its charge and the 3rd component of isospin which is given by Gell-Mann-Nishijima formula i.e.

$$Y = 2(Q - I_3)$$

Where $I_3$ is the 3rd component of isospin and $Q$ denotes the charge of the particle.
The particles in the SM interact with each other, and the strength of interaction is measured by the coupling constants. These coupling constants evolve with the energy scale and how these couplings evolve termed as the "beta functions" or "Renormalization Group Equations". The gauge couplings show the strength of interaction of the scalars and fermions with the gauge-bosons, and it is of three types corresponding to the three gauge groups of the SM. Next, Yukawa coupling shows the interaction between the fermionic LH doublet,

Higgs and RH fermionic singlet. The quartic coupling refers to the interaction among the Higgs field itself.

In this thesis, the initial chapters are on deriving the RGE's for the general theory taking the most general form of Lagrangian. Also, the SM results for the corresponding coupling is derived from the general results in the same chapter. The plots for the couplings with logarithmic energy scale is also shown for one-loop results and are compared with the two-loop RGE's. Then we have extended the SM by adding Clockwork fermions at different scales and have analyzed their effects on the SM RGEs.

## 1.1 Gauge Group Generators and Invariants in the Standard Model

To calculate the beta functions one needs to know the gauge group generators and calculate the corresponding invariants. We are considering a theory associated with non-commuting local symmetry, i.e., non-abelian gauge theory where unlike the abelian case the kinetic term for the fields has a covariant derivative term here which is defined for the fermion and scalar field as:

$$\begin{aligned} D_\mu \psi_j &= (\partial_\mu - igT^a_{jk}A^a_\mu)\psi_k \\ D_\mu \Phi_c &= (\partial_\mu - ig\Theta^a_{cb}A^a_\mu)\Phi_b \end{aligned} \tag{1.1}$$

where $g$ is the gauge coupling, $A_\mu$ corresponds to the gauge field and superscript "a" is summed over all the generators of the gauge-group. The generators $T^a$ and $\Theta^a$ corresponds to gauge group generators for fermions and scalars respectively. The numbers of generators for a Special Unitary group(SU(N)) are equal to $N^2 - 1$ corresponding to the independent parameters of the group.

Starting with the color group i.e. SU(3) gauge group, corresponding to eight independent gluons we have eight Gell-Mann matrices as the generators in the fundamental representation defined by the relation:

$$[\frac{\lambda_i}{2}, \frac{\lambda_j}{2}] = if_{ijk}\frac{\lambda_k}{2}$$

The structure constants are given as: $f^{123} = 1$ , $f^{147} = f^{165} = f^{246} = f^{257} = f^{345} = f^{376} = \frac{1}{2}$ , $f^{458} = f^{678} = \frac{\sqrt{3}}{2}$ . The invariants for this group are given as:

$$\begin{aligned} C_2(F) = T^a T^a &= \frac{4}{3} I \\ C(F) = Tr(T^a T^b) &= \frac{1}{2} I \\ C_2(G) = f^{acd} f^{bcd} &= 3 \end{aligned} \tag{1.2}$$

For SU(2) group, we have three weak gauge-bosons and the generators are three Pauli matrices satisfying the commutation relation:

$$[\frac{\sigma_i}{2}, \frac{\sigma_j}{2}] = i\epsilon_{ijk} \frac{\sigma_k}{2}$$

where $\epsilon_{ijk}$ is an Levi-Civita anti-symmetric tensor. The invariants here will be:

$$\begin{aligned} C_2(F) = C_2(S) = T^a T^a = \Theta^a \Theta^a &= \frac{3}{4} I \\ C(F) = C(S) = Tr(T^a T^b) = Tr(\Theta^A \Theta^B) &= \frac{1}{2} I \\ C_2(G) = f^{acd} f^{bcd} &= 2 \end{aligned} \tag{1.3}$$

For the hypercharge group, the generator is the hypercharge diagonal matrix, which gives

$$\begin{aligned} \Theta^a = \frac{1}{2} I &\Longrightarrow C(S) = \frac{1}{2} \\ C(F) &= \sum_{gen} (\frac{Y}{2})^2 \times color \end{aligned} \tag{1.4}$$

## 1.2 The Standard Model Particles

The table below lists all the Standard Model particles(quarks, leptons, and Higgs) and their corresponding charge(Q), 3rd isospin component($I_3$) and hypercharge(Y).

The scalar field in the SM, i.e., Higgs is a $SU(2)$ complex doublet. Higgs is responsible for generating mass to the particles in the SM by taking a non-zero vacuum expectation value, and this phenomenon is called Spontaneous Symmetry Breaking(SSB). For example, Glashow-Weinberg-Salam model([Xin 07]) explains Higgs Mechanism(when a gauge-boson acquires mass, and a massless Goldstone-boson is produced) with broken symmetry $SU(2) \times U(1)_Y \rightarrow U(1)_{EM}$, which predicts the massive vector bosons $W^+, W^-, Z$ and mass-less photon. This theory also unifies the weak and electromagnetic interaction between elementary particles.

| | Generation I | Generation II | Generation III | $I_3$ | Y | Q |
|---|---|---|---|---|---|---|
| Leptons | $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ | $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$ | $\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$ | +1/2<br>-1/2 | -1<br>-1 | 0<br>-1 |
| | $e_R$ | $\mu_R$ | $\tau_R$ | 0 | -2 | -1 |
| | | | | | | |
| Quarks | $\begin{pmatrix} u \\ d \end{pmatrix}_L$ | $\begin{pmatrix} c \\ s \end{pmatrix}_L$ | $\begin{pmatrix} t \\ b \end{pmatrix}_L$ | +1/2<br>-1/2 | +1/3<br>+1/3 | +2/3<br>-1/3 |
| | $u_R$ | $c_R$ | $t_R$ | 0 | +4/3 | +2/3 |
| | $d_R$ | $s_R$ | $b_R$ | 0 | -2/3 | -1/3 |
| | | | | | | |
| Higgs | $\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$ | | | 1/2<br>-1/2 | 1<br>1 | 0<br>1 |

Figure 1.1: The Standard Model Particles

# Chapter 2

# Gauge Beta Functions

In this chapter, we will be deriving the gauge beta functions taking the most general form of Lagrangian for both the scalar and fermionic fields. The RGEs for the SM are derived from the general results.

## 2.1 Gauge Beta Functions for Fermions - General Background

We will start with the general bare Lagrangian(subscript '0' shows bare fields and couplings) for non-abelian gauge theory with Dirac fermions which can be written as:

$$L = -\frac{1}{4}(F^a_{0\mu\nu})^2 - \frac{1}{2\zeta}(\partial^\mu A^a_{0\mu})^2 + \overline{\Psi}_0 i\gamma^\mu(\partial_\mu - ig_0 A^a_{0\mu}T^a)\Psi_0 - \overline{c^a_0}(\partial^2\delta^{ac} + g_0\partial^\mu f^{abc}A^b_\mu)c^c_0 \tag{2.1}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$

and is the gauge-invariant kinetic term for the gauge field in non-abelian gauge theory. The fields $c^a$ are the Fadeev-Poppov ghosts which appear in the non-abelian gauge theory, (see Chapter-16[Peskin 95]) and interact with the gauge-fields only. In abelian case, since $f^{abc} = 0$, there is only kinetic term for ghost fields and hence contribute nothing to beta functions. In the renormalized theory the above bare Lagrangian is modified to:

$$L = -\frac{1}{4} Z_A (F^a_{\mu\nu})^2 + i Z_\Psi \overline{\Psi} \gamma^\mu \partial_\mu \Psi + g_0 Z_\Psi Z_A^{1/2} \overline{\Psi} \Psi A^a_\mu T^a + ... \tag{2.2}$$

where

$$Z_\Psi = 1 + \delta_\Psi \tag{2.3}$$

$$Z_A = 1 + \delta_A \tag{2.4}$$

$$g_0 Z_\Psi Z_A^{1/2} = g \mu^\epsilon Z_g \tag{2.5}$$

$$Z_g = 1 + \delta_g \tag{2.6}$$

$$Z_c = 1 + \delta_c \tag{2.7}$$

$$\frac{4-d}{2} = \epsilon \tag{2.8}$$

The bare Lagrangian can be written in terms of renormalized plus the counter terms:

$$L = L_{ren} + L_{ct}$$

where

$$L_{ren} = \frac{-1}{4} (F^a_{\mu\nu})^2 + \overline{\Psi} i \gamma^\mu (\partial_\mu - i g \mu^\epsilon A^a_\mu T^a) \Psi - \overline{c}^a (\partial^2 \delta^{ac} + g \mu^\epsilon \partial_\mu f^{abc} A^b_\mu) c^c \tag{2.9}$$

$$L_{ct} = \frac{-1}{4} \delta_A (F^0_{\mu\nu})^2 + \delta_\Psi \overline{\Psi} i \gamma^\mu \partial_\mu \Psi + \mu^\epsilon g \delta_g \gamma^\mu \overline{\Psi} A^a_\mu \Psi T^a + ... \tag{2.10}$$

From (2.5) we can write,

$$\ln (Z_g Z_{\overline{\Psi}}^{-1} Z_A^{-1/2}) = \sum_{n=1}^{n=\infty} \frac{G_n(g)}{\epsilon^n} \tag{2.11}$$

which further gives,

$$\ln g_0 = \sum_{n=1}^{n=\infty} \frac{G_n(g)}{\epsilon^n} + \ln g + \epsilon \ln \mu \tag{2.12}$$

Since the bare parameters don't depend on $\mu$, we have

$$0 = \sum_{n=1}^{n=\infty} (g\frac{\partial G_n}{\partial g}\frac{dg}{d\ln\mu})\frac{1}{\epsilon^n} + \frac{dg}{d\ln\mu} + g\epsilon \tag{2.13}$$

and in a renormalizable theory the rate of coupling evolution has to be finite for $\epsilon \to 0$ which gives,

$$\frac{dg}{d\ln\mu} = -\epsilon g + \beta_g(g) \tag{2.14}$$

From equations (2.13) and (2.14) matching the term with the coefficient of $1/\epsilon$ for the one loop contribution to the beta function we have ,

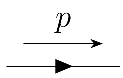
$$\beta_g(g) = g^2\frac{\partial G_1}{\partial g}$$

which after putting value of $G_1$, expanding logarithmic terms using equations (2.3) to (2.8) we have the gauge beta functions general formula in terms of the counter-terms i.e.,

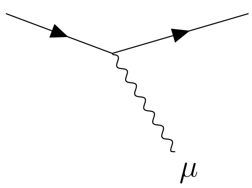
$$\beta_g(g) = g^2\frac{\partial}{\partial g}(\delta_g - \delta_\Psi - \frac{\delta_A}{2}) \tag{2.15}$$

### 2.1.1 Calculating Feynman Rules and Vertex Corrections

The Feynman rules are defined as:



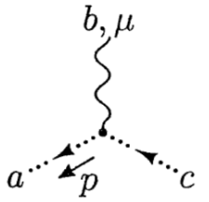

$$= \frac{ip^\mu\gamma_\mu}{p^2} \tag{2.16}$$



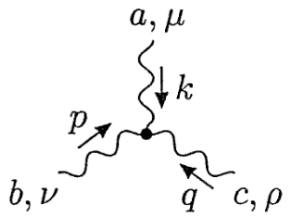

$$= ig\gamma^\mu T^a \tag{2.17}$$

$$\mu \sim\!\sim\!\sim \nu \; = \frac{-ig_{\mu\nu}}{p^2} \qquad [Feynman - gauge] \tag{2.18}$$

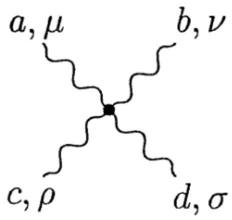

$$= \frac{i\delta^{ab}}{p^2}$$




$$= g f^{abc} p^{\mu}$$

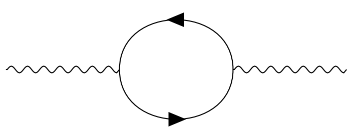



$$= g f^{abc}\big[g^{\mu\nu}(k-p)^{\rho} + g^{\nu\rho}(p-q)^{\mu} + g^{\rho\mu}(q-k)^{\nu}\big]$$



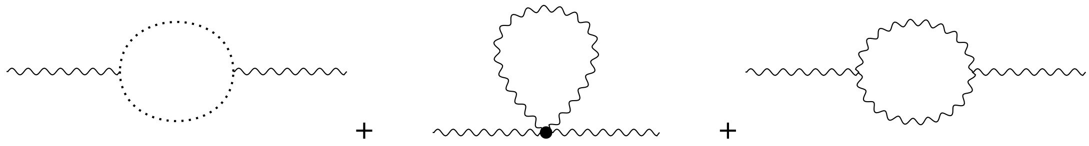

$$= -ig^2\big[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})\big]$$

The Feynman rules above were used to calculate the loop corrections. We have used dimensional regularization and $\overline{MS}$ scheme to solve the loop integrals as is explained in Appendix A. The diagrams contributing to gauge boson propagator are:



$$= -i(q^2 g^{\mu\nu} - q^{\mu}q^{\nu})\delta^{ab}\left(\frac{g^2}{(4\pi)^2}\frac{4}{3\epsilon}n_f C(F)\right) \tag{2.19}$$

where $n_f$ are the number of fermions interacting with gauge field.



