Study of Magnetic Traps and Radio Frequency Dressed State Potentials

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A dissertation submitted for the partial fulfilment of BS-MS dual degree in Science

Supervised by

Dr. Mandip Singh

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Certificate of Examination

This is to certify that the dissertation titled “Study of Magnetic Traps and Radio Frequency Dressed State Potentials ” submitted by Swadheen Dubey (Reg.No. MS14029) for the partial fulfilment of BS-MS dual degree programme of the Institute has been examined by the thesis committee duly appointed by the Institute. The committee finds the work done by the candidate satisfactory and recommends that the report be accepted.

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Declaration

The work presented in this dissertation has been carried out by me under the guidance of Dr. Mandip Singh at the Indian Institute of Science Education and Research Mohali.

This work has not been submitted in part or in full for a degree, a diploma, or a fellowship to any other university or institute. Whenever contributions of others are involved, every effort is made to indicate this clearly, with due acknowledgement of collaborative research and discussions. This thesis is a bonafide record of original work done by me and all sources listed within have been detailed in the bibliography.

Swadheen Dubey

(Candidate)

Dated: April 20, 2019

In my capacity as the supervisor of the candidate's project work, I certify that the above statements by the candidate are true to the best of my knowledge.

Dr. Mandip Singh

(Supervisor)

Acknowledgements

While writing this thesis I realized that my time at IISER was one of the most exciting in my life. First I would like to thank Dr. Mandip Singh for providing me an opportunity to work with him. It was a nice experience to have him as a supervisor and to learn from him. I want to thank my parents for their love and constant support. I thank all of my friends (both inside and outside the institute) for inspiring me in different ways during my stay at IISER. In particular, I want to thank Shashank Prakash and Gouri Goverdhan very much for his encouragement. I really enjoyed those endless walks, chats, and coffee. I thank Adarsh, Priyasha, Navketan, Shruti, Irteza, Tuba, Ramandeep, Rishabh, Misbha, Akansha, Jaideep, Ritika, etc. for all those discussions. This thesis is dedicated to each and every one of my friends for making this time a memorable one.

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Abstract

In this thesis a theoretical study of Magnetic traps and Radio Frequency dressed state is presented. RF-Dressed state can produce a double well, a ring trap and in general polarization dependent potentials. RF- dressed state potentials are controlled by RF amplitudes, RF detuning and Rf polarization state. This thesis presents detailed calculations of RF dressed state potential for a Bose Einstein Condensate trapped in a Magnetic trap.

Contents

1 Introduction

Bose Einstein Condensate (BEC) was first predicted in 1925 by Alberst Einstein on the basis of works of Satyendra Nath Bose. Einstein predicted that when you cool an ideal bose gas to a point where the de-Broglie wavelength of the atoms is comparable to the inter-atomic separation a macroscopic occupation of ground state occurs. First time BEC was experimentlly observed in 1995 by groups at JILA, RICE and MIT. Eric A. Cornell, Carl E. Wieman from JILA and Wolfgang Ketterle from MIT awarded Nobel Prize for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms. It was by combining magnetic trapping techniques with laser cooling and evaporative cooling that lead to achievement of BEC in dilute gasses of Alkali atoms. The achievement of BEC is itself was the begining of new research field that has expanded rapidly over the last decade.

In this chapter we describe some theoretical and experimental techniques used to produce condensate of ^{87}Rb atoms in the state of $|F = 2, m_F = 1, 2\rangle$. To reach BEC atoms are kept in ultra high vacuum while laser and evaporative cooling is employed to reach ulta cold regime.

1.1 Methods of cooling

1.1.1 Laser Cooling

When an atom interacts with a laser which is on resonance with atom it absorbs a photon and get excited. This absorption leads to transfer of momentum. Shortly afterwards, atoms jumps back to the ground state emitting a photon in random direction due to spontaneous emission. This results in zero net change in momentum of atom due to averaging over many absorption and emission cycles. Therefore, shinning shinning a laser beam over an atom results in change in momentum of atom in the direction of laser beam. The maximum force on the atom per photon can be given by

$$F = \frac{\Delta p}{\Delta t} = \frac{\hbar k}{2\tau}$$

where $\hbar K$ is the momentum transfer to atom during each absorption-emission cycle and τ is exited state life time.

1.1.2 Doppler Cooling

For cooling a gas cloud which follows Maxwell Boltzman velocity distribution one needs to slow down a range of velocities. As explained above lasers are used to slow down atoms with force which is velocity dependent. Consider an atom moving in the the presence of light such that the light is slightly red-detuned from resonance. In atoms frame of reference, it will see light moving towards as closer to resonance and light which moves away further from resonance. Hence an atom moving in opposite direction from laser is slowed down because of scattering of photon. The scattering force explained in previous section becomes dissipative because of spontaneous emission. This technique is know as Doppler Cooling and was first suggested in the paper. For simplicity as an example take one dimensional case i.e, pair of laser beam along only one of the axes but it can be easily generalized to three dimensions. Let I be the beam intensity, the the total force (including the dissipation due to spontaneous emission) is given by

$$F = \frac{\hbar k \tau}{2}\left(\frac{I_0}{1 + 2I_0 + 4(\Delta - kv)^2/\tau^2} - \frac{I_0}{1 + 2I_0 + 4(\Delta + kv)^2/\tau^2}\right)$$

where $I_0 = I/I_s$, τ is the line width, and Δ is the detuning from resonance. For the case of red detuning and in the regime of velocity tends to zero the force can be written as

$$F = \alpha v$$

Where α is

$$\alpha = -4\hbar K^2 s\left(\frac{2\Delta/\tau}{(4\Delta^2/\tau^2 + 2s + 1)^2}\right)$$

Using this cooling technique atoms can be cooled upto

$$T_D = \frac{\hbar \tau}{2k_b}$$

Where T_D is known as the Doppler temperature.

1.1.3 Sub-Doppler Cooling

In the previous method we are considering atoms as two level system instead of multiple level and also we are not considering the effects polarization of light. These things will be included in this section. When we shine a light of particular polarisation on multi-level atom (at rest) and if the light field is in resonance with the sub-levels the population will redistribute between the sub-levels. In other words atom's dipole will orient relative to the polarisation of the field which introduces a spatially varying polarisation pattern resulting in redistribution of population such that the atomic dipole follows the light field. The time lag between the orientation of dipole with the polarisation is finite due to time taken by optical pumping. This give rise to sub-Doppler Cooling effects.

The limit on the lowest temperature can be achieved is set by the single photon recoil velocity. Therefore, we can define recoil temperature

$$T_R = \frac{(\hbar k)^2}{2mk_B}$$

1.2 Magneto Optical Trapping

To understand how MOT is used to trap particles consider a simple example of two level atom with ground state angular momentum $J = 0$ and excited state $J = 1$. In presence of magnetic field upper level splits into three sub levels with $m_j = -1, 0, 1$. For trapping we use a quadrapole field created by two coils in anti helmoltz configuration as explained in next chapter. Near the trap center the energies of these states varies linearly. Now, circular polarized light is applied to these atoms such they will produce non diagonal terms in hamiltonian causing spin flips. Atoms which are away from center are pushed towards the trap center. The magnetic fields used in magneto-optical trapping are much weaker than those required to confine the atoms in a purely magnetic potential and without the scattering forces of the laser beams the atoms would not be trapped.

1.3 Evaporative Cooling

Based on the same principle by which a hot drink cools down as vapor rises its surface the whole system rethermalizes. In this method a laser-cooled cloud in trapped in a magnetic trap. Consider its temperature to be T_1. It is evaporatively cooled in steps. In the first step, atoms with energy greater then a threshold are removed from the trap. the cloud rethermalizes to a new temperature say T_2 via elastic collisions and so on. For maximum cooling efficiency, the evaporation should be fast enough to overtake the effects of inelastic cooling and must be slow enough for the cloud to rethermalize as a net result number of low energy atoms increases. Higher density clouds permit faster evaporation. In harmonic potential as the cloud cools down the density of atoms increases allowing the speed of evaporation to further increase. One of the methods used for evaporation in magnetic traps is by applying a radio frequency to drive spin-flip changing trapped states to untrapped states. It is a highly controlled method by tuning the RF radiation to correspondingly larger frequencies relatively higher energy atoms can selectively be removed from the trap.

