Measurement of Earth-Moon Distance by Parallax Method Using Digital Photography

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Certificate of Examination

This is to certify that the dissertation titled "*Measurement of Earth-Moon Distance by Parallax Method Using Digital Photography*" submitted by Mr. Kanha Ram Khator (Reg. No. MS13030) for the partial fulfilment of BS-MS dual degree programme of the Institute, has been examined by the thesis committee duly appointed by the Institute. The committee finds the work done by the candidate satisfactory and recommends that the report be accepted.

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Declaration

The work presented in this dissertation has been carried out by me under the guidance of Prof. Arvind at the Indian Institute of Science Education and Research Mohali. This work has not been submitted in part or in full for a degree, a diploma, or a fellowship to any other university or institute. Whenever contributions of others are involved, every effort is made to indicate this clearly, with due acknowledgement of collaborative research and discussions. This thesis is a bonafide record of original work done by me and all sources listed within have been detailed in the bibliography.

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Dated: April 26, 2019

In my capacity as the supervisor of the candidate's project work, I certify that the above statements by the candidate are true to the best of my knowledge.

Prof. Arvind

(Supervisor)

DEDICATED TO MY PARENTS

To all free thinkers out there who can genuinely appreciate and constructively criticise this work

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Contents

Chapter 1

Introduction

It has always been an attractive and fascinating topic in physics, mathematics, and other disciplines of science to find distances; Whether be the distances between points, lines, curves, and surfaces, or the distances between places on earth and the distances to planets and stars, we humans throughout the history of science always remained fascinated to investigate length and distances. In the process of evolving methods to measure distances, today we have a lot of techniques which can efficiently measure any distance from a few micrometers or even less to the kilometers and light years. Today, in our daily life we have many critical applications of distance measuring techniques, the distance between two cities being the most common instance of it. In this work, we will be talking about a popular method of distance measurement called the parallax method.

1.1 Parallax Method

The parallax method of distance measurement is a popular technique in physics, particularly in astronomy and astrophysics. This is a common phenomenon which we observe in our routine life knowingly or unknowingly. An easy way to observe parallax is to hold a finger in front of the eyes, with a background object, let us say some mark on a wall, and seeing that finger from alternative eyes, means seeing from one eye at a time and then another. We can easily observe that the finger moves right and left with respect to the background mark, while the background does not move or moves less in comparison to the finger. It is possible for us to tell how far away the finger is from our eyes just by measuring the distance between our eyes and the distance the finger appeared to move while altering the eyes.

The other phenomenon of parallax is observed when we travel in a bus or car and look out through the window, the nearby trees and poles go back very fast, but far away objects do not seem to move or move significantly slow. Objects like the sun and the moon do not appear to move at all with our daily movements.

Analogue meters are yet another example where parallax can be observed, In an analogue ammeter, one person may see a different reading of current than other person looking from a different angle.

1.1.1 Parallax Shift

Parallax shift is the apparent shift or movement of an object against a distant background, when viewed from two different positions. A relative motion between two observers and an object causes this, while passing by from any object, one observe that it moves in opposite direction. The speed by which the object moves depends on the distance from the observer. The more the distance from observer is, the slower the object is. For a shift in observer's position, there is lesser shift in farther object comparative to the nearer object. This property of the parallax phenomenon is useful to find distances to distant objects.

1.1.2 Parallax Shift using Digital Photography

The Phenomenon of parallax can be observed with digital photos. All we need to do for it is to click two shots of an object from two different positions with same background. There will be shift in the object in the direction in which the camera was moved from one position to other. To see this clearly we can overlay the two pictures overlapping the distant background. Here is an example of visualization of parallax shift, the two photos in the example were taken from two different positions 5 meters apart.

Figure 1.1: Photo from position1

Figure 1.2: Photo from position2

This shows that the objects nearer to camera have more shift in their positions than the farther objects.

Figure 1.3: overlay of the above two pictures

Chapter 2

Measurement of Earth-Moon Distance:Various Methods

2.1 Distance from the duration of a lunar eclipse

Around 270 BC Aristarchus derived the Earth-Moon's distance from the duration of a lunar eclipse.

It was popularly accepted in those days that the Earth was a sphere and Astronomers also believed that the Earth is the center of the universe and Moon, planets, and stars including sun orbit around it. Aristarchus assumed that the Moon moves in a large circle around the Earth.

Let us know how Aristarchus led to the derivation of earth-moon distance, Let R be the radius of that circle in which the moon orbits around the earth and take the time T to go around the earth once, that is equal to about one month. In this time the Moon covers a distance of $2\pi R$, where $\pi \sim$ 3.1415926... (pronounced as "pi") is a well known constant today, the ratio (circumference/diameter) in a circle.

A lunar eclipse always occurs when the Moon passes through the shadow of the Earth, That means the moon and sun has to be in the opposite side of each other with respect to the earth and therefore it has to be a full moon. If we take the radius of the earth r, then the shadow's width is close to 2r. Let t be the time taken by the mid-point of the Moon to cross the center of the shadow of the earth, about 3 hours (observed in eclipses of the longest duration).

If the Moon orbits around Earth at a constant speed and it takes time T to cover $2\pi R \sim$ 6.28R, and time t to cover 2r, then

6.28R/2r = T/t

From this Aristarchus obtained R/r $\sim$ 60

which fits the average distance of the Moon, well accepted today, 60 tims of Earth radii.