$$= i(q^2 g^{\mu\nu} - q^\mu q^\nu)\delta^{ab}(\frac{g^2}{(4\pi)^2}\frac{5}{3\epsilon}C_2(G)) \tag{2.20}$$

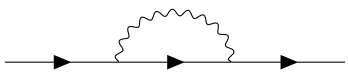
And for finite propagator, the counter-term factor $\delta_A$ should cancel out the divergences of the gauge propagator which gives,

$$= -i\delta^{ab}(q^2 g^{\mu\nu} - q^\mu q^\nu)\delta_A \tag{2.21}$$

The counter term for gauge propagator by combining all the above corrections is :

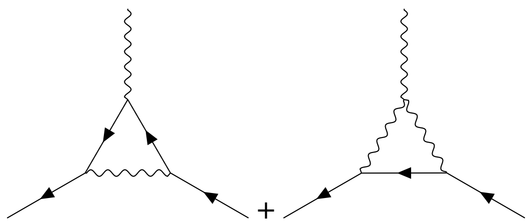
$$\delta_A = \frac{g^2}{(4\pi)^2}(\frac{5}{3}C_2(G) - \frac{4}{3}n_f C(F)) \tag{2.22}$$

Similarly, for fermion self energy we have,



$$\rightarrow \delta_\Psi = -\frac{g^2}{(4\pi)^2}C_2(F) \tag{2.23}$$

The diagrams whose divergences are cancelled out by the gauge-fermion vertex counter-term are:



$$\delta_g = -\frac{g^2}{(4\pi)^2}(C_2(G) + C_2(F)) \tag{2.24}$$

Putting all these counter-terms vertices in (2.15), we have the final expression for the gauge beta functions:

$$\beta_g(g) = -\frac{g^3}{(4\pi)^2}(\frac{11}{3}C_2(G) - \frac{4}{3}n_f C(F)) \tag{2.25}$$

## 2.2 Gauge Beta Functions for Scalar Field

The renormalized Lagrangian for non-abelian gauge theory with complex scalar field will be:

$$L_{ren} = \frac{-1}{4}(F^a_{\mu\nu})^2 + (D_\mu\phi)^\dagger(D^\mu\phi) - \bar{c}^a(\partial^2\delta^{ac} + g\partial_\mu f^{abc}A^b_\mu)c^c + ... \tag{2.26}$$

The (2.3) and (2.5) here are:

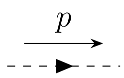
$$Z_\phi = 1 + \delta_\phi \tag{2.27}$$

$$g_0 Z_\phi Z_A^{1/2} = g\mu^\epsilon Z_g \tag{2.28}$$

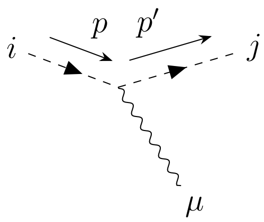
which implies that the beta function for gauge-scalar coupling is:

$$\beta_g(g) = g^2\frac{\partial}{\partial g}(\delta_g - \delta_\phi - \frac{\delta_A}{2}) \tag{2.29}$$

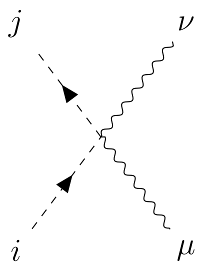
The new Feynman rules will be:




$$= \frac{i}{p^2} \tag{2.30}$$

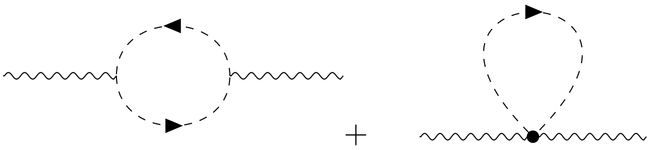



$$= ig(p+p')^\mu\Theta^a_{ji} \tag{2.31}$$




$$= ig^2\{\Theta^a, \Theta^b\}g^{\mu\nu} \tag{2.32}$$

The diagrams including contribution of scalar field to gauge self energy are:

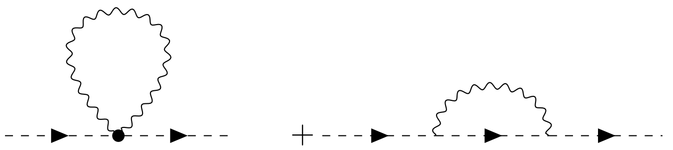


$$= -i(q^2 g^{\mu\nu} - q^\mu q^\nu)\frac{g^2}{3\epsilon(4\pi)^2} n_s C(S) \tag{2.33}$$

where $n_s$ is the number of scalar fields. The other diagrams shown in equation (2.20) contributing to gauge fields will also be there and the contribution will remain same as earlier. Combining the contribution from scalar field, the counter term for gauge-propagator becomes,

$$\delta_A = \frac{g^2}{16\pi^2}(\frac{5}{3}C_2(G) - \frac{1}{3}n_s C(S)) \tag{2.34}$$

Now the diagrams with One-Particle Irreducible(1PI) correction to scalar propagator are:

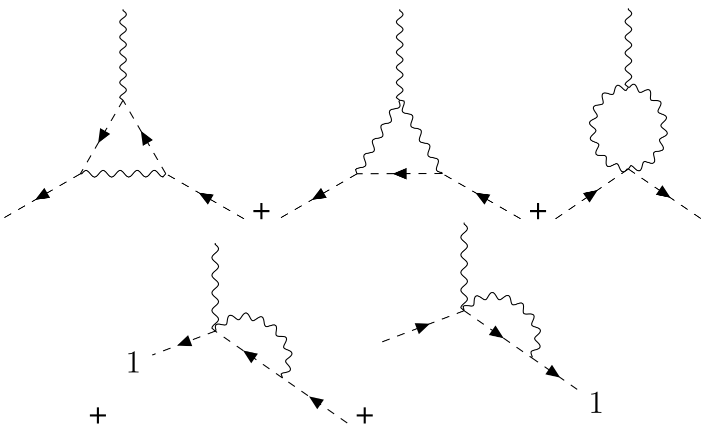


$$= -ik^2 \frac{2g^2}{\epsilon(4\pi)^2} C_2(S)$$

This gives the scalar propagator correction $\delta_\phi$ to be

$$\delta_\phi = \frac{2g^2}{(4\pi)^2} C_2(S) \tag{2.35}$$

The scalar-gauge vertex correcting diagrams at one-loop contributing to $\delta_g$ are:




$$\delta_g = \frac{-g^2}{16\pi^2}(C_2(G) - 2C_2(S)) \tag{2.36}$$

Now putting (2.34), (2.35) and (2.36) in eq. (2.29) gives the general expression for scalar - gauge coupling beta function which is,

$$\beta_g(g) = \frac{-g^3}{16\pi^2}(\frac{11}{3}C_2(G) - \frac{1}{3}n_s C(S)) \tag{2.37}$$

Now, for the total general gauge coupling beta functions for complex scalars and fermions can be calculated by combining results in eq.(2.15) to eq. (2.37) which gives,

$$\beta_g(g) = \frac{-g^3}{16\pi^2}(\frac{11}{3}C_2(G) - \frac{4}{3}n_f C(F) - \frac{1}{3}n_s C(S)) \tag{2.38}$$

### 2.2.1 From General to the SM Gauge Beta Functions

Now for deriving the SM gauge beta function from general theory, we know that SM is a semi-simple gauge theory so we will need to do it separately for all 3 gauge groups. For fermions of definite helicity one needs to take 1/2 factor for fermionic loop(1 for Dirac fermion).

- **SU(3)** : We have four SU(3) triplets($N_t$) $(u_L, u_R, d_L, d_R)$ in each generation. The invariant $C(F) = 1/2$(from Chapter 1). And for total 3 generations it gives $n_f = 4 \times 3 = 12$.

$$\beta_g(g_3) = \frac{-g_3^3}{16\pi^2}(\frac{11}{3} \times 3 - \frac{4}{3} \times 4 \times 3 \times \frac{1}{4}) \Longrightarrow -\frac{7}{16\pi^2}g_3^3 \tag{2.39}$$

- **SU(2)** : We have four SU(2) doublets($N_d$) $((q_L)_{r,g,b}, l_L)$ in each generation. The invariant $C(F) = 1/2$ here also(from Chapter 1). And for total 3 generations it gives $n_f = 4 \times 3 = 12$. And we have one Higgs doublet also.

$$\beta_g(g_2) = \frac{-g_2^3}{16\pi^2}(\frac{11}{3} \times 2 - \frac{4}{3} \times 4 \times 3 \times \frac{1}{4} - \frac{1}{3} \times \frac{1}{2}) \Longrightarrow -\frac{-19}{6}\frac{g_2^3}{16\pi^2} \tag{2.40}$$

- **U(1)** : The table corresponding to hypercharge is shown in chapter 1. The sum of $(Y/2)^2$ for one generation of fermions is $\frac{10}{3}$. Also transforming $g_Y \longrightarrow \sqrt{\frac{3}{5}}g_1$, and summing over all generations and scalar fields, we get

$$\beta_g(g_1) = \frac{g_1^3}{16\pi^2}(\frac{4}{3} \times \frac{10}{3} \times 3 \times \frac{1}{2} + \frac{1}{3} \times 2 \times \frac{1}{4}) \times \frac{3}{5} \Longrightarrow \frac{41}{10}\frac{g_1^3}{16\pi^2} \tag{2.41}$$

In the Figure 2.1, $\alpha = \frac{g^2}{4\pi}$ and the initial values for the couplings are taken from PDG webpage where at Top Quark mass scale i.e. $M_t = 173.1$ GeV we have $g_1(M_t) = 0.4630$, $g_2(M_t) = 0.6538$, $g_3(M_t) = 1.1628$.

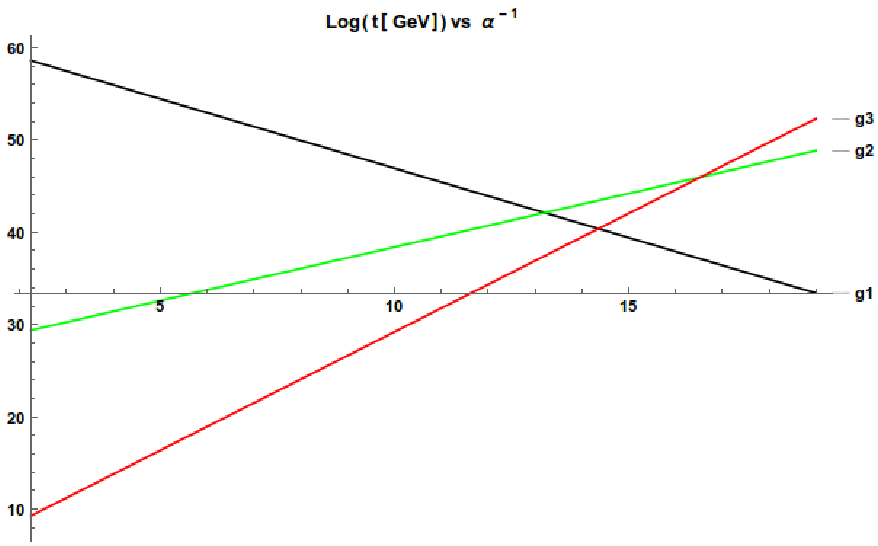



Figure 2.1: $\alpha^{-1}$ vs $Log(t(GeV))$ for the SM.

One can see from the figure that $g_2$ and $g_3$ couplings in the Standard Model behave asymptotically free(decreases with energy scale) unlike $g_1$.

## 2.3 Qualitative Picture of Asymptotic Freedom

One can understand the sign of beta function of $g_1$ coupling as due to charge screening effect by the vacuum, where electron-positron pairs fluctuate into existence and thus respond to the presence of a source in such a way as to decrease its field at a long distance. And this leads to coupling grow at larger energy scales and vice-versa.

For $g_2$ and $g_3$ beta functions the sign is opposite for finite number of $N_d$ and $N_t$ as in the SM case. So, how are they can produce anti-screening? Since the gauge bosons and fermions both are charged so must lead to screening as in the abelian case, but the opposite sign shows that the anti-screening effect due to gauge fields must be a dominating factor here.