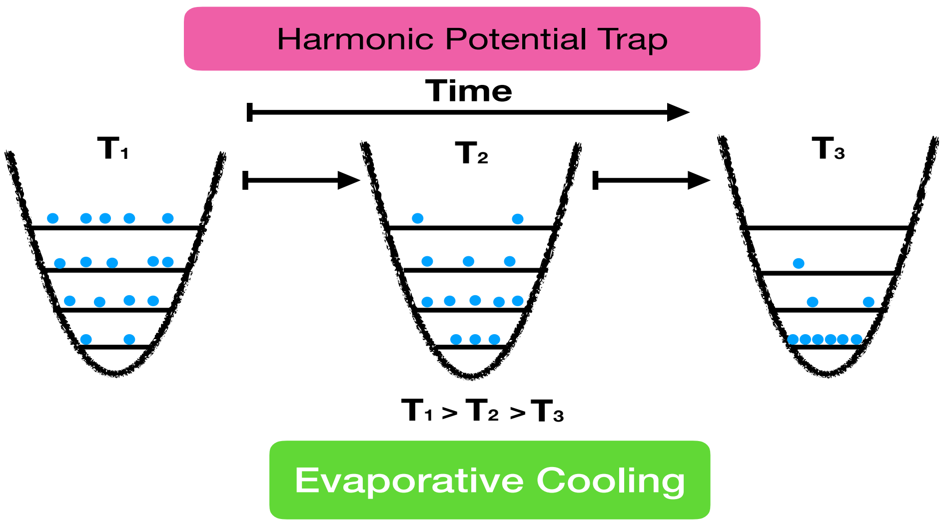


Figure 1.1: With each step length parameter is decreased and cloud density increases

1.4 Bose Einstein Condensate (BEC) in a uniform ideal gas

BEC is used to study a variety of research areas. After the first BEC was experimentally observed in 1995, BEC has been used for fundamental experiments, atom precision measumrment, atom interferometry and simulation of condensed matter system. This section provides a short introduction of the theory behind BEC.In this section the phase transition properties of a uniform ideal Bose gas is derived.

The cloud of N noninteracting Bosons in the trap after going transition to BEC behaves like a giant matter wave. Because of properties of Bosons, there is a macroscopic occupation of a single particle quantum state, i.e ground state. An atom's spatial occupation can be described by its de-Broglie wavelength λ_{DB}, for the uniform bose gas of atoms with the number density n, temperature T, it's given by

$$\lambda_{DB} = \frac{\hbar}{2\pi m k_B T}$$

The de-Broglie wavelength of an atom at room temperature is much shorter than the interparticle separation between the atoms of a cloud with density n which is proportional to $n^{1/3}$. But as the temperature is decreased, the wavelength increases and at the point where $n^{1/3} = \lambda_{dB}$, the wavelength of the atoms start to overlap and the system undergoes a phase transition to a Bose-Einstein Condensation. A single wavefunction of the system is created by the overlap of all atoms. In terms of phase space density of the gas, this transition is attained at $\rho = 1$ where $\rho = n\lambda_{DB}^3$ The potential for trapped atoms in a 3-D harmonic oscillator potential is given as

$$V_{ad} = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$$

The energy associated with this potential

$$E(n_x, n_y, n_z) = \hbar\Big(n_x + \frac{1}{2}\Big)\omega_x + \hbar\Big(n_y + \frac{1}{2}\Big)\omega_y + \hbar\Big(n_z + \frac{1}{2}\Big)\omega_z$$

In BEC all particles occupy the ground state so

$$\Phi(\vec{r_1}, \vec{r_2}, ..., \vec{r_N}) = \prod_{i=1}^{N} \Phi_0(\vec{r_i})$$

where

$$\Phi_0(\vec{r_i}) = \Big(\frac{m\omega'}{\pi\hbar}\Big) \exp\Big(\frac{-m}{2\hbar}(\omega_x^2 + \omega_y^2 + \omega_z^2)\Big)$$

where ω' is the relevant geometric mean of the trap frequencie, i.e

$$\omega' = (\omega_x \omega_y \omega_z)^{\frac{1}{3}}$$

Most of the properties of the BEC phase transition can be calculated from the partition function of the grand canonical ensemble. This function is given by

$$Z_{gc} = Tr(\exp{(-\beta(\hat{H} - \mu\hat{N}))})$$

Where $\beta = \frac{1}{k_b T}$, $\hat{H}$ is hamilotian, $\hat{N}$ is number operator and μ is chemical potential which amounts to number fluctuation. we can see from that Z_{gc} is not defined for the ground state for $\mu = 0$ The most probable N which we were initially referring to the total number of atoms can be calculated using the grand canonical function

$$N = \sum_i N_i = \left(\sum \Big(\frac{1}{\exp(\beta(E_i - \mu)) - 1}\Big)\right)_{(T,V)}$$

If V is large the spacing between the energy levels will be much less than $k_b T$ and this discrete sum can be converted to an integral to give

$$N = N_0 + N_{ex} = N_0 + \int dE \Big(\frac{\rho(E)}{\exp(\beta(E - \mu)) - 1}\Big)$$

where $\rho(E)$ is the density of states with energy E and the number of particles in the ground state has been separated out since it is not accounted by the integral. The density of states for a harmonic potential of volume V is given by

$$\rho(E) = \frac{V(2m)^{\frac{3}{2}}\sqrt{E}}{\hbar^3 (2\pi)^2}$$

From the integral number of excited states are

$$N_{ex} = \frac{V}{(2\pi)^2}\left(\frac{2mKT}{\hbar^2}\right)^{\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\zeta\left(\frac{3}{2}\right)$$

Where Γ and ζ are gamma and Riemann zeta function respectively. The critical temperature at which particles can accommodate in the excited states with no particles in the ground state can be calculated using

$$T_c = \frac{2\pi\hbar^2}{mK}\left(\frac{n}{\zeta\left(\frac{3}{2}\right)}\right)$$

where $n = \frac{N}{V}$ and $\zeta\left(\frac{3}{2}\right) = 2.612$

At temperature T, the total number of particles in the ground state in terms of T_c will be

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c}\right)^{\frac{2}{3}}$$

Now, to achieve BEC all we need to do is to devise a method to trap a finite number of particles and cool them below T_c. Particles are cooled using methods which are mentioned above and the structure of different traps and trapping methods are explained in the next chapter.

2 Magnetic Traps for Neutral Atoms

2.1 Principle behind magnetic trapping

Magnetic traps are based on two principles

2.1.1 Earnshaw Theorem

Eanshaw theorem tells us that using any configuration of sources which produce static fields we can not achieve a local maxima.

2.1.2 Zeeman effect

According to classical electromagnetism the energy of magnetic dipole moment acted by external magnetic field B is

$$V = -\mu.B$$

Quantum mechanically this interaction produces splitting of particles energy levels. Total angular momentum is specified by spin and obital angular momentum. The quantity reads as

$$V(r) = -g_s\mu_B S - g_l\mu_b L$$

For atoms in magnetic traps potential will be

$$V(r) = g_F\mu_B F.B(r)$$

where

$$g_F = g_j\frac{F(F+1)+J(J+1)I(I+1)}{2F(F+1)} - \frac{\mu_N}{\mu_B}g_I\frac{F(F+1)J(J+1)+I(I+1)}{2F(F+1)}$$

The second term is neglected because it is of the order of 10^{-3}. For ^{87}Rb the hyperfine levels that are used throughout the thesis will be $|F=2\rangle$ and $|F=1\rangle$ which have $g_F = 1/2$ and $g_F = -1/2$, respectively.