From [Peskin 95] sec:16.7, considering Coulomb gauge i.e. $d_i A^{ai} = 0$ in non-abelian case the Gauss' law takes the gauge-invariant form as :

$$D_i E^{ai} = g\rho^a = \partial_i E^{ia} + g f^{abc} A_i^b E^{ic}$$

where $E^{ai} = F^{a0i}$ and $\rho^a$ is the charge density of the global symmetry current of the fermions. For $SU(2)$ gauge group we have $f^{abc} = \epsilon^{abc}$ and $a = 1, 2, 3$. Considering a static charge particle we write $\rho^a(x) = \delta^{(3)}(\vec{x})\delta^{a1}$ where $a = 1$ gives a source at the origin. From the covariant derivative equation we can write the equation that we want to solve is

$$\partial_i \vec{E}^{ia} = g\delta^{(3)}(\vec{\vec{x}})\delta^{a1} + g\epsilon^{abc}\vec{A}^{ib}\vec{E}^{ic} \tag{2.42}$$

1. At leading order, the source produces a Coulomb field: $\vec{E}^a(x) = g\frac{\delta^{1a}(\vec{x})}{x^2}$
2. Now considering a fluctuation of the vector potential with $a = 2$ direction at $\vec{x}_0$ away from origin and is aligned at angle to the source field as in Figure 2.2.
3. The second term in RHS in (2.42) is $g\epsilon^{abc}\vec{A}^b\vec{E}^c$. This fluctuation with $E^1$ creates a sink for the electric field $\vec{E}^3$ is created at $\vec{x}_0$ as we see in Figure 2.3.
4. Now again looking at the second term in (2.42):

   $$\nabla.\vec{E}^1 = ... + g\epsilon^{123}\vec{A}^2\vec{E}^3$$

   This $a = 2$ potential creates a source(sink) for the field in 1 direction where $\vec{A}^2$ and $\vec{E}^3$ are parallel(anti-parallel) as can also be seen in the figure. Thus, the dipole formed by this way points towards the original fields and anti-screens it.

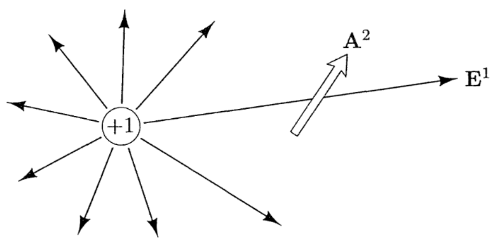



Figure 2.2: Vacuum fluctuation at a distance from source.

It has been shown in [T. Appelquist 77] that quantitatively anti-screening effect is 12 times higher than screening effect. Due to this domination of anti-screening effect the gauge fields

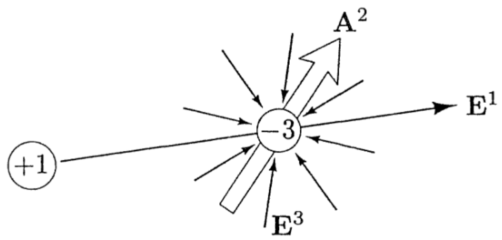



Figure 2.3: A sink of $\vec{E}^3$ is created at $\vec{x}_0$.

have opposite sign from fermions in gauge beta functions and when this anti-screening dominates the screening of fermions it causes the asymptotic free behaviour for non-abelian gauge theories.

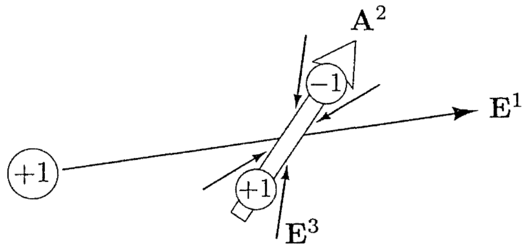



Figure 2.4: Figure showing a dipole formation due to vacuum fluctuation.

# Chapter 3

# Yukawa Coupling Beta Functions

For the derivation of general Yukawa beta functions, we have used the Lagrangian mentioned in [M. Luo 03]. The general results were derived for two-component chiral fermions and then were used to find the SM RGEs for Yukawa.

## 3.1 General Yukawa Beta Functions

The general Lagrangian with gauge fields $A_\mu^a$, scalar fields $\Phi_a$ and 2-component fermion fields $\psi_j$ given in [M. Luo 03] is

$$L = L_0 + L_1 + (gauge - fixing + ghost - terms)$$

$$L_0 = \frac{-1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + \frac{1}{2}D^\mu\Phi_a D_\mu\Phi^a + i\psi_j^\dagger\sigma_\mu D_\mu\psi_j - \frac{1}{2}(Y_{jk}^a\psi_j\zeta\psi_k\Phi_a + h.c.) - \frac{1}{4!}\lambda_{abcd}\Phi_a\Phi_b\Phi_c\Phi_d \tag{3.1}$$

where $\sigma_\mu = (1, \vec{\sigma})$, $\overline{\sigma}_\mu = (1, -\vec{\sigma})$ and $\zeta = i\sigma_2$, and it ensures the Lorentz-invariance of Yukawa term The wave function renormalization constant $Z_i$ is given as: $Z_i = 1 + \sum_{n=1}^{\infty}\delta_i^n\frac{1}{\epsilon^n}$.

The corresponding anomalous dimension is $\gamma_i = \frac{-1}{2}\sum_i \rho_i x_i \frac{\partial\delta_i^{(1)}}{\partial x_i}$ where $\rho_i = 1(2)$ for gauge and Yukawa(scalar quartic) couplings.

Beta function for Yuakwa coupling is given in terms of anomalous dimensions is given as:

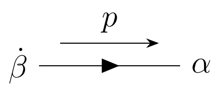
$$\beta(Y^a) = \gamma^a + Y^a\gamma^F + \gamma^{F\dagger}Y^a + \gamma^S_{ab}Y^b \tag{3.2}$$

where $\gamma^a$ are the anomalous dimensions of the operators $\psi_j\zeta\psi_k\Phi_a$, $\gamma^F$ and $\gamma^S$ for scalar and fermion fields respectively. Further, the constraint on Yukawa coupling matrices by gauge-invariance for complex fermions here is given by the equation

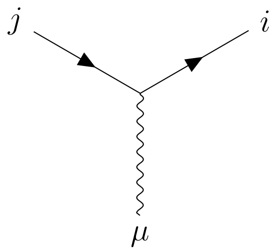
$$Y^a_{jk}\Theta^A_{ab} + Y^b_{jl}T^A_{lk} + T^{A*}_{jl}Y^b_{lk} = 0 \tag{3.3}$$

Since the fermions are considered to be complex, $T^{aT}$ is replaced by $T^{a*}$in the above equation.

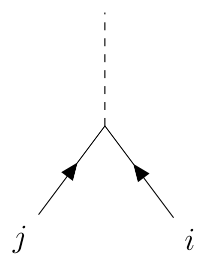
Now for calculating the anomalous dimensions the Feynman rules for the above Lagrangian were defined first. Then we found the divergences in the vertex correcting loop diagrams as did earlier. The Feynman rules for 2-component fermions are derived here using [Martin 12]. Here, $\alpha$ and $\dot{\beta}$ refers to the chiral components showing LH incoming and RH outgoing particle respectively. The rest rules remains the same as earlier.




$$= \frac{ip^\mu.\overline{\sigma}_{\mu\alpha\dot{\beta}}}{p^2} \quad or \quad -\frac{ip^\mu.\sigma^{\mu\dot{\beta}\alpha}}{p^2} \tag{3.4}$$

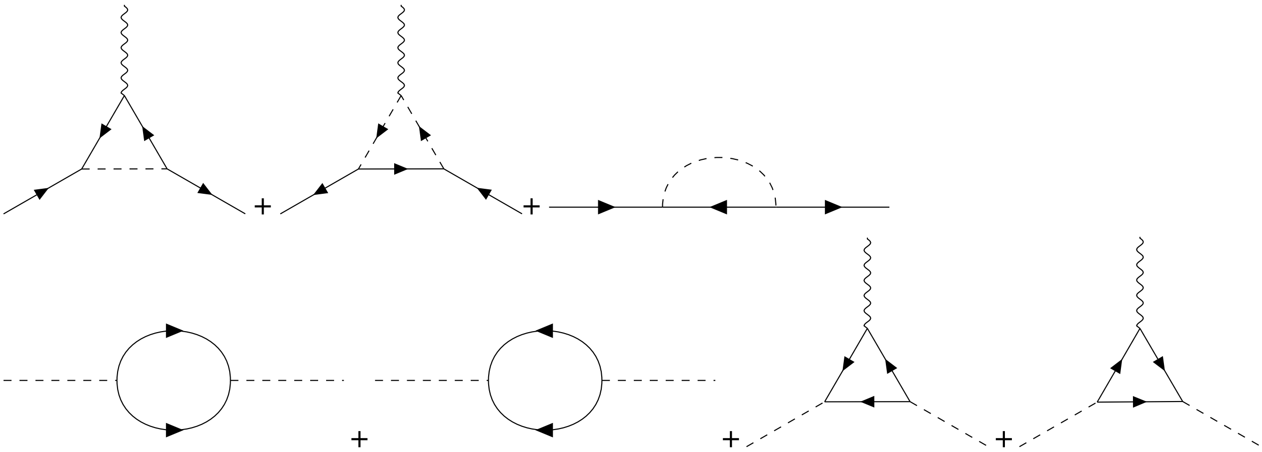



$$= ig\sigma^{\mu\dot{\beta}\alpha}T^a_{ij} \quad or \quad -ig\overline{\sigma}^{\mu\alpha\dot{\beta}}T^a_{ij} \tag{3.5}$$

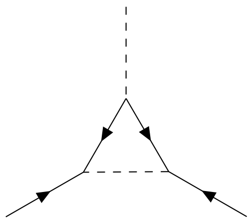



$$= -i\frac{Y_{ij}}{2}\delta^\alpha_\beta \tag{3.6}$$

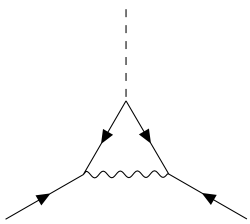
At one-loop level the Yukawa interaction has no contribution to the gauge coupling so the results for gauge-beta functions calculated in previous chapter remains the same. One can check the below diagrams cancels out each other divergences at one-loop level.



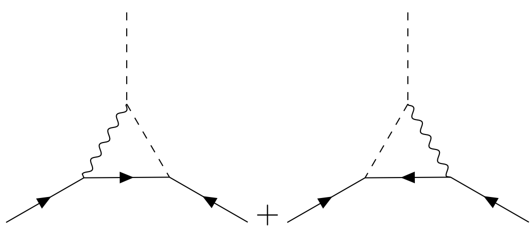
The diagrams and their contribution to Yukawa vertex calculated via d-dimensional regularization(putting d=4 later) are given as:



$$= \quad i\frac{Y^a Y^{b\dagger} Y^a}{2(4\pi)^2}\frac{1}{\epsilon} \tag{3.7}$$



$$= \quad i4g^2\frac{T^{A*}Y^b T^A}{2(4\pi)^2}\frac{1}{\epsilon} \tag{3.8}$$



$$= \quad ig^2\frac{(T^{A*}Y^b\Theta^A_{ba} + Y^b T^A\Theta^A_{ba})}{2(4\pi)^2}\frac{1}{\epsilon} \tag{3.9}$$

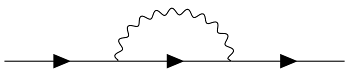
Using all these calculations, the counter-term and hence the anomalous dimension($\gamma^b$) is

$$\frac{1}{16\pi^2}[2(Y^aY^{b+}Y^a) + g^2(T^{A*}Y^bT^A + (T^{A*}Y^b\Theta_{ba} + Y^bT^A\Theta_{ba}))]$$

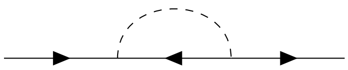
which is further simplified using (3.3) to

$$\gamma^b = \frac{1}{16\pi^2}[2(Y^aY^{b\dagger}Y^a) + 2g^2(2T^{A*}Y^bT^A - C_2(F)Y^b - Y^bC_2(F))] \tag{3.10}$$

The diagrams contributing to fermion propagator at one-loop is:



$$= -ig^2(\overline{\sigma}.k)\frac{T^AT^A}{(4\pi)^2\epsilon} \tag{3.11}$$

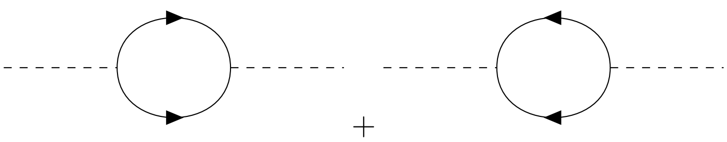


$$= -i(\overline{\sigma}.k)\frac{Y^aY^{a\dagger}}{(4\pi)^2 2\epsilon} \tag{3.12}$$

These two combined gives

$$\gamma^F = (g^2C_2(F) + \frac{Y^aY^{a\dagger}}{2})\frac{1}{16\pi^2} \tag{3.13}$$

The quartic coupling doesn't contribute to any divergence at one-loop order for scalar propagator. We have the diagrams for scalar-self energy divergences are:



$$= ik^2\frac{Tr(Y^{a\dagger}Y^b + Y^{b\dagger}Y^a)}{(4\pi)^2 2\epsilon} \tag{3.14}$$

$$= -ik^2g^2\frac{\Theta^A\Theta^A}{\epsilon}\frac{2}{(4\pi)^2} \tag{3.15}$$

And this gives,

$$\gamma^S_{ab} = (\frac{Tr(Y^{a\dagger}Y^b + Y^{b\dagger}Y^a)}{2} - 2g^2\Theta^A\Theta^A)\frac{1}{16\pi^2} \tag{3.16}$$

Putting equations (3.10), (3.13) and (3.16) in (3.2) and using
$Y^b\Theta^A_{bc}\Theta^A_{ca} = 2(T^{A*}Y^aT^A) + C_2(F)Y^a + Y^aC_2(F)$
gives the expression for general beta function i.e:

$$\beta(Y^b) = \frac{1}{16\pi^2}[(2Y^aY^{b\dagger}Y^a) - 3\{C_2(F), Y^b\} + \frac{1}{2}(Y^bY^{b\dagger}Y^b + Y^bY^bY^{b\dagger}) + Y^a\frac{Tr(Y^{a\dagger}Y^b + Y^{b\dagger}Y^a)}{2}] \tag{3.17}$$

which is the general result given in [M. Luo 03].

### 3.1.1 From General to the SM Yukawa Coupling

For the Standard Model Yukawa coupling, the top-quark coupling is the strongest. Hence, we will be calculating beta function for only this. For SM the top-quark Yukawa term can be written as:

$$L_{yuk,SM} = -y_t\overline{Q}_L\tilde{\Phi}t_R \tag{3.18}$$

where $t_R$ is the right-handed top quark and $\overline{Q}_L{}^T = \begin{bmatrix} t^*_L \\ b^*_L \end{bmatrix}$ and $\tilde{\Phi} = i\sigma_2\Phi^* = \frac{1}{\sqrt{2}}\begin{bmatrix} \Phi_3 - i\Phi_4 \\ -\Phi_1 + i\Phi_2 \end{bmatrix}$
Now defining top-Yukawa coupling $y_t$ and expanding the SM Yukawa coupling form ,

$$L_{yuk,SM} = -\frac{y_t}{\sqrt{2}}(t^*_Lt_R\Phi_3 - it^*_Lt_R\Phi_4 - b^*_Lt_R\Phi_1 + ib^*_Lt_R\Phi_2)$$

To derive from general to the SM we define $\psi = \begin{bmatrix} t^*_L \\ b^*_L \\ t_R \\ b_R \end{bmatrix}$ and for each scalar field we define a Yukawa matrix, where each matrix will be symmetric in this basis i.e.

$$Y_{\Phi_1} = -\frac{y_t}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, Y_{\Phi_2} = \frac{y_t}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, Y_{\Phi_3} = \frac{y_t}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Y_{\Phi_4} = -\frac{y_t}{\sqrt{2}} \begin{bmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now all the invariants in the general Yukawa beta function (3.17) can be calculated

- $Y^a Y^{a\dagger} = y_t^2 Diag.(1, 1, 2, 0)$
- $\frac{1}{2}Tr(Y^{a\dagger}Y^b + Y^{b\dagger}Y^a) = 3\delta^{ab}$
- For $Y_\Phi$, $\frac{1}{2}(Y^b Y^{b\dagger} Y^b + Y^b Y^b Y^{b\dagger}) = \frac{3}{2}y_t^2[Y_\Phi]$
- $Y^a Y^{b\dagger} Y^a = 0$, where a is summed over all Yukawa matrices

Also, since the SM is semi-simple , implies $g^2 C_2(R) \to \sum_k g_k^2 C_2^k(R)$ where $R$ is sum over all the gauge-groups in the theory and $g_k$ is the corresponding gauge coupling. Using this we calculated the gauge factors,

- $\mathbf{U(1)} \implies C_2(F) = \frac{1}{2}((\frac{1}{6})^2 + (\frac{2}{3})^2) \times 6 \times \frac{3}{5}g_1^2 = \frac{17}{20}g_1^2$
- $\mathbf{SU(2)} \implies$ Only L-H forms the doublet so, $C_2(F) = \frac{1}{2}(\frac{3}{4} + 0) \times 6g_2^2 = \frac{9}{4}g_2^2$
- $\mathbf{SU(3)} \implies$Both LH and RH quarks are triplets under $SU(3), C_2(F) = \frac{1}{2}(\frac{4}{3} + \frac{4}{3}) \times 6g_3^2 = 8g_3^2$

Since $Y^b = \frac{y_t}{\sqrt{2}}$ implies, $\beta(y_t) = \sqrt{2}\beta(Y)$. Putting all the above calculations in the general beta form (3.2), we get the expression for the SM top-Yukawa coupling

$$\beta(y_t) = \frac{9}{2}y_t^3 - (\frac{9}{4}g_2^2 + 8g_3^2 + \frac{17}{20}g_1^2)y_t \tag{3.19}$$

which is in agreement with the SM top-Yukawa beta function given in [Machacek 84]. One can also calculate the beta functions for $y_b$ i.e. bottom-Yukawa and $y_\tau$ tau-Yukawa in the SM by defining matrices as we did above. In the SM we have,

$$L_{yuk} = -y_b \overline{Q}_L \Phi b_R - y_\tau \bar{l}_L \Phi \tau_R$$

For contribution of $y_b$ in Top-Yukawa, there will be mixing of couplings and in the above defined invariants in (3.17) we have

- $2Y^aY^{b\dagger}Y^a = -2y_ty_b^2$
- $\frac{Y^bY^{b\dagger}Y^b + Y^bY^bY^{b\dagger}}{2} = \frac{1}{2}y_ty_b^2$
- $Y^a\frac{Tr(Y^{a\dagger} + Y^{b\dagger})}{2} = 3y_ty_b^2$

which gives $\frac{3}{2}y_ty_b^2$ for the contribution of $y_b$ in the Top-Yukawa coupling. The contribution of $y_\tau$ to $y_t$ beta function will be only through the fermionic loop in scalar propagator which gives $y_ty_\tau^2$ as the factor in beta function. The couplings $y_b$ and $y_\tau$ are very small and both remains negligible till the Planck scale as shown in Figure 3.1, and so, we will not be considering them in our calculations.

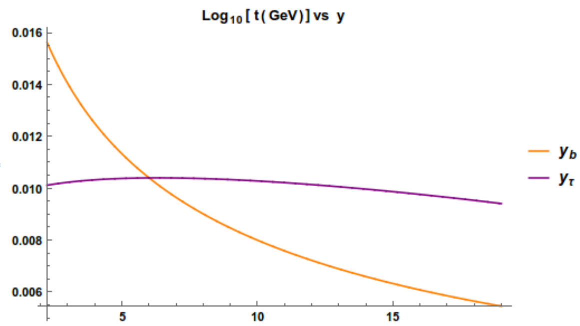



Figure 3.1: $y_b$ and $y_\tau$ couplings in the SM

The SM couplings(top-Yukawa and gauge) beta functions are plotted below with $M_t = 173.1GeV$, and Higgs mass $M_h(M_t) = 125GeV$. The vacuum expectation value of Higgs is $v = 246GeV$ which gives, $y_t(M_t) = \sqrt{2}M_t/v = 0.99497$.

One can see in the Figure 3.2 that the Yukawa coupling $y_t$ decays with the energy scale. This is due to negative contribution from the gauge couplings as can be seen by (3.19), where $g_3$ being the strongest coupling accounts more for this decay. The first plot is for one loop RGE's and the second is for two-loop. In two loop plot the gauge coupling appears more closer than one-loop plot. Well, the behaviour of the couplings with energy scale remains the same in both.

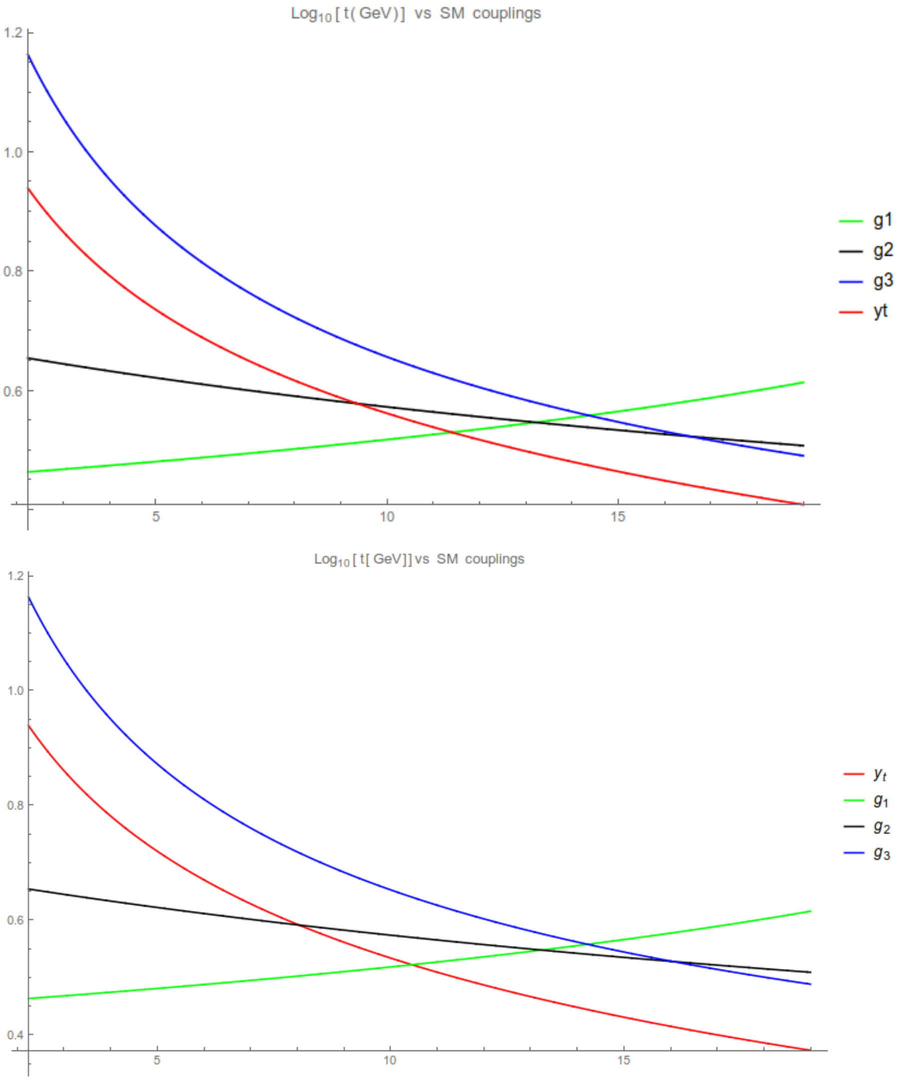



Figure 3.2: SM couplings vs $Log(t(GeV))$ for one-loop(above) and two-loop(below).

# Chapter 4

# Quartic Coupling Beta Functions

In this chapter we have derived the results for General Quartic beta functions using the same Lagrangian as previous chapter. The SM RGE for quartic coupling is derived using general results and the Yukawa matrices defined in previous chapter.

## 4.1 General Quartic Beta Functions

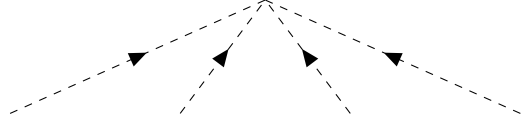
The general Lagrangian with gauge fields $A^a_\mu$, scalar fields $\Phi_a$ and 2-component fermion fields $\psi_j$ is

$$L = L_0 + L_1 + (gauge - fixing + ghostterms)$$

$$L_0 = \frac{-1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} D^\mu \Phi_a D_\mu \Phi^a + i\psi_j^\dagger \sigma_\mu D_\mu \psi_j - \frac{1}{2}(Y^a_{jk}\psi_j \zeta \psi_k \Phi_a + h.c.) - \frac{1}{4!}\lambda_{abcd}\Phi_a\Phi_b\Phi_c\Phi_d \tag{4.1}$$

where $\zeta = i\sigma_2$.