2.2 Potential in a Static Magnetic Trap

The potential of an atom with hyperfine spin F in a magnetic field B(r) is given by

$$V(r) = g_F \mu_B F.B(r)$$

2.2.1 The Adiabatic Hamiltonian

The Hamiltonian for the atom in the trap

$$H = \frac{P^2}{2m} - \mu.B(r) = T + h(r)$$

The first term T is kinetic energy of the center of mass of the atom and the second term is the the interaction of magnetic moment with magnetic field. The eigenstate of $h(r)$ will be $|m(r)\rangle$ and its eigenvalues will be

$$h(r)\,|m(r)\rangle = e_m(r)\,|m(r)\rangle$$

It is useful to introduce a r independent unitary operator $U(r)$ which rotates $B(r)$ parallel to z axis and acts on states such that

$$|m(r)\rangle = U(r)\,|m_z\rangle$$

We define the transformed Hamiltonian to be

$$H' = U(r)HU^\dagger(r) = U(r)TU^\dagger(r) + U(r)h(r)U^\dagger(r)$$

$$H' = T' + h'(r) = T + h'(r) + \Delta T = H_{ad} + \Delta T$$

here H_{ad} is the adiabatic hamiltonian and the adiabatic approximation corresponds to neglecting $\Delta T = T' - T$. In this picture the adiabatic potential will be simply

$$V_{ad} = g_F \mu_B m_F \mid B(r) \mid$$

$e_m(r)$ are diagonal elements of $h'(r)$ in a r independent basis $|m\rangle$. A simplified example of magnitude of magnetic field is $\mid B(r) \mid = \sqrt{B_0^2 + \alpha^2(x^2 + y^2)} = B$.

For such a case $U(r)$ will be

$$U(r) = \beta x\sigma_x - \beta y\sigma_y + \gamma\sigma_z$$

where $\gamma^2 = \frac{B_0+B}{2B}$ and $\beta^2 = \frac{\alpha^2}{2B(B_0+B)}$. From above we can see there will exist only types of state

- Strong-field seeking state: If the direction of the magnetic moment and magnetic field are parallel for $g_F > 0$ then an atom will seek for the higher magnitude of the magnetic field as Minima of the potential are found at the maxima of the field and vice versa. As the maximum of B corresponds to a minimum of potential energy. But according to Earnshaw theorem maximum of the magnetic field is forbidden in free space. So using static magnetic field one can not trap positive spin particle.
- Weak-field seeking state: If the direction of the magnetic moment and magnetic field are anti-parallel for $g_F < 0$ then an atom will seek for the lower magnitude of the magnetic field as minima of the potential energy correspond to minima of a magnetic field and vice versa. The minima of the magnetic field are allowed by Earnshaw theorem. These types of traps are most common for neutral atoms and will trap only negative spin particles. Although an important point to note is that these states are not the state of lowest energy as positive spin can energies are negative in signature for any $|B| > 0$.

2.3 Quadrupole Trap

Quadrupole trap is obtained by positioning two parallel coils in anti- helmoltz configuration, i.e, with their currents flowing in opposite direction. The magnetic field because in cartesian co-ordinates reads as

$$B = \frac{3\mu_B I_c a^2 b}{2}(a^2 + b^2)^{2.5}(xe_x + ye_y - 2ze_z)$$

Magnitude of magnetic field at the trap center

$$|B| = \frac{3\mu_B I_c a^2 b}{2}(a^2+b^2)^{2.5}(\sqrt{x^2+y^2+4z^2})$$

Here, we can see at the trap center magnetic field is zero. Hence, if the trapped particles reached the minimum the energy spacing between the zeeman level rather reduce increasing the probability of non adiabatic transitions among these energy levels, leading to loss of particles because of spin-flip.

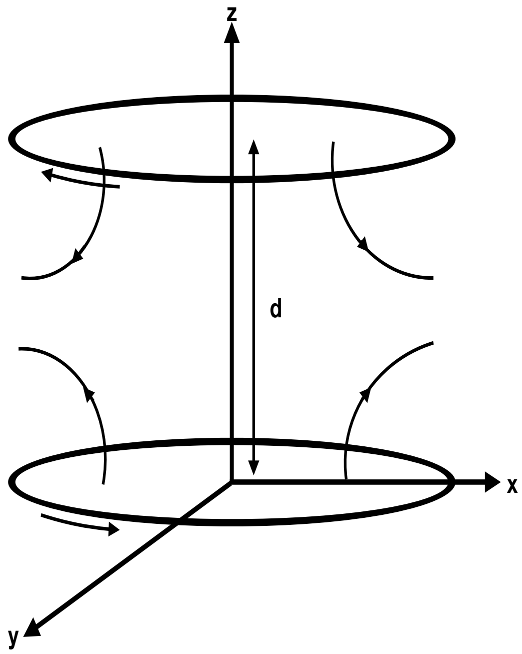


Figure 2.1: Two current carrying rings in anti- helmoltz configuration.

2.4 Ioffe-Pritchard Trap

2.4.1 Geometry and Magnetic Field Profile

Ioffe-Pritchard trap contains four parallel wires and two coils. The configuration of wires is such that they produce a quadrapole field (there is a azimuthal symmetry). The coils are in Helmholtz configuration around the wires as shown in figure 2.2

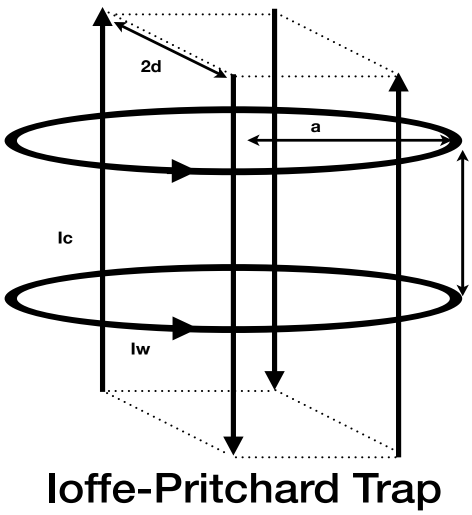


Figure 2.2: From here on we will use this trap only for our static field.

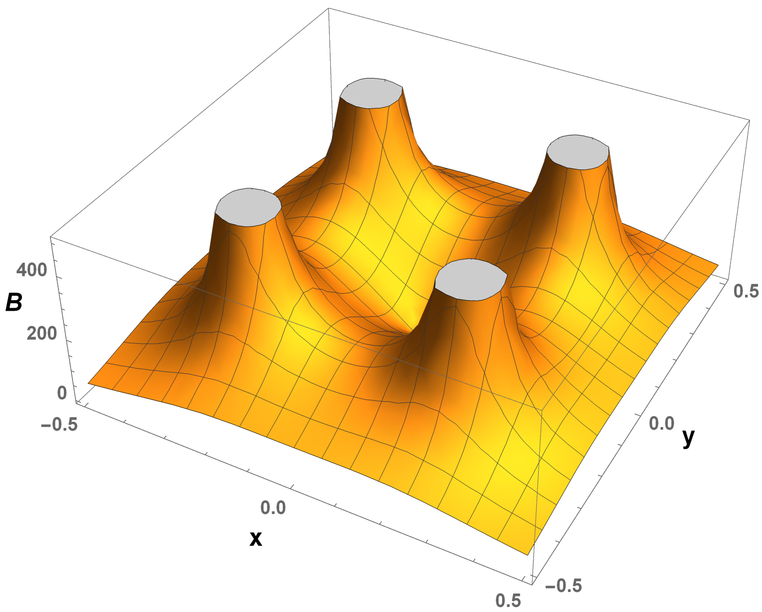


Figure 2.3: Variation of magnitude of magnetic field in x-y plane.