The Feynman rules mentioned earlier remains the same and for quartic coupling it is:



$$= -i\lambda_{abcd} \tag{4.2}$$

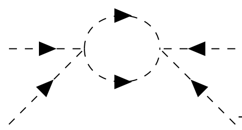
Here, the constraint on quartic coupling by gauge-invariance is given as:

$$\Theta^A_{ii'}\lambda_{i'jkl} + \Theta^A_{jj'}\lambda_{ij'kl} + \Theta^A_{kk'}\lambda_{ijk'l} + \Theta^A_{ll'}\lambda_{ijkl'} = 0 \tag{4.3}$$

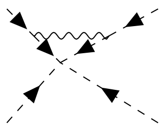
The beta function for quartic coupling is given by

$$\beta(\lambda_{abcd}) = \gamma_{abcd} + \sum_i \gamma^s(i)\lambda_{abcd} \tag{4.4}$$

where $\gamma_{abcd}$ are the anomalous dimensions of the quartic operator and $\gamma^s$ of scalars. The diagrams contributing to the correction of quartic coupling at one-loop are:



$$+\ \ 2\ \ others = i\frac{1}{2}(\lambda_{iji'j'}\lambda_{i'j'kl}+\lambda_{iki'j'}\lambda_{i'j'jl}+\lambda_{kji'j'}\lambda_{i'j'jl})\frac{1}{16\pi^2\epsilon} \tag{4.5}$$



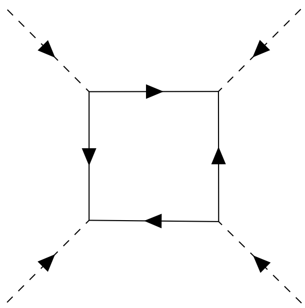
$$\begin{aligned}+\ \ 5\ \ others = i(&\Theta^A_{ii'}\Theta^A_{jj'}\lambda_{i'j'kl} + \Theta^A_{ii'}\Theta^A_{kk'}\lambda_{i'jk'l} + \Theta^A_{ii'}\Theta^A_{ll'}\lambda_{i'jkl'} + \\ &\Theta^A_{jj'}\Theta^A_{kk'}\lambda_{ij'k'l} + \Theta^A_{jj'}\Theta^A_{ll'}\lambda_{ij'kl'} + \Theta^A_{kk'}\Theta^A_{ll'}\lambda_{ijk'l'})\frac{1}{16\pi^2\epsilon}\end{aligned} \tag{4.6}$$

The terms in bracket can be simplified using eq. 4.3 as:

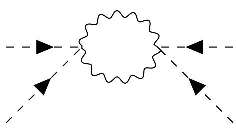
$$\begin{aligned}
&= \frac{1}{2}(\Theta^A_{ii'}\Theta^A_{jj'}\lambda_{i'j'kl} + \Theta^A_{ii'}\Theta^A_{kk'}\lambda_{i'jk'l} + \Theta^A_{ii'}\Theta^A_{ll'}\lambda_{i'jkl'} + \Theta^A_{jj'}\Theta^A_{kk'}\lambda_{ij'k'l} + \Theta^A_{jj'}\Theta^A_{ll'}\lambda_{ij'kl'} + \\
&\quad \Theta^A_{jj'}\Theta^A_{kk'}\lambda_{ij'k'l} + \Theta^A_{ll'}\Theta^A_{ii'}\lambda_{i'jkl'} + \Theta^A_{ll'}\Theta^A_{jj'}\lambda_{ij'kl'} + \Theta^A_{ll'}\Theta^A_{kk'}\lambda_{ijk'l'} + \Theta^A_{kk'}\Theta^A_{ll'}\lambda_{ijk'l'} + \\
&\quad \Theta^A_{kk'}\Theta^A_{ii'}\lambda_{i'jk'l} + \Theta^A_{kk'}\Theta^A_{jj'}\lambda_{ij'k'l}) \\
&= \frac{1}{2}(\Theta^A_{ii'}(\Theta^A_{jj'}\lambda_{i'j'kl} + \Theta^A_{kk'}\lambda_{i'jk'l} + \Theta^A_{ll'}\lambda_{i'jkl'}) + \Theta^A_{jj'}(\Theta^A_{kk'}\lambda_{ij'k'l} + \Theta^A_{ll'}\lambda_{ij'kl'} + \Theta^A_{kk'}\lambda_{ij'k'l}) + \\
&\quad \Theta^A_{ll'}(\Theta^A_{kk'}\lambda_{ij'k'l} + \Theta^A_{jj'}\lambda_{ij'kl'} + \Theta^A_{kk'}\lambda_{ijk'l'}) + \Theta^A_{kk'}(\Theta^A_{ll'}\lambda_{ijk'l'} + \Theta^A_{ii'}\lambda_{i'jk'l} + \Theta^A_{jj'}\lambda_{ij'k'l}) \\
&= -\frac{1}{2}(\Theta^A_{ii'}\Theta^A_{i'i''}\lambda_{i''jkl} + \Theta^A_{jj'}\Theta^A_{j'j''}\lambda_{ij''kl} + \Theta^A_{kk'}\Theta^A_{k'k''}\lambda_{ijk''l} + \Theta^A_{ll'}\Theta^A_{l'l''}\lambda_{ijkl'}) \\
&= -\frac{1}{2}4C_2(S)\lambda_{ijkl} = -2C_2(S)\lambda_{ijkl}
\end{aligned} \tag{4.7}$$

and this gives the final expression for the diagrams which is:

$$= -i\frac{2}{16\pi^2\epsilon}g^2C_2(S)\lambda_{ijkl} \tag{4.8}$$

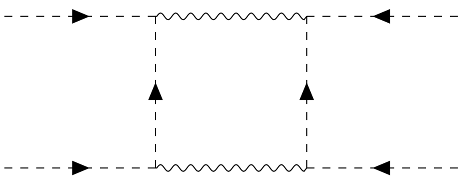


$$+ \quad 5 \quad others = -i\frac{2}{\epsilon 16\pi^2}\Big(\sum_{permut} Tr(Y^{a\dagger}Y^bY^{c\dagger}Y^d)\Big) \tag{4.9}$$

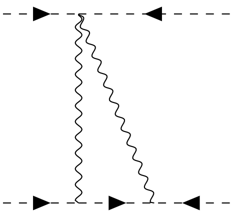


+ 2 others

$$\begin{aligned} &= i\frac{2}{16\pi^2\epsilon}(\{\Theta^A,\Theta^B\}_{ij}\{\Theta^A,\Theta^B\}_{kl}+ \\ \{\Theta^A,\Theta^B\}_{ik}\{\Theta^A,\Theta^B\}_{jl} &+ (\{\Theta^A,\Theta^B\}_{il}\{\Theta^A,\Theta^B\}_{jk})g^4 \\ &= i\frac{2}{16\pi^2\epsilon}g^4A_{ijkl} \end{aligned} \tag{4.10}$$



$$+ \quad 5 \quad others = \frac{i}{2}\frac{1}{16\pi^2\epsilon}g^4A_{ijkl} \tag{4.11}$$



$$+ \quad 5 \quad others = -i\frac{1}{16\pi^2\epsilon}g^4A_{ijkl} \tag{4.12}$$

Now using the results we can determine the anomalous dimension of quartic term which gives:

$$16\pi^2\gamma_{abcd} = \frac{1}{8}\sum_{permut}\lambda_{abef}\lambda_{efcd} - 4g^2C_2(S)\lambda_{abcd}+$$
$$3g^4\frac{1}{8}\sum_{permut}\{\Theta^A,\Theta^B\}_{ab}\{\Theta^A,\Theta^B\}cd - 4Tr(Y^{a\dagger}Y^bY^{c\dagger}Y^d) \tag{4.13}$$

The anomalous dimension for a scalar propagator is(as calculated in previous chapter):

$$16\pi^2\gamma^S(i) = Tr(\frac{Y^aY^{b\dagger}+Y^{a\dagger}Y^b}{2}) - 2g^2\Theta^A\Theta^A = Y_2(i) - 2g^2C_2(S) \tag{4.14}$$

Now putting the above equations in (4.4) gives the final expression for general beta functions for quartic coupling i.e.

$$\beta_{\lambda_{abcd}} = \Lambda^2_{abcd} - 4H_{abcd} + \Lambda^Y_{abcd} - 3g^2\Lambda^S_{abcd} \tag{4.15}$$

where the newly introduced expressions above are:

$$\Lambda^2_{abcd} = \frac{1}{8}\sum_{permut}\lambda_{abef}\lambda_{efcd}$$
$$H_{abcd} = \frac{1}{4}\sum_{permut}Tr(Y^{a\dagger}Y^bY^{c\dagger}Y^d)$$
$$\Lambda^Y_{abcd} = \sum_i Y_2(i)\lambda_{abcd}$$
$$\Lambda^S_{abcd} = \sum_i C_2(i)\lambda_{abcd}$$
$$A_{abcd} = \frac{1}{8}\sum_{permut}\{\Theta^A,\Theta^B\}_{ab}\{\Theta^A,\Theta^B\}_{cd}$$

### 4.1.1 From General to the SM Couplings

In SM the Higgs coupling looks like,

$$L_{Higgs} = -\lambda(H^\dagger H)^2 = -\lambda\frac{(\phi_1^2+\phi_2^2+\phi_3^2+\phi_4^2)}{4}$$

whereas in general theory it was

$$L_{\phi^4} = -\lambda'_{abcd}\phi_a\phi_b\phi_c\phi_d$$

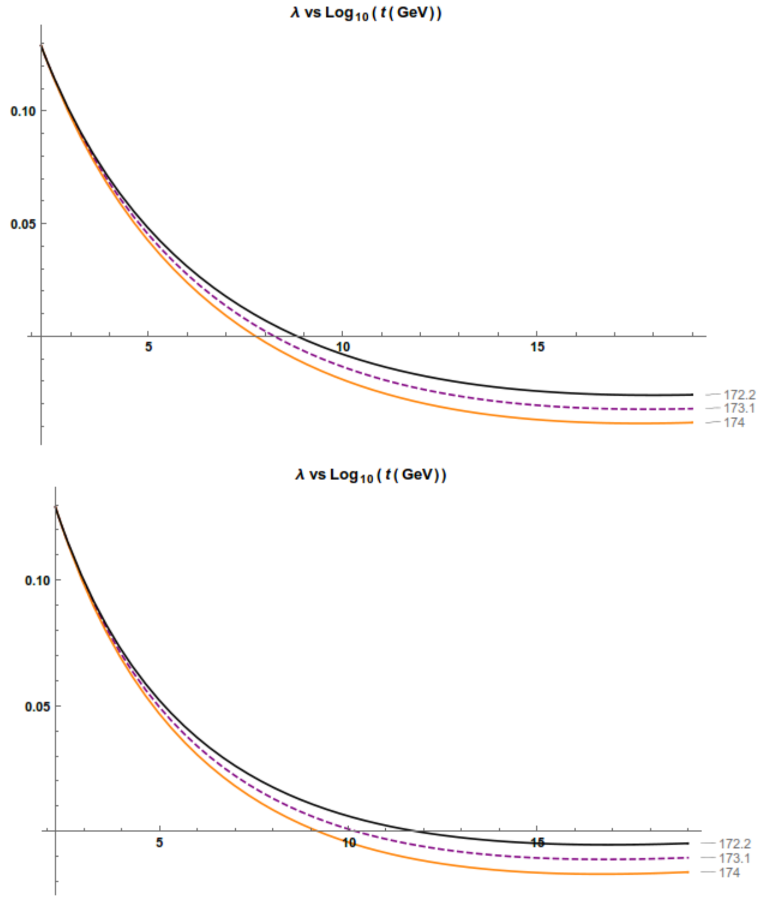



Figure 4.1: $\lambda$ vs $Log(t(GeV))$ for One-loop(above) and Two-Loop(below) for 3 different masses of Higgs.

This implies $\frac{\lambda'}{4!} \rightarrow \frac{\lambda}{4} \implies \frac{\beta(\lambda')}{6} = \beta(\lambda)$. Also since SM is semi-simple, $g^4 A_{abcd} = \sum_{n,m} g_n^2 g_m^2 \tilde{A}^{mn}_{abcd}$ where $m$ and $n$ are the gauge-group indices. Using Yukawa matrices defined in Chapter 3 and substituting $\lambda$ as above one can find all the invariants defined above. For corrections to quartic operator (4.5), now we have charged fields in the SM so one needs to count for 2 permutations of inner legs, and also the $1/2$ factor taken for real scalar has to be ignored, which gives contribution after doing substitution in $\beta(\lambda)$ as $24\lambda^2$.

The final result for the SM quartic coupling beta function is:

$$(4\pi)^2\beta(\lambda) = 24\lambda^2 + 12\lambda y_t^2 - 6y_t^4 - \lambda(9g_2^2 + \frac{9}{5}g_1^2) + \frac{9}{8}g_2^2 + \frac{27}{200}g_1^4 + \frac{9}{20}g_1^2 g_2^2 \quad (4.16)$$

which takes the same form as given in [Machacek 85] after making transformation as $\lambda = \lambda'/2$ implies $\beta(\lambda) = \beta(\lambda')/2$ and gives 12 as the coefficient for $\lambda^2$ and vice-versa for other terms. Figure 4.1 shows the RGE of quartic coupling at one-loop(above) and two-loop(below) level plotted for 3 different masses of Higgs. The value of $\lambda$ at $M_t$ is given as

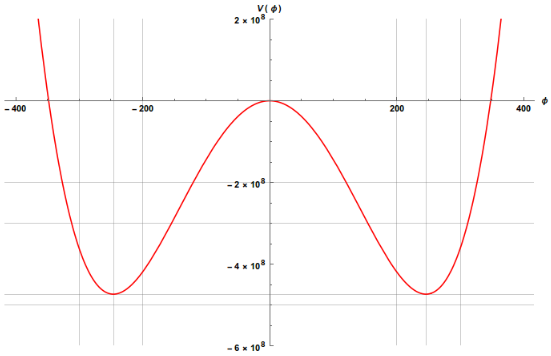



Figure 4.2: Higgs potential at 173.1 GeV.