The magnetic field due to the four wires under the assumption $d << x, y$ in x-y plane

$$B_w = \frac{\mu_0 I_1}{d^2 \pi}(yj - xi)$$

And because of the coils assuming $a, b >> x, y, z$ in x-y plane

$$B_c = 3C_3 z(xi + yj) + (C_1 + 1.5C_3(x^2 + y^2 - 2z^2))k$$

where

$$C_1 = \frac{\mu_0 I_2 a^2}{(a^2+b^2)^{3/2}}, C_3 = \frac{\mu_0 I_2 a^2(a^2-4b^2)}{2(a^2+b^2)^{7/2}}, \alpha = \frac{\mu_0 I_1}{d^2\pi}$$

Magnitude of magnetic field for our trap in x-y plane

$$|B| = \sqrt{(\alpha^2 + 3C_1C_3)(x^2+y^2) + C_1^2}$$

Near the trap center $C_1^2 >>> (\alpha^2 + 3C_1C_3)$

$$|B| = C_1 + \frac{1}{2}\frac{(\alpha^2+3C_1C_3)}{C_1}(x^2+y^2)$$

2.4.2 Adiabatic potential for atoms in Ioffe-Pritchard trap

In trap potential for atoms in x-y plane

$$V_{ad} = \mu|B| = \mu\sqrt{(\alpha^2 + 3C_1C_3)(x^2+y^2) + C_1^2}$$

$$V_{ad} = \mu C_1 + \frac{1}{2}\frac{\mu(\alpha^2+3C_1C_3)}{C_1}(x^2+y^2)$$

Near the trap center first term is causing Zeeman splitting and the second term behaves like a 2-D harmonic oscillator potential.

$$V_{ad} = \hbar\omega_0 + \frac{M\omega_x^2 x^2}{2} + \frac{M\omega_y^2 y^2}{2}$$

where energy splitting and trap frequencies in x and y direction is given by

$$\omega_0 = \frac{\mu C_1}{\hbar}, \omega_x = \omega_y = \sqrt{\frac{\mu(\alpha^2+3C_1C_3)}{MC_1}}$$

respectively. The adiabatic condition is given by

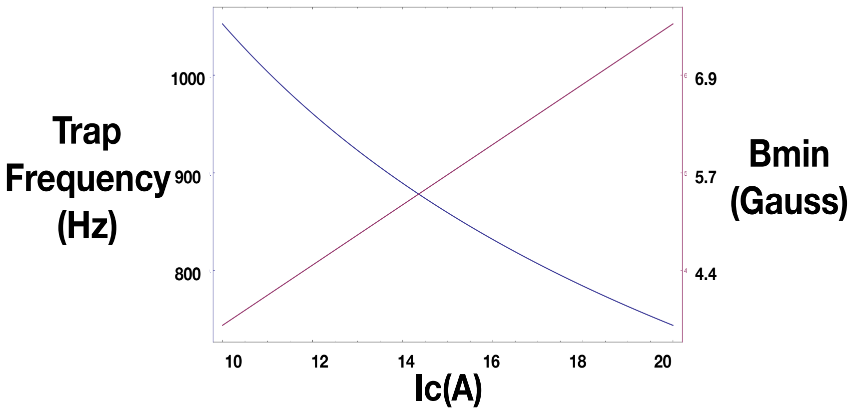
$$X_0 = \frac{\omega_x}{\omega_0} <<<< 1$$

Trap frequency along the z-axis is given by

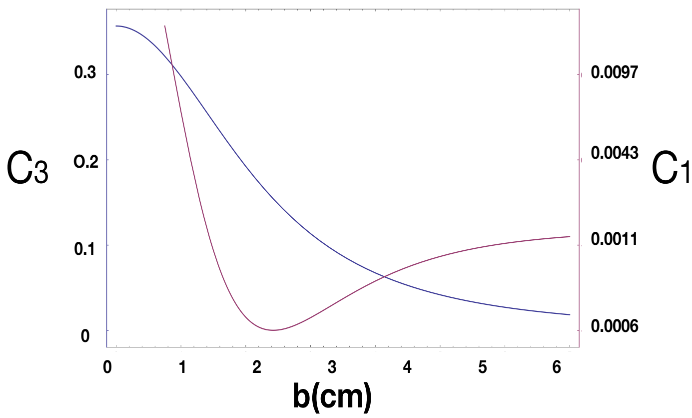
$$V_{ad} = \mu C_1 - 3\mu C_3 Z^2$$

$$\omega_z = \sqrt{\frac{6\mu C_3}{M}}$$

. The distance between the coils and the current in the wire and the coils are decided such that we get the maximum trap frequency and B_{min}.



(a) for Ic=14.36 A both Radio Frequency and Bmin are maximum



(b) for b=1 cm both C3 and C1 are maximum

Figure 2.4: Different Parameters for Ioffe-Pritchard Trap

3 Radio Frequency Dressed State Potentials

In this chapter for simplicity, we will consider atom to be two levels. The same procedure can be extended for atoms with three level and higher. The two levels are labeled as $|1\rangle$ and $|2\rangle$ with energy difference $\hbar\omega_0$. In presence of oscillating magnetic field the Hamiltonian changes to $H = H_0 + H'$ where H' and H_0 are the Hamiltonian due to interaction and atom respectively. We will first introduce a semiclassical approach to find new states. Later on, we will devise a method to understand the effects of different polarization on the dressed state.

3.1 Interaction of a two level atom with a oscillating magnetic field

In this section, we are using a semi-classical approach where the atomic levels are quantized while the field is classical. Because of our static trap, there is Zeeman splitting in the direction along the static magnetic field. Now, if we apply an oscillating magnetic field orthogonal to the direction of static traps field the potential for two states changes as shown below

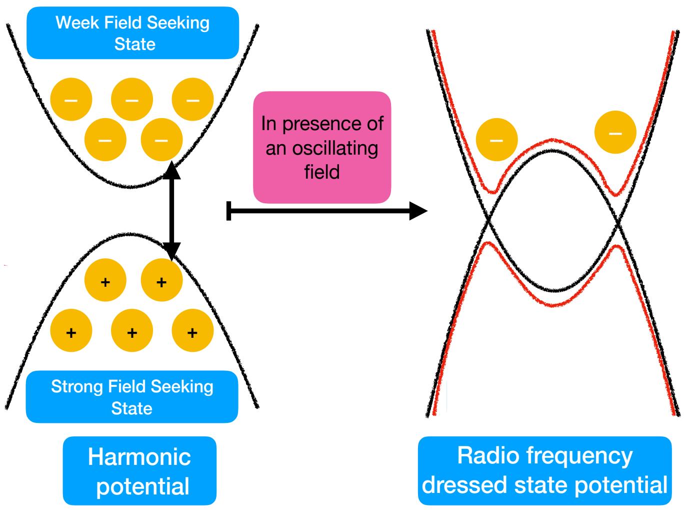


Figure 3.1: Change in potential profile of static field in presence of oscillating field.

3.1.1 A semi classical approach

The radio frequency magnetic field is taken to be of the form

$$B_{rf} = B_{rf}^0 \begin{bmatrix} \cos(\omega t) \\ \lambda \cos(\omega t - \gamma) \\ 0 \end{bmatrix}$$

The two level atom state for this system will be

$$|\psi\rangle = c_1(t)\,|1\rangle + c_2(t)\,|2\rangle$$

And in the presence of only static magnetic field in the z direction the hamiltonian for the atom was

$$H = \frac{\hbar}{2} \begin{bmatrix} -\omega_0 & 0 \\ 0 & \omega_0 \end{bmatrix}$$

Where $\omega_0 = g_F \mu_B m_z B_s$ and B_s is static magnetic field because of Ioffe-pritchard trap.