$\lambda = \frac{M_h^2}{2v^2}$.

It can be observed from the plots that the $\lambda$ becomes negative after a certain scale. The scale is pushed forward for two-loop calculations and for lighter Higgs mass also. For one-loop(left) the scale of vacuum stability is at $\approx 10^8$ GeV whereas for two-loop(right) it is shifted to $\approx 10^{10}$ GeV.

The Higgs potential is given as :

$$V = m^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

where $m^2 < 0$ and the vacuum expectation value is $v = \sqrt{-m^2/\lambda}$.

The Higgs potential at $M_t$ where $\lambda > 0$ is plotted in the figure 4.2. One can see that the potential at $M_t$ is bounded and has a minimum of $\approx 246$ GeV. At higher field values this potential behaves as $V \approx \frac{\lambda}{4}\phi^4$. The coupling $\lambda$ going to negative causes a problem with the vacuum stability of the Higgs field. At higher scales where $\lambda < 0$ before the Planck scale, Higgs can take high expectation values making the potential $< 0$ and hence unbounded from below. This makes the present Higgs vacuum metastable. For Higgs to be stable at every scale till $M_P$, it gives an indication of the Physics beyond the SM. Some new phenomena or particles are needed to stabilize Higgs potential through their interactions. This is the motivation for our further work where we have extended the SM by CW fermions as described in the next chapters.

# Chapter 5

# Applications: Study of RGE in the Context of Fermionic Clockwork Theory

This chapter explains the clockwork mechanism and its implications to the fundamental theory. We start by writing the Lagrangian for CW taken from [Patel 17] and then explain how it affects the couplings in the effective theory. Later, we have extended the Standard Model by adding Clockwork fermions taking different cases. The effects of this extension on all the SM couplings is analyzed.

## 5.1 Clockwork Theory

An underlying theory in Clockwork(CW) phenomena includes multiple fields, namely the clockwork gears, and a potential containing interactions between only the adjacent CW gears. When an interaction is introduced between an external sector and the field at one of the end sites of the CW chain, the arrangement generates interaction between the external sector and the field at the other end with exponentially enhanced or suppressed coupling. This mechanism has been studied for various phenomenological applications to Higgs sector, dark matter, neutrino masses, inter-generational mass hierarchy and other sectors.
So as to write the Lagrangian of this theory we consider N+1 2-component left-handed fermionic fields $f^c_a(a = 0, 1, 2..., N)$ and N other such right-handed fields $f_b(b = 1, 2, ..., N)$ with same Quantum numbers for gauge groups but opposite hypercharge from $f^c$ fields.

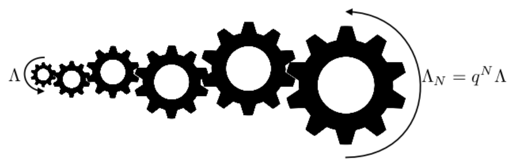



Figure 5.1: A representation of Clockwork mechanism resulting in enhanced or suppressed coupling.[Giudice 17]

Also considering a global symmetry which is a product of several $U(1)$ factors: $G = \Pi_{a,b} U(1)_{L,a} \times U(1)_{R,b}$. Under the $G$, the fields $f_a$ and $f_b^c$ have charges $(1,0)$ and $(0,1)$, respectively. The symmetry $G$ is then broken by N mass terms giving rise to N massive Dirac fermions and leaving one linear combination of $f_b^c$ as a massless fermion. The Lagrangian for the CW is given as

$$L_{CW} = i\overline{f_0^c}\gamma^\mu \partial_\mu f_0^c + \sum_{a=1}^{N} (i\overline{f_a}\gamma^\mu \partial_\mu f_a + i\overline{f_a^c}\gamma^\mu \partial_\mu f_a^c - M f_a f_a^c + m f_a f_{a-1}^c + h.c.) \quad (5.1)$$

where m and M are the mass parameters. Using Euler-Lagrange equation one can integrate out the fields at $b^{th}$ site for small momentum to find out that,

$$f_b^c = \frac{m}{M} f_{b-1}^c = q f_{b-1}^c \quad (5.2)$$

The above equation implies that the field at $b^{th}$ site is suppressed by a factor q from its precedent field. One can write the effective Lagrangian as

$$L_{eff} = i\overline{f_0^c} z \gamma^\mu \partial_\mu f_0^c = i\overline{f^c} z \gamma^\mu \partial_\mu f^c$$

where $z = 1 + q^2 + q^4 + ... + q^{2N} = \frac{1-q^{2(N+1)}}{1-q^2}$ and $\sqrt{z} q^{-N} f_c^N = \sqrt{z} f_0^c = f^c$

If the Yukawa interaction between $f_b^c$ and an another field $\psi$ is introduced at the $N_{th}$ site in the fundamental theory with a coupling of natural size then

$$L_Y = -y\psi\phi f_N^c = -yq^N \psi\phi f_0^c = -yq^N \sqrt{z^{-1}} \psi\phi f^c \quad (5.3)$$

This implies that the Yukawa coupling in effective theory is suppressed by $q^N$.

### 5.1.1 Flavored Clockworks

The CW theory can be applied simultaneously to different generations, called as Flavored CWs where the CW gears of different generations are allowed to interact with each other, maintaining the nearest neighbour interaction structure of the CW mechanism. Say for $n_f$ generations of a given type of fermion, the FCW gears consist of the fields: $f_a^{(i)}(a = 1, 2, ..., N; i = 1, 2, ..., n_f)$ and $f_b^{c(i)}(b = 0, 1, 2, ..., N; i = 1, 2, ..., n_f)$. The Lagrangian in equation will now be modified as :

$$L = \sum_{i,j=1}^{n_f}(i\overline{f}_0^{c(i)}\gamma^\mu\partial_\mu f_0^{c(i)}+\sum_{a=1}^{N}(i\overline{f}_a^{c(i)}\gamma^\mu\partial_\mu f_a^{c(i)}+i\overline{f}_a^{(i)}\gamma^\mu\partial_\mu f_a^{(i)}-(M_{ij}f_a^{(i)}f_a^{c(j)}-m_{ij}f_a^{(i)}f_{a-1}^{c(j)}+h.c.)) \tag{5.4}$$

where $M$ and $m$ are now $n_f \times n_f$ matrices in generation space. Integrating out $f_b^{c(i)}$ and $f_b^{(i)}$ as earlier gives us

$$f_b^{(i)} = 0, f_b^{c(i)} = \sum_{k=1}^{n_f}(mM^{-1})_{ik}f_{b-1}^{c(k)} = \sum_{k=1}^{n_f}(Q)_{ik}f_{b-1}^{c(k)}$$

The effective here Lagrangian here takes the form

$$L_{eff} = i\overline{f_0^c}Z\gamma^\mu\partial_\mu f_0^c$$

where $f_0^c = (f_0^{c(1)}, f_0^{c(2)}, ...,, f_0^{c(n_f)})^T$ for $n_f$ generations and
$Z = 1 + Q^\dagger Q + Q^{\dagger 2}Q^2 + ... + +Q^{\dagger N}Q^N$. The matrices $Z$ and $Q$ can be diagonalised as

$$\begin{aligned} V_Q Q U_Q^\dagger &= Diag(q_1, q_2, ..., q_{n_f}) = \tilde{Q} \\ U_Q Z U_Q^\dagger &= Diag(z_1, z_2, ..., z_{n_f}) = \tilde{Z} \end{aligned} \tag{5.5}$$

The Yukawa interaction defined at $N_{th}$ site here is

$$L_Y = -\psi Y\phi f_N^c = -\psi(YV_Q^\dagger\tilde{Q}^N\sqrt{\tilde{Z}^{-1}})\phi f^c$$

where$\psi = (\psi_{(1)}, \psi_{(2)}, .., \psi_{(n_f)})^T$ and $f^c = \sqrt{\tilde{Z}U_Q f_0^c}$. Here too it can be seen that there is a suppression to Yukawa coupling matrix $Y$ from the diagonal matrix $\tilde{Q}^N\sqrt{\tilde{Z}^{-1}}$.

In our work we have extended the SM with clockwork fermions as done in [Patel 17]. It has been shown in [Patel 17] through the simulations that the hierarchies in the masses and mixing of different SM fermions generated by $N = 4$ for $f = q, u^c, e^c$and $N = 2$ for $f = l, d^c$ produces the same Froggatt-Nielsen(FN) charge as produced by FN mechanism and so are in good agreement with the current observations. In our work we have used the same number of gears for the generations and analyzed the RGEs for SM correspondingly.

## 5.2 Extending The Standard Model via Clockwork Fermions

In our extension of the SM, we assume that only the $N_{th}$ gear interact with the SM LH doublets through Yukawa interaction, the SM Yukawa terms is $q^N$ order suppressed i.e. $y_{u,d,l} = yq^N$ where $y$ is the Yukawa coupling of $\mathcal{O}(1)$ in the fundamental theory. The Lagrangian of the modified theory can be written for one generation as:

$$\begin{aligned} L_{SM+CW} = -\frac{1}{4}(F^{a\mu\nu})^2 + D_\mu\Phi(D^\mu\phi)^\dagger + i\overline{f_0^c}\gamma^\mu D_\mu f_0^c + \\ \sum_{a=1}^{N}(i\overline{f_a}\gamma^\mu D_\mu f_a + i\overline{f_a^c}\gamma^\mu D_\mu f_a^c - M f_a f_a^c + m f_a f_{a-1}^c + h.c.) \\ +(yq_d^{N_{d^c}} q_0^c \Phi d_0^c + yq_u^{N_{u^c}} q_0^c \tilde{\Phi} u_0^c + yq_e^{N_{e^c}} l_0^c \Phi e_0^c + h.c.) + \lambda(\Phi\Phi^\dagger)^2 \end{aligned} \tag{5.6}$$

where $N$ is added over the number of gears for each flavor as mentioned in Table 5.1, $f_0^c$ is the SM fermion, $N_{d^c}, N_{u^c}$ and $N_{e^c}$ are the number of gears for corresponding flavor shown by the subscript. The Yukawa couplings in the SM, suppressed from the fundamental couplings, are given as $y_f = yq_f^{N_f}$. Among the SM Yukawa couplings, the top-Yukawa coupling is $\mathcal{O}(1)$, and all others Yukawa couplings are less than 1, so for third generation there are no CW Gears for $q$, $u^c$ and $e^c$ because if they are, then $y_t$ is not expected to have such a higher value.

The table below shows the number and the gauge quantum factors for the CW fermions added to each generation:

| CW fermions | I | II | III |
|---|---|---|---|
| q(4) | (3,2,-1/3) | (3,2,-1/3) | |
| q$^c(4)$ | $(\overline{3}, 2, 1/3)$ | $(\overline{3}, 2, 1/3)$ | |
| u(4) | (3,1,-4/3) | (3,1,-4/3) | |

| | | | |
|---|---|---|---|
| u$^c$(4) | $(\overline{3}, 1, 4/3)$ | $(\overline{3}, 1, 4/3)$ | |
| e(4) | (1,1,2) | (1,1,2) | |
| e$^c$(4) | (1,1,-2) | (1,1,-2) | |
| l(2) | (1,2,1) | (1,2,1) | (1,2,1) |
| l$^c$(2) | (1,2,-1) | (1,2,-1) | (1,2,-1) |
| d(2) | (3,1,2/3) | (3,1,2/3) | (3,1,2/3) |
| d$^c$(2) | $(\overline{3}, 1, -2/3)$ | $(\overline{3}, 1, -2/3)$ | $(\overline{3}, 1, -2/3)$ |

Table 5.1: Table shows the gauge quantum factors and the number of CW gears(shown in bracket) added to the SM in each generation.

From the gauge beta functions in Chapter 1, one can see that each new particle added to SM contributes more positively to the $g_1$ coupling beta functions. The asymptotic behaviour of $g_2$ and $g_3$ is dependent on the number of doublets and triplets in the theory. One can calculate that in extending the SM with new particles, say for adding doublets$(N_d) > 9$, and triplets$(N_t) > 20$ the asymptotic freedom of $g_2$ and $g_3$ is broken respectively. We have extended the SM by taking three cases as:

- First we have added the CW gears at 1 TeV and have analyzed how the gauge beta functions behave. In the picture below the corresponding gauge coupling beta functions are plotted. The Landau pole for $g_3$ can be derived using $g_3$ beta function in Chapter 2, which gives:

$$\Lambda = exp(-\frac{2\pi}{\alpha_3(M)(7 - \frac{N_t}{3})})M \tag{5.7}$$

where $M$ is the scale at which the CW gears are introduced. From (5.9) one can calculate that the $\Lambda(g_3)$ occurs at $2.28 \times 10^5$ GeV which can also be seen in the Figure 5.2. Since gauge couplings are real and $\alpha^2 < 0$ implies them to be imaginary, so the perturbation theory becomes non-valid after $\Lambda(g_3)$.