The z-direction here is pointing towards the direction of the static field because of the unitary transformation that we made in the previous section. Assuming that the contribution of the two orthogonal components of B_{rf} from B_s be $x_1 \cos(\omega t - \phi_1)$ and $x_2 \cos(\omega t - \phi_2)$. Then the hamiltonian will become

$$H = g_F \mu_B F.(B_s e_z + x_1 \cos(\omega t - \phi_1) e_x + x_2 \cos(\omega t - \phi_1) e_y)$$

$$H = g_F \mu_B (F_z B_s + F_x x_1 \cos(\omega t - \phi_1) + F_y x_2 \cos(\omega t - \phi_1))$$

replacing F_x and F_y by $\frac{F_+ + F_-}{2}$ and $\frac{F_+ - F_-}{2i}$ respectively. Where F_+ and F_- are ladder operators. The final hamiltonian in $(|1\rangle, |2\rangle)$ basis will look like

$$H = \frac{\hbar}{2} \begin{bmatrix} -\omega_0 & \Omega_+(x_1 \cos(\omega t - \phi_1) + i x_2 \cos(\omega t - \phi_2)) \\ \Omega_-(x_1 \cos(\omega t - \phi_1) - i x_2 \cos(\omega t - \phi_2)) & \omega_0 \end{bmatrix}$$

where $\Omega_\pm = \frac{g_F \mu_B F_\pm}{2}$. How to find x_1, ϕ_1 and x_2, ϕ_2 is given in section 3.1.3.

3.1.2 Rotating wave approximation

To make calculations easy we will make a unitary transformation to go to the rotating frame of B_{rf}.

$$U = \begin{bmatrix} \exp\left(-\frac{i\omega t}{2\hbar}\right) & 0 \\ 0 & \exp\left(\frac{i\omega t}{2\hbar}\right) \end{bmatrix}$$

Such that $U^\dagger |\psi\rangle = |\psi'\rangle$. For this new state the schrodingers equation will transform as

$$i\hbar \frac{d\,|\psi'\rangle}{dt} = (U^\dagger H U + i\hbar \frac{d(U^\dagger)}{dt} U)\,|\psi'\rangle$$

Making the rotating wave approximation and ignoring the terms of the form of $\exp(\pm 2\omega)$. The new hamiltonian for $|\psi'\rangle$ state will look like

$$H'' = \frac{\hbar}{2}\begin{bmatrix} \omega - \omega_0 & \Omega_+(x_1 \exp(-\phi_1) + i x_2 \exp(-\phi_2)) \\ \Omega_-(x_1 \exp(\phi_1) - i x_2 \exp(\phi_2)) & \omega_0 - \omega \end{bmatrix}$$

The energies of these new states are the eigenvalues of the above hamiltonian

$$E = \pm\frac{\hbar}{2}\sqrt{(\omega - \omega_0)^2 + \Omega^2(x_1^2 + x_2^2 + 2x_1x_2\sin(\phi_1 - \phi_2))}$$

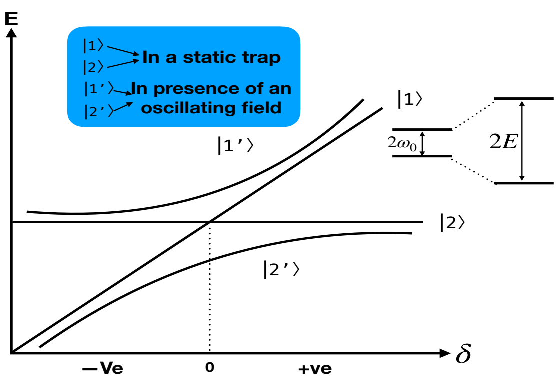


Figure 3.2: Variation in E with detuning.

Now all we have to do is to find x_1 and x_2.

3.1.3 Contribution of B in E for different polarization

From the previous section

$$B_s = \begin{bmatrix} -\alpha x \\ \alpha y \\ B_0 \end{bmatrix}$$

A unit vector in direction of B$_s$

$$e_z = \frac{1}{\sqrt{x^2+y^2+z^2}} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

We will replace x by $-\alpha x$, y by αy and z by B_0 when needed. This basis in polar co-ordinates along with the other two orthogonal basis will be

$$e_z = \begin{bmatrix} \cos\phi\sin\theta \\ \sin\phi\sin\theta \\ \cos\theta \end{bmatrix}, e_x = \begin{bmatrix} \sin\phi \\ -\cos\phi \\ 0 \end{bmatrix}, e_y = \begin{bmatrix} -\cos\phi\cos\theta \\ -\sin\phi\cos\theta \\ \sin\theta \end{bmatrix}$$

The inverse of corresponding rotation matrices

$$R^{-1} = \begin{bmatrix} -\cos\phi\cos\theta & -\sin\phi\cos\theta & \sin\theta \\ \sin\phi & -\cos\phi & 0 \\ \cos\phi\sin\theta & \sin\phi\sin\theta & \cos\theta \end{bmatrix}$$

Contribution of B_{rf} in this rotated frame

$$\begin{bmatrix} x_1\cos(\omega t-\phi_1) \\ x_2\cos(\omega t-\phi_2) \\ x_3\cos(\omega t-\phi_3) \end{bmatrix} = B^0_{rf} \begin{bmatrix} -\cos\phi\cos\theta & -\sin\phi\cos\theta & \sin\theta \\ \sin\phi & -\cos\phi & 0 \\ \cos\phi\sin\theta & \sin\phi\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos(\omega t) \\ \lambda\cos(\omega t-\gamma) \\ 0 \end{bmatrix}$$

From above

$$x_1 \cos\phi_1 = B^0_{rf}(-\cos\theta\cos\phi - \lambda\cos\theta\sin\phi\cos\gamma)$$
$$x_1 \sin\phi_1 = B^0_{rf}(-\lambda\cos\theta\sin\phi\sin\gamma)$$
$$x_2 \cos\phi_2 = B^0_{rf}(\sin\phi - \lambda\cos\phi\cos\gamma)$$
$$x_2 \sin\phi_2 = B^0_{rf}(-\lambda\cos\phi\sin\gamma)$$
$$x_1^2 + x_2^2 + 2x_1x_2\sin(\phi_1 - \phi_2) = (B^0_{rf})^2 h(\theta, \phi, \gamma, \lambda)$$

$$h(x,y,\gamma,\lambda) = \frac{B_0^2}{\alpha^2(x^2+y^2)+B_0^2}\frac{x^2+\lambda^2y^2}{x^2+y^2} + \frac{y^2+\lambda^2x^2}{x^2+y^2} + \frac{2\lambda\alpha^2 xy\cos\gamma}{\alpha^2(x^2+y^2)+B_0^2}$$
$$-\frac{2\lambda B_0\sin\gamma}{\sqrt{\alpha^2(x^2+y^2)+B_0^2}}$$

So the Rf dressed state potential will be

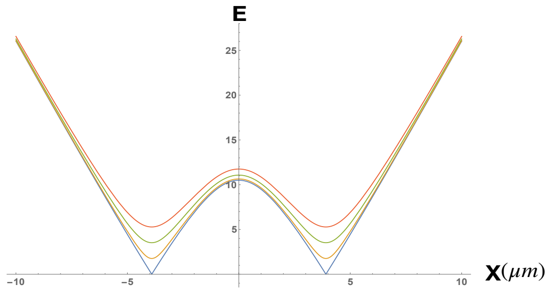
$$E = \pm\frac{\hbar}{2}\sqrt{(\omega-\omega_0)^2 + (\Omega B^0_{rf})^2 h(x,y,\lambda,\gamma)}$$