- Next, we tried to find the minimum scale at which the CW gears can be added to avoid the Landau poles in the theory till Planck Scale. Figure 5.3 shows that at a scale of $2.8 \times 10^{14}$ GeV, the first Landau pole occurs in $g_1$ at $1.0024 \times 10^{19}$ GeV. This gives the maximum limit at which all the CW gears can be added such that the perturbation theory remains valid till $M_P$.

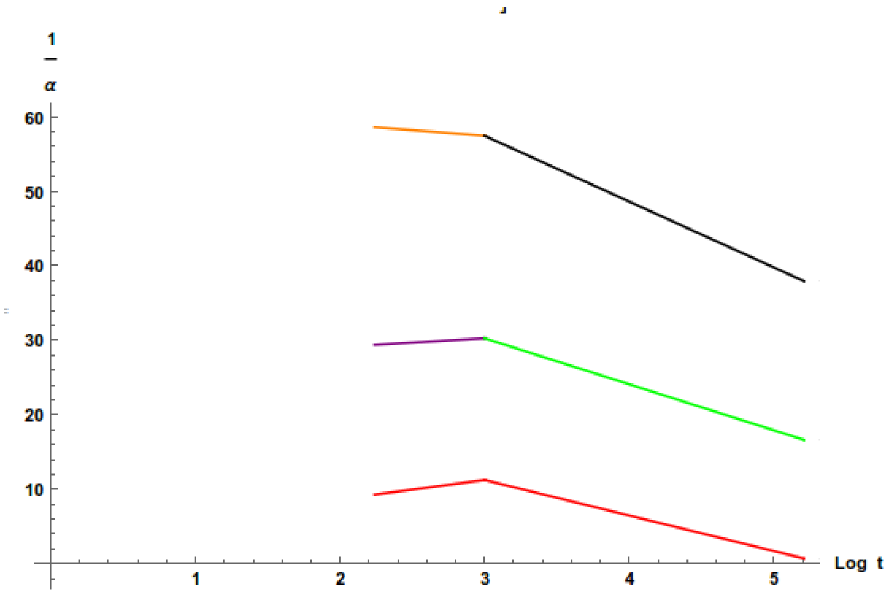



Figure 5.2: Gauge beta functions for $M = 10^3$ GeV.(Color code:Black-$g_1$, Green-$g_2$ and Red - $g_3$)

- Then, we tried to split the scales till Planck scale keeping two motives in mind. One, the theory should remain perturbative upto $M_P$ and second was to unify the gauge couplings. The total number of gears for $f, u^c$ and $e^c$ are 16 and for $l$ and $d^c$ are 12. For this we did the following splittings and extensions of CW gears at energy scales as,

| scale(GeV) | q | u$^c$ | e$^c$ | l | d$^c$ |
|---|---|---|---|---|---|
| $4 \times 10^4$ | 6 | 0 | 4 | 2 | 4 |
| $10^{10}$ | 0 | 6 | 0 | 2 | 2 |
| $4 \times 10^{14}$ | 0 | 2 | 0 | 0 | 2 |
| $3 \times 10^{18}$ | 10 | 8 | 12 | 8 | 4 |

Table 5.2: CW fermions added at different scales. The number shown is for the pair added i.e. $(f, f^c)$ .

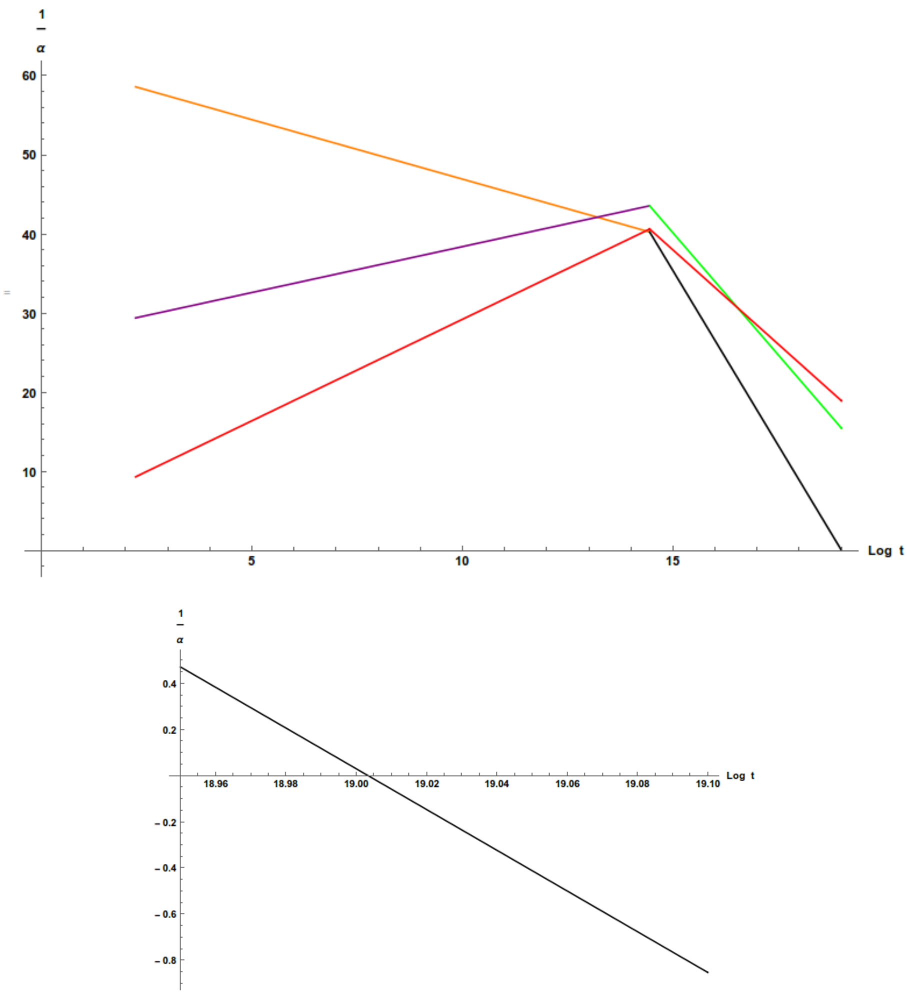



Figure 5.3: Gauge beta functions for $M = 2.8 \times 10^{14}$ GeV. The plot below shows the Landau pole for $g_1$.(Color code:Black-$g_1$, Green-$g_2$ and Red - $g_3$)

The gauge couplings changes with the above CW extension as:

$$\beta_{g_1} = \frac{41}{10} g_1^3 \rightarrow \frac{211}{30} g_1^3 \rightarrow \frac{109}{10} g_1^3 \rightarrow \frac{367}{30} g_1^3 \rightarrow \frac{241}{10} g_1^3 \tag{5.8}$$

$$\beta_{g_2} = -\frac{19}{6} g_2^3 \rightarrow \frac{7}{2} g_2^3 \rightarrow \frac{25}{6} g_2^3 \rightarrow \frac{25}{6} g_2^3 \rightarrow \frac{101}{6} g_2^3 \tag{5.9}$$

$$\beta_{g_3} = -7 g_3^3 \rightarrow -\frac{5}{3} g_3^3 \rightarrow g_3^3 \rightarrow \frac{7}{3} g_3^3 \rightarrow 13 g_3^3 \tag{5.10}$$

The sign of beta functions for $g_2$ and $g_3$ depicts the scale at which they started behaving non-asymptotic free. The $U(1)_Y$ gauge factor $\sum (Y/2)^2$ for this new theory is $\frac{241}{10}$, and $\alpha(3 \times 10^{18})^{-1} = 5.3611$ which gives the first landau pole $\Lambda(g_1)$ after $M_P$ at $1.213 \times 10^{19}$ for $g_1$ gauge coupling and this can also be seen from the Figure 5.4. Also, the gauge couplings appears to unify at a scale of $\approx 10^{16}$ GeV and it lies within the $3\sigma$ error range.

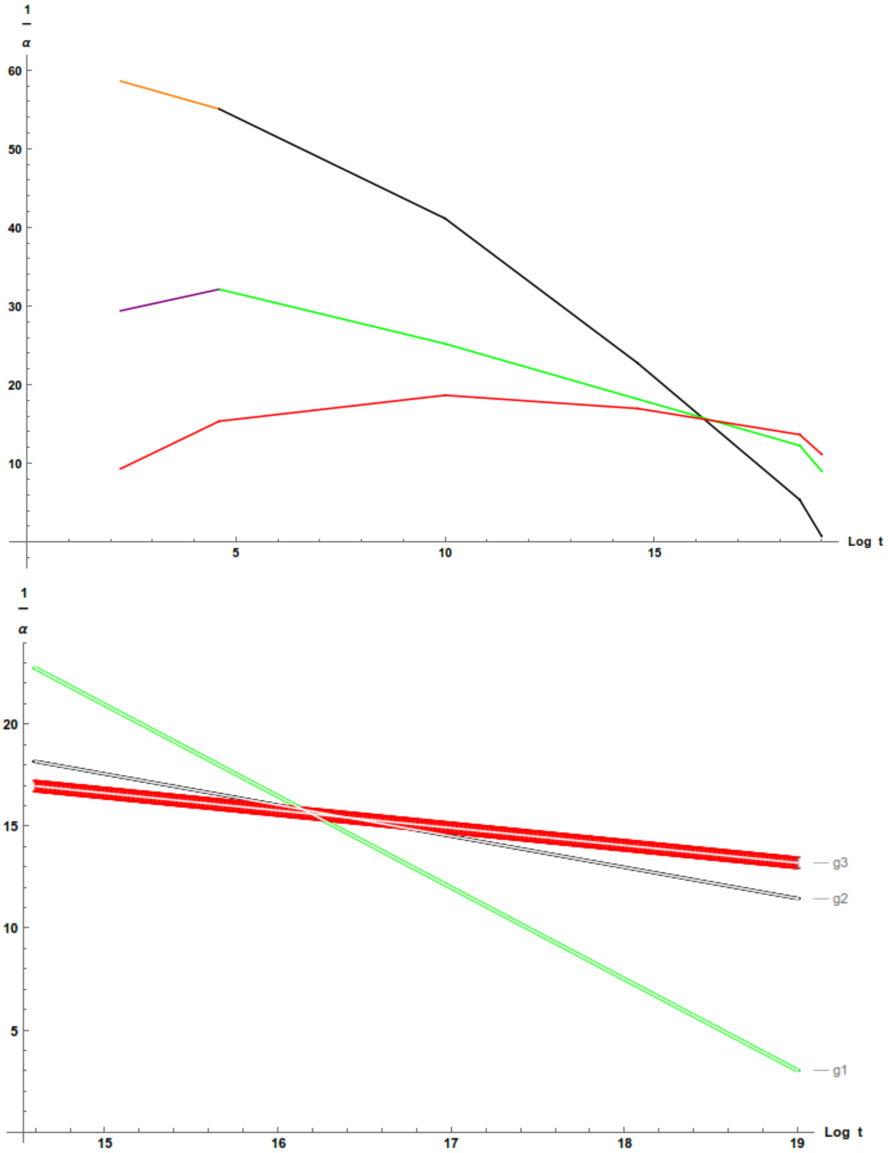



Figure 5.4: Gauge couplings for splitting case. The plot below shows the $3\sigma$ error bars i.e. for $\Delta{\alpha_1}^{-1} = 0.00912, \Delta{\alpha_2}^{-1} = 0.01524, \Delta{\alpha_3}^{-1} = 0.25954$

We analyzed the affects of the new gauge-beta functions on all other SM couplings also. We have only the $N_{th}$ fermion interacts with the SM doublet to give Yukawa coupling, which can be reduced to give the effective SM Yukawa i.e. suppressed by order($q^N$) from the fundamental couplings. The plots further will be showing how the new CW gears added as shown in Table 5.2 affects the SM Yukawa, gauge and Quartic couplings with energy scale. The orange colored line is for the SM case and the colored ones correspond to the SM couplings after extending the SM at different scales.
The Figure 5.5 shows how the CW affects the $g_1$ beta functions changes. As expected the

new fermions add more contribution to the $g_1$ coupling increasing it further more than the SM case as can be seen in eq. (5.8).

For $g_2$ beta function as can be seen in the Figure 5.5 and also from eq. (5.9), the asymptotic freedom is broken as the number of doublets added exceeds 9 in our theory. This makes the $g_2$ coupling to start increasing unlike the SM case.