3.2 Linear Polarized light

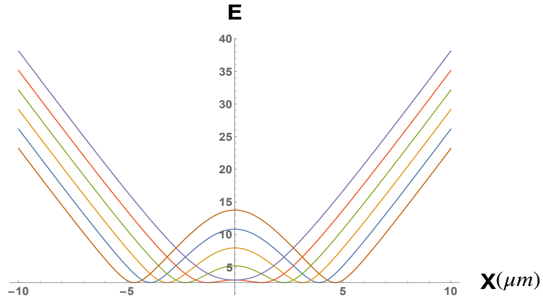
For linearly polarized light $\lambda = 0$. Energy for the particles in this trap will be

$$E = \pm\frac{\hbar}{2}\sqrt{(\omega-\omega_0)^2 + (\Omega B^0_{rf})^2\frac{B_0^2+\alpha^2y^2}{\alpha^2(x^2+y^2)+B_0^2}}$$

In figure 3.1 (b), with an increase in detuning above a critical value the center of trap minima splits into two minima symmetric along the y-axis. With further increase in detuning distance between two minima increases. In figure 3.1 (a), for the case of maximum detuning, as we increase the B^0_{rf} the sharpness of minima decreases and B_{min} increases. Contour plots for the linearly polarized trap of different frequencies are given below. Color distribution for all the contour plots follows VIBGYOR from minima to maxima.



(a) Effects of changing magnitude of RF field



(b) Effects of changing frequency of RF field

Figure 3.3: Different cases of RF potential for linear polarization

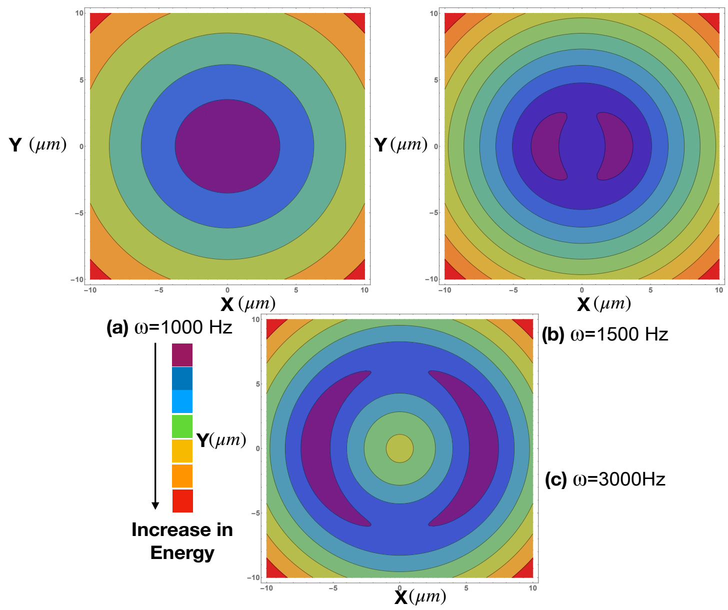


Figure 3.4: Linear polarization at different radio frequency.

Therefore, depending on the polarization either the potential remains at the same position with a slight change in shape or it can shift the minimum position and convert into a double well potential.

3.3 Circular Polarized Light

For circular polarized light $\lambda = 1$ and $\gamma = \pm\frac{\pi}{2}$

$$E = \pm\frac{\hbar}{2}\sqrt{(\omega-\omega_0)^2 + (\Omega B_{rf}^0)^2\left(\frac{B_0^2}{\alpha^2(x^2+y^2)+B_0^2} + 1 \pm \frac{2B_0}{\sqrt{\alpha^2(x^2+y^2)+B_0^2}}\right)}$$

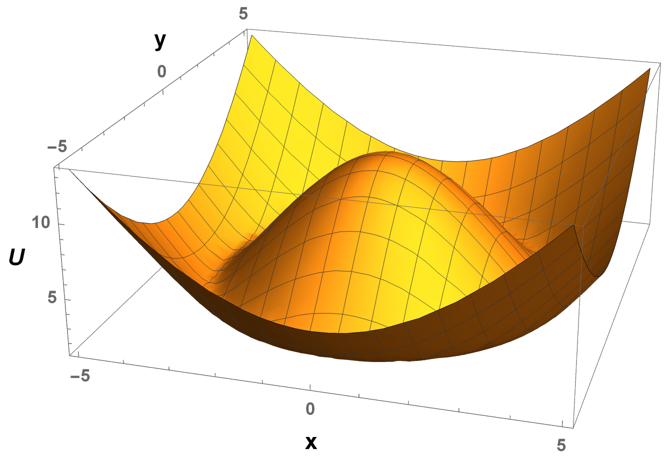


Figure 3.5: Ring Trap

Here again, there are two possible scenarios: Left circular polarization and right circular polarisation. In the case when $B_{rf} = 0$ which is independent of γ, we get the same plot for left circular polarisation and right circular polarisation, the plot obtained is shown in figure 3.6a. The rf potential for the case in which $\sin = 1$ is shown in figure 3.6, with increase in detuning we get a ring trap while in the case of $\sin\gamma = -1$ we get the same plot.

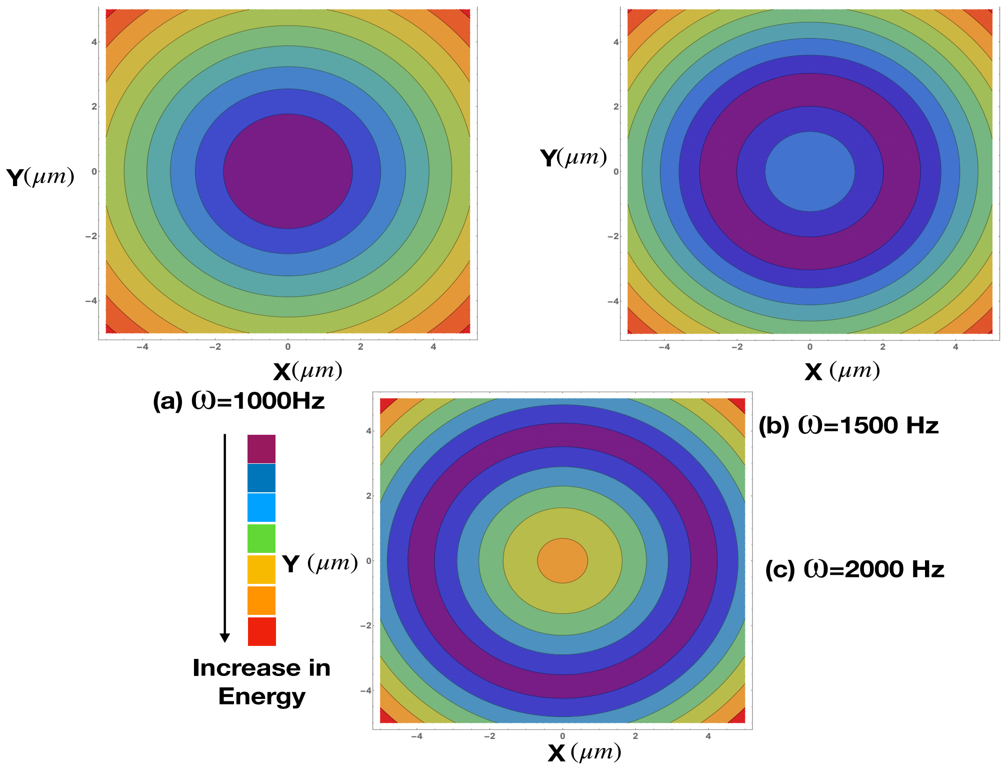


Figure 3.6: Ring trap at different radio frequencies.

3.4 Elliptical Polarized Light

In the second case when detuning is negative there is always a minima at non-zero x(similar to the case of linear polarization) and is independent of γ. In this case the rf potential is ring shaped for any value of rf field strength.

For elliptical polarized light $\gamma = \frac{\pi}{2}$

$$E = \pm\frac{\hbar}{2}\sqrt{(\omega-\omega_0)^2 + (\Omega B^0_{rf})^2\left(\frac{(1+\lambda^2)B_0^2 + \alpha^2(x^2+\lambda^2 y^2)}{\alpha^2(x^2+y^2)+B_0^2} - \frac{2\lambda B_0}{\sqrt{\alpha^2(x^2+y^2)+B_0^2}}\right)}$$

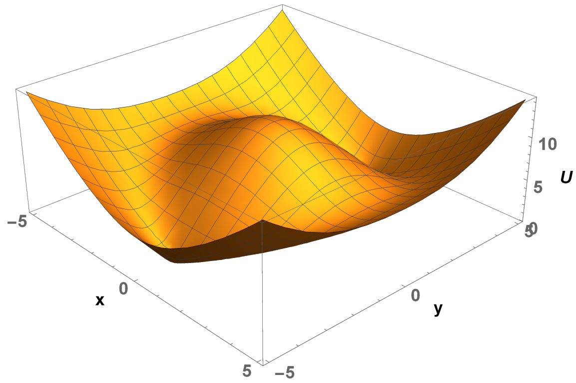
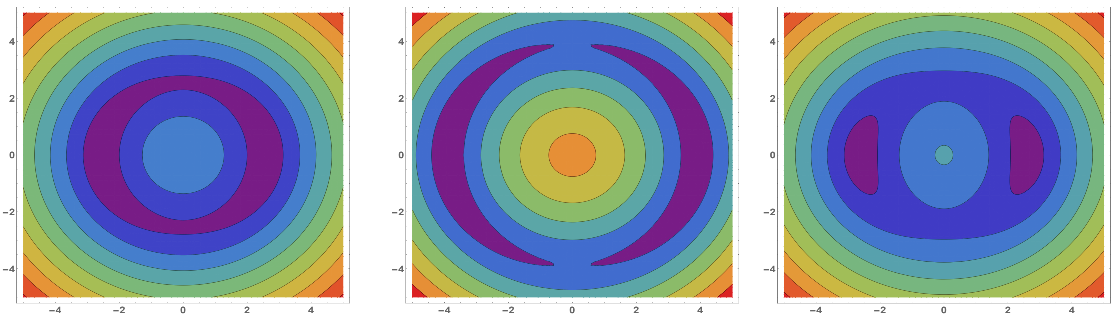
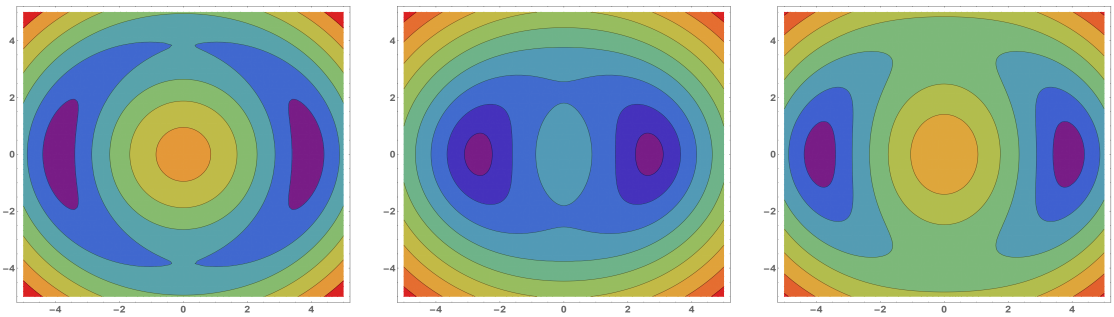


Figure 3.7: A trap with elliptical polarization.



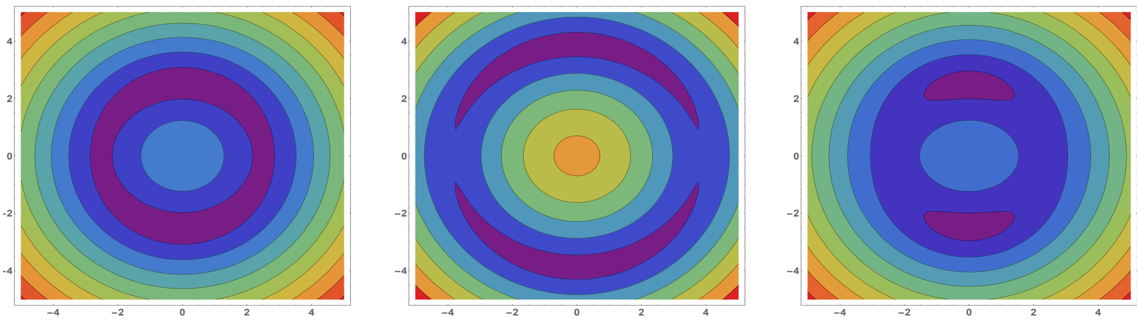
(a) $\omega = 15, B_{rf}^0 = 3$ (b) $\omega = 20, B_{rf}^0 = 3$ (c) $\omega = 15, B_{rf}^0 = 6$



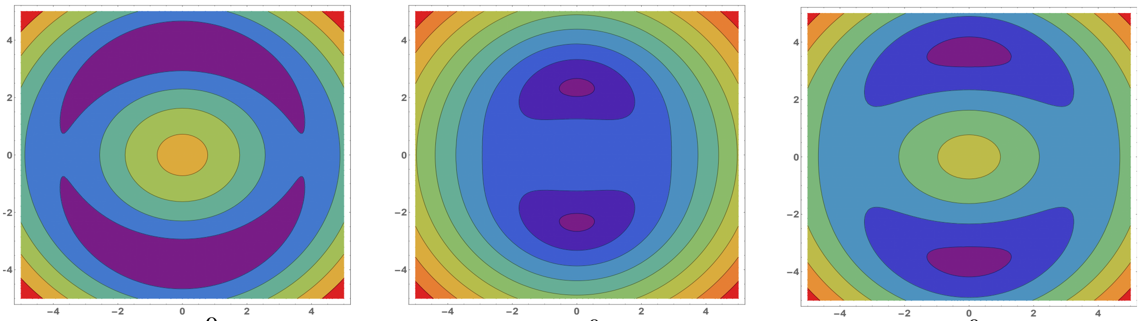
(d) $\omega = 20, B_{rf}^0 = 6$ (e) $\omega = 15, B_{rf}^0 = 10$ (f) $\omega = 20, B_{rf}^0 = 10$

Figure 3.8: Different cases of elliptical trap for $\lambda = 0.5$ here frequency is ω100Hz and b_{rf} in gauss.

In figure 3.8, we can see that above a threshold value of B_{rf}^0 we get two separate regions of minima and with increase in B_{rf}^0 they become more isolated or the well becomes more stepper. On the other hand one can achieve the same effects with just increasing the detuning.