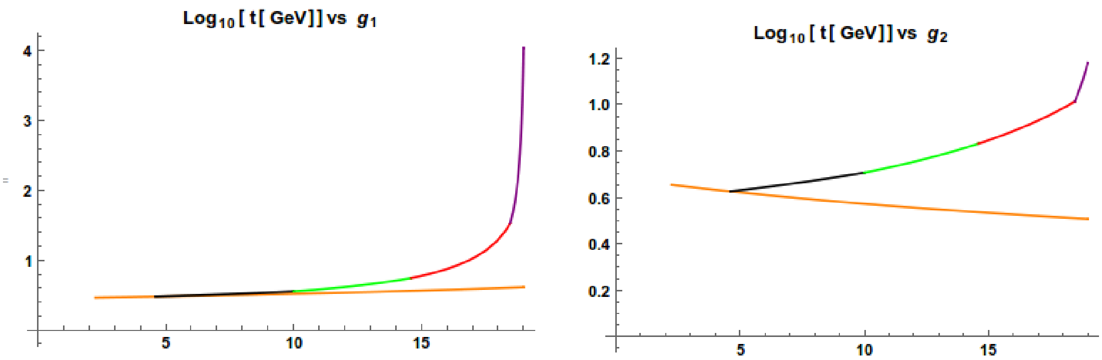



Figure 5.5: $g_1$(left) and $g_2$(right) coupling with with CW extension of the SM plotted vs logarithmic energy scale.

Similarly here for $g_3$ beta function, the asymptotic freedom is broken as the number of triplets exceeds 20 as shown in eq. (5.10). The decay of $g_3$ is gradual as compared to SM case at earlier scale. The coupling $g_3 \approx \mathcal{O}(1)$ is a strong coupling in the SM and one can see from Figure 5.6 that it remains at higher values till large scale. Later it starts rising up as the asymptotic freedom breaks.

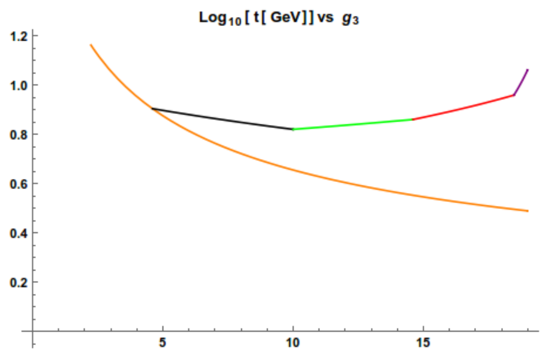



Figure 5.6: $g_3$ coupling with CW extension of the SM.

Figure 5.7 shows the top-Yukawa coupling in SM and after adding CW fermions. Since all the gauge couplings have a negative effect on the beta function of top-Yukawa as can be seen from SM beta functions in Chapter 2. And as the above plots depicts that the gauge couplings are now at higher values than the SM so leading to faster decay of top-Yukawa coupling.

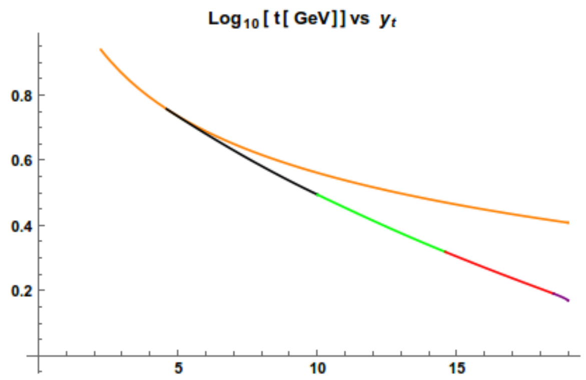



Figure 5.7: $y_t$ coupling with CW extension of the SM.

The top-Yukawa and gauge couplings affects the quartic coupling behaviour along the energy scales. In the SM case, at one-loop level, the negative contribution from the top-Yukawa loops dominates which leads to the decay of the quartic coupling and hence creates the problem of vacuum stability as seen in Chapter 4. The top-Yukawa dominates dominates at lower scales and leads to decay of the quartic coupling, but sooner the new fermions start contributing to the gauge couplings positively as we saw above and leads to faster decay in top-Yukawa and this slows down the declination of quartic coupling. The $g_1$ and $g_2$ couplings also starts rising faster and their contribution leads to a direct positive rise in the quartic coupling.

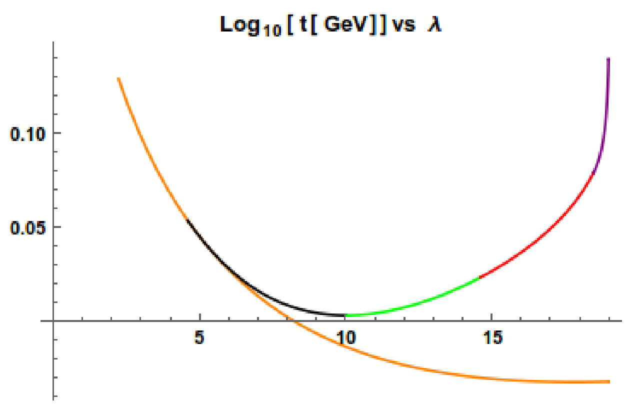



Figure 5.8: $\lambda$ coupling with CW extension of the SM.

# Chapter 6

# Summary and Conclusion

In our work, we first derived the general Renormalization Group Equations for gauge, Yukawa and quartic coupling. The general results were then used to derive the SM beta functions for the corresponding coupling in Chapter 2, 3 and 4 respectively for gauge, Yukawa and quartic coupling. The quartic coupling in the SM goes to negative before the Planck scale which creates the issue of the instability of the Higgs vacuum at higher field values. This implies that the Higgs boson is trapped in a false vacuum and thus indicates Physics beyond the SM to stabilize Higgs vacuum. This was our motivation, and we worked on extending the SM via clockwork fermions as done in [Patel 17].

We observed that adding CW fermions to theory can break the asymptotic freedom of the $g_3$ and $g_2$ gauge couplings for added triplets, $N_t > 20$ and doublets, $(N_d) > 9$ respectively which leads to major effects on the behavior of Yukawa and quartic coupling. Fermionic contribution can also lead to produce Landau poles before $M_P$ making the perturbation theory valid afterward. We found that for CW fermions added at splitting scales, the Landau poles can be avoided. Also, we have shown that one can unify the gauge couplings in $3-\sigma$ error-bar and along with this can get rid of the vacuum stability issue also by start adding CW fermions at lower scales $\approx 10^4$ GeV.

Our work on CW extension of the SM is one application of using the general RGEs to solve for Beyond the SM case. One can work on any other model of BSM, and the general results can be used conveniently for that. We did the calculations for the one-loop case, extending for two or higher loop calculations can give more insight into the behavior of SM couplings for BSM.

There are many other remarkable problems like Higgs Mass anomaly, Dark Matter problem, etc., that the SM fails to elucidate and motivates contemporary physicists to explore beyond the SM theories. What model describes the Universe as a whole, i.e., unify all the fundamental interactions, is still an unplumbed sector that needs closer attention. There is a need to garner substantial information about the Universe to provide answers to deep-rooted problems such as, what caused the Big Bang? Source of Dark Energy? What is Dark Matter? etc., that still appears aloof and under continuous scrutiny from present-day physicists.

# Appendix A

## A.1 Feynman Parameters

To combine the propagator denominators, we used Feynman parameters as:

$$\frac{1}{A_1A_2...A_n} = \int_0^1 dx_1dx_2...dx_n\delta(\sum x_i - 1)\frac{(n-1)!}{[x_1A_1 + x_2A_2 + ...]^n} \tag{A.1}$$

For one-loop calculations we dealt with only with two propagators in the denominator i.e.
,

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{[xA + (1-x)B]^2} \tag{A.2}$$

Since A and B in above equations are quadratic in momenta $p^\mu$ so the term inside the bracket is also quadratic. Next, complete the squares and shifting the variables to absorb the terms linear in momentum $p^\mu$, which is now defined as a new variable of integration , $l^\mu$.

For example,

$$I(q,m) = \int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2-m^2)((p+q)^2-m^2)} \tag{A.3}$$

Here we have $A = (p+q)^2 - m^2$, and $B = p^2 - m^2$.
Using Feynman parameters we can write,

$$I(q,m) = \int_0^1 dx \int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2 + 2x.p.q + xq^2 - m^2)^2} \tag{A.4}$$

Now substituting $l^\mu = p^\mu + xq^\mu$, we have

$$I(q,m) = \int_0^1 \int \frac{d^4l}{(2\pi)^4}\frac{1}{(l^2 - \Delta(x))^2} \tag{A.5}$$

where $\Delta(x) = m^2 - x(1-x)$. The terms in numerator with odd momenta vanishes by symmetric integration. By symmetry we can write, $l^\mu l^\nu = g^{\mu\nu}\frac{l^2}{d}$. After transforming the integral into this form, we can use the Minkowski table integrals to solve it in d-dimensions.

## A.2 Loop Integrals and Dimensional Regularization

For our calculations we used the following integrals in Minkowski table for d-dimensional integration given as:

$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2 - \Delta)^n} = \frac{i(-1)^n}{(4\pi)^{d/2}} \frac{\Gamma(n - \frac{d}{2})}{\Gamma(n)} (\frac{1}{\Delta})^{n-\frac{d}{2}} \tag{A.6}$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l^2}{(l^2 - \Delta)^n} = \frac{i(-1)^{n-1}}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)} (\frac{1}{\Delta})^{n-\frac{d}{2}-1} \tag{A.7}$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l^\mu l^\nu}{(l^2 - \Delta)^n} = \frac{i(-1)^{n-1}}{(4\pi)^{d/2}} \frac{g^{\mu\nu}}{2} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)} (\frac{1}{\Delta})^{n-\frac{d}{2}-1} \tag{A.8}$$

For $d = 4$, the diverging integral can be expanded as,

$$(\frac{1}{\Delta})^{2-\frac{d}{2}} = 1 - (2 - \frac{d}{2}) log\Delta + ... \tag{A.9}$$

Also expanding $\Gamma(x)$ near its poles:

$$\Gamma(x) = \frac{1}{x} - \gamma + \mathcal{O}(x) \tag{A.10}$$

near $x = 0$, and

$$\Gamma(x) = \frac{(-1)^n}{n!(x+n)} - \gamma + 1... + \frac{1}{n} + \mathcal{O}(x+n) \tag{A.11}$$

near $x = -n$. Here $\gamma$ is the Euler- Mascheroni Constant, $\gamma \approx 0.5772$. The following combination of terms often appearing in calculations is simplified as:

$$\frac{\Gamma(2 - \frac{d}{2})}{(4\pi)^{d/2}} (\frac{1}{\Delta})^{2-\frac{d}{2}} = \frac{1}{\epsilon} - (log\Delta + \gamma - log(4\pi)) + \mathcal{O}(\epsilon)$$

with $\epsilon = \frac{4-d}{2}$.

In $\overline{MS}$ Scheme the coefficient of $1/\epsilon$ term gives the counter-term factor $\delta$ in the Lagrangian.

# Appendix B

# Two Loop SM RGEs

For completeness we give the two-loop SM RGEs taken from [Buttazzo 13].
Here $\beta(X) = \beta_1(X) + \beta_2(X) + ..$

$$\beta_2(g_1) = \frac{g_1^3}{(4\pi)^4}[\frac{44g_3^2}{5} + \frac{27g_2^2}{10} + \frac{199g_1^2}{50} - \frac{17y_t^2}{10}] \tag{B.1}$$

$$\beta_2(g_2) = \frac{g_2^3}{(4\pi)^4}[12g_3^2 + \frac{35g_2^2}{6} + \frac{9g_1^2}{10} - \frac{3y_t^2}{2}] \tag{B.2}$$

$$\beta_2(g_3) = \frac{g_2^3}{(4\pi)^4}[-26g_3^2 + \frac{9g_2^2}{2} + \frac{11g_1^2}{10} - 2y_t^2] \tag{B.3}$$

$$\begin{aligned}\beta_2(\lambda) = \frac{1}{(4\pi)^4}(&-312\lambda^3 - 144\lambda^2 y_t^2 + 36\lambda^2[3g_2^2 + \frac{3}{5}g_1^2] - 3\lambda y_t^4 + \lambda y_t^2[80g_3^2 + \frac{45}{2}g_2^2 + \frac{17}{2}g_1^2] \\ &-\frac{73}{8}\lambda g_2^2 + \frac{117}{20}\lambda g_2^2 g_1^2 + \frac{1887}{200}\lambda g_1^4 + 30y_t^6 - 32y_t^4 g_3^2 - \frac{8}{5}y_t^4 g_1^2 - \frac{9}{4}y_t^2 g_2^2 + \frac{63}{10}y_t^2 g_2^2 g_1^2 \\ &-\frac{171}{100}y_t g_1^4 + \frac{305}{16}g_2^6 - \frac{289}{80}g_2^4 g_1^2 - \frac{1677}{400}g_2^2 g_1^4 - \frac{3411}{2000}g_1^6)\end{aligned} \tag{B.4}$$

$$\begin{aligned}\beta_2(y_t) = \frac{y_t}{(4\pi)^4}(&-12y_t^4 + y_t^2[\frac{393}{80}g_1^2 + \frac{225}{16}g_2^2 + 36g_3^2 - 12\lambda] + \frac{1187}{600}g_1^4 - \frac{9}{20}g_2^2 g_1^2 \\ &+\frac{19}{15}g_1^2 g_3^2 - \frac{23}{4}g_2^4 + 9g_2^2 g_3^2 - 108g_3^4 + 6\lambda^2)\end{aligned} \tag{B.5}$$

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