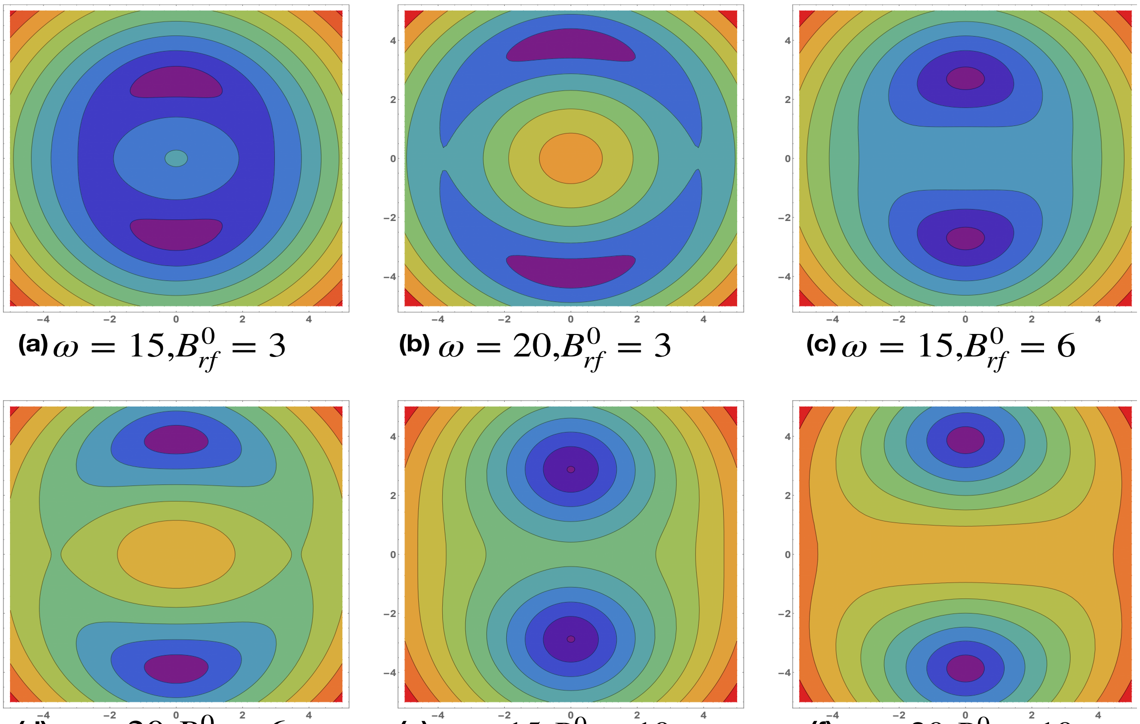
(a) $\omega = 15, B^0_{rf} = 3$ **(b)** $\omega = 20, B^0_{rf} = 3$ **(c)** $\omega = 15, B^0_{rf} = 6$



(d) $\omega = 20, B^0_{rf} = 6$ **(e)** $\omega = 15, B^0_{rf} = 10$ **(f)** $\omega = 20, B^0_{rf} = 10$

Figure 3.9: Different cases of elliptical trap for $\lambda = 1.25$ here frequency is ω100Hz and b_{rf} in gauss.

Now, if we change the polarization of light to $\lambda = 1.25$ as in figure 3.9. the orientation of minima changes. In case of $\lambda = 0.5$ center of minima's was situated along with $y = 0$ while in this case these are situated along the x-axis. For circular polarization, we get a ring trap which is symmetrical in the x-y plane and has $\lambda = 1$. For linear polarization that is $\lambda = 0$, we have a center of minima's along the x-axis. For $\lambda = 2$ (Figure 3.10) center of minimas are along the y-axis. So one can state that center of minima's will be along the x-axis for $0 < \lambda < 1$ and will be along the y-axis for $\lambda > 1$. The most efficient elliptical trap we get is at $\omega = 20$ and $B^0_{rf} = 6$, if we are talking in terms of the spatial extent of minima.



(a) $\omega = 15, B_{rf}^{0} = 3$
(b) $\omega = 20, B_{rf}^{0} = 3$
(c) $\omega = 15, B_{rf}^{0} = 6$
(d) $\omega = 20, B_{rf}^{0} = 6$
(e) $\omega = 15, B_{rf}^{0} = 10$
(f) $\omega = 20, B_{rf}^{0} = 10$

Figure 3.10: Different cases of elliptical trap for $\lambda = 2$

4 Correction to the above

4.1 Contribution along the direction of static field

The above calculation is first done in [13] and was experimentally realized in [8]. The problem with the calculation in [13] is that they have not included a time-dependent contribution of x_3 in E i.e, the effect of radio frequency in the direction of the static field. From the rotation matrix mentioned in section 3.1.3 in chapter we can see that

$x_3 \cos(\phi_3) = \cos(\phi)\sin(\theta) + \sin(\phi)\sin(\theta)\lambda\cos(\gamma)$

$x_3 \sin(\phi_3) = \sin(\phi)\sin(\theta)\lambda\sin(\gamma)$

In cartesian co-ordinates

$$x_3^2 = \frac{\alpha^2(x^2 + y^2\lambda^2 - 2xy\lambda\cos\gamma)}{\alpha^2(x^2 + y^2) + B_0^2}$$

In other words in case of linear polarization only along $x = 0$ the contribution of x_3 is zero. For the case of circular polarization x_3 becomes

$$x_3^2 = \frac{\alpha^2(x^2 + y^2)}{\alpha^2(x^2 + y^2) + B_0^2}$$

and for elliptical polarization x_3 becomes

$$x_3^2 = \frac{\alpha^2(x^2 + y^2\lambda^2)}{\alpha^2(x^2 + y^2) + B_0^2}$$

For both the cases the contribution of x_3 is zero only along z axis. So instead of equation..... the general equation for the energy should be

$$E = \pm\frac{\hbar}{2}\sqrt{(\omega - (\omega_0 + x_3\cos(\omega t - \phi_3)))^2 + (\Omega B_{rf}^0)^2 h(x, y, \lambda, \gamma)}$$

For $\lambda = 1$ and $\gamma = 0$, x_3 will be zero along x=y.

We can find a general 3D polarization for which x_3 will be zero.

$$\begin{bmatrix} x_1\cos(\omega t-\phi_1) \\ x_2\cos(\omega t-\phi_2) \\ x_3\cos(\omega t-\phi_3) \end{bmatrix} = B^0_{rf} \begin{bmatrix} -\cos\phi\cos\theta & -\sin\phi\cos\theta & \sin\theta \\ \sin\phi & -\cos\phi & 0 \\ \cos\phi\sin\theta & \sin\phi\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos(\omega t) \\ \lambda\cos(\omega t-\gamma_1) \\ k\cos(\omega t-\gamma_2) \end{bmatrix}$$

$x_1\cos\phi_1 = B^0_{rf}(-\cos\theta\cos\phi - \lambda\cos\theta\sin\phi\cos\gamma + k\sin\theta\cos\gamma_2)$
$x_1\sin\phi_1 = B^0_{rf}(-\lambda\cos\theta\sin\phi\sin\gamma + k\sin\theta\sin\gamma_2)$
$x_2\cos\phi_2 = B^0_{rf}(\sin\phi - \lambda\cos\phi\cos\gamma_1)$
$x_2\sin\phi_2 = B^0_{rf}(-\lambda\cos\phi\sin\gamma_1)$
$x_1^2 + x_2^2 + 2x_1x_2\sin(\phi_1-\phi_2) = (B^0_{rf})^2 h(\theta,\phi,\gamma,\lambda)$
But, E will be true only along

$$\cot\phi\sin\gamma_2 = \lambda\sin(\gamma_1-\gamma_2)$$

and function h for this polarization along the line....will be

$$h(x,y,\lambda,k,\gamma_1,\gamma_2) = 1-\sin^2\theta\cos^2\phi+k^2\sin^2\theta\cos^2\gamma_2+\lambda^2\cos^2\gamma_1(1+\sin^2\phi\sin^2\theta)$$

$$+2\lambda(\cos\theta\sin{}_1-\cos\phi\sin\phi\cos\gamma_1\sin^2\theta)+2k\sin(\sin\phi\sin\gamma_2-\cos\theta\cos\phi\cos\gamma_2)+$$

$$2k\lambda\sin\theta(\cos\phi\sin(\gamma_1-\gamma_2) - \sin\phi\cos\theta\cos\gamma_1\cos\gamma_2)$$

4.2 Summary and Conclusion

Different cooling and trapping methods are discussed to achieve BEC. Ioffe Pritchard Trap was studied in detail. Different parameters are calculated to make Ioffe pritchard trap most efficient. Further, Rf dressed state theory is used to study different traps with complex potentials in detail for linear, circular and elliptical polarization. Found that in [9] experimental realization of ring trap is claimed without including the time-dependent term x_3 which is mentioned above.

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