2.2 Parallax Method

It takes a year for the Earth to complete an orbit around the Sun. The sun is 1 Astronomical unit away from the earth on an average. If we look at a star in June and again look at it in December, the shift in our position will be equal to 2 AU; We call it Baseline. A baseline is a distance between the two locations of the observer. We will see that the concerned star has shifted from its position if it is a few light years away, the stars which are too far away will not change their positions with this baseline. We can tell the distance to the star with measurement of the parallax angle. The following figure illustrates this situation

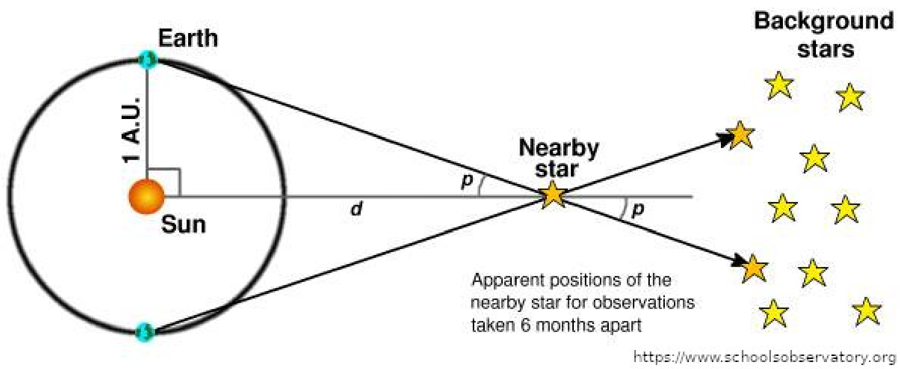


Figure 2.1: A Visualisation of Parallax Method

The apparent angular shift in the star is equal to twice the 'parallax angle' (p). Since this is a small angle, we can use the 'small angle approximation,' and conclude that tan(p) is approximately equal to the p (p in radians):

$$p\text{=Parallax Angle} \cong \tan p = \frac{\text{radius of the Earth's orbit}}{\text{distance to the star (d)}}$$

We can rearrange this formula to solve for the distance to the star:

$$\text{distance(d)=}\frac{\text{radius of the Earth's orbit(1 AU)}}{\text{Parallax Angle}(p)}$$

We get the distance in 'Parsec' if the parallax angle from the Earth at six-month intervals is measured in arcseconds, the distance to the star is in parsecs, where 1 parsec=206,265 AU = $3.0857\text{X}10^{16}$ m.

2.3 Aristarchus' Method of Determining the Relative Distances from the Earth to the Moon and to the Sun

Aristarchus (310 - 230 B.C.) figured out how to measure the relative distances from the Earth (E) to the Sun (S) and the Moon (M). When the Moon is exactly in half full phase, the angle E-M-S must be exactly 90 degrees.

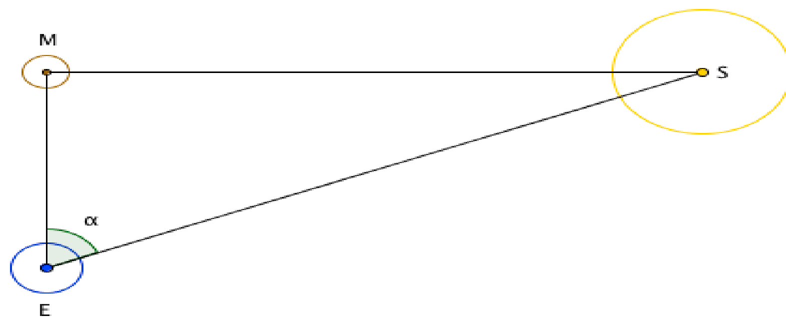


Figure 2.2: Form of EMS Triangle when the Moon is Exactly half Lit

Therefore, a measurement of angle MES, when the Moon is half full will give the ratio of the Earth-Sun distance to the Earth-Moon distance. Aristarchus measured the angle MES to be 87 degrees, giving the ratio to be 1/19. But the actual angle is 89 degrees, 51 minutes, giving a value of 1/400, means the Sun is around 400 times farther away from the Earth than the Moon.

The reason behind Aristarchus' off measurement may be that it is hard to determine the centers of the Moon and the Sun, and secondly, it is difficult to know that when the Moon is exactly half full, Although his estimate showed us that the Sun is much farther away from Earth than the Moon.

2.4 Laser Ranging Retroreflector

Astronauts on the Apollo 11(July 16, 1969- Neil Armstrong, Michael Collins, Edwin E. Aldrin Jr.), Apollo 14(January 31, 1971), and Apollo 15(26 July 1971) missions deployed retroreflectors (a bit like the cat's eyes) on the surface of the Moon. To measure that how far away the moon, physicists send a pulse of the laser to one of these reflectors and take the time this pulse takes to make the return trip which is about two and a half seconds. Since we know the speed of light, it can be worked out how far away the Moon is and to the accuracy of a few millimeters.

This project is called The Laser Ranging Retroreflector experiment, and it took many measurements every year between 1969 and 2009. The information collected has led us, for instance, to the discovery that the Moon creeps 38 mm further away from us every year. We also got to know about variations in the rotation of the Moon which are related to the distribution of mass inside it and allowed physicists to deduce that the Moon has a small liquid core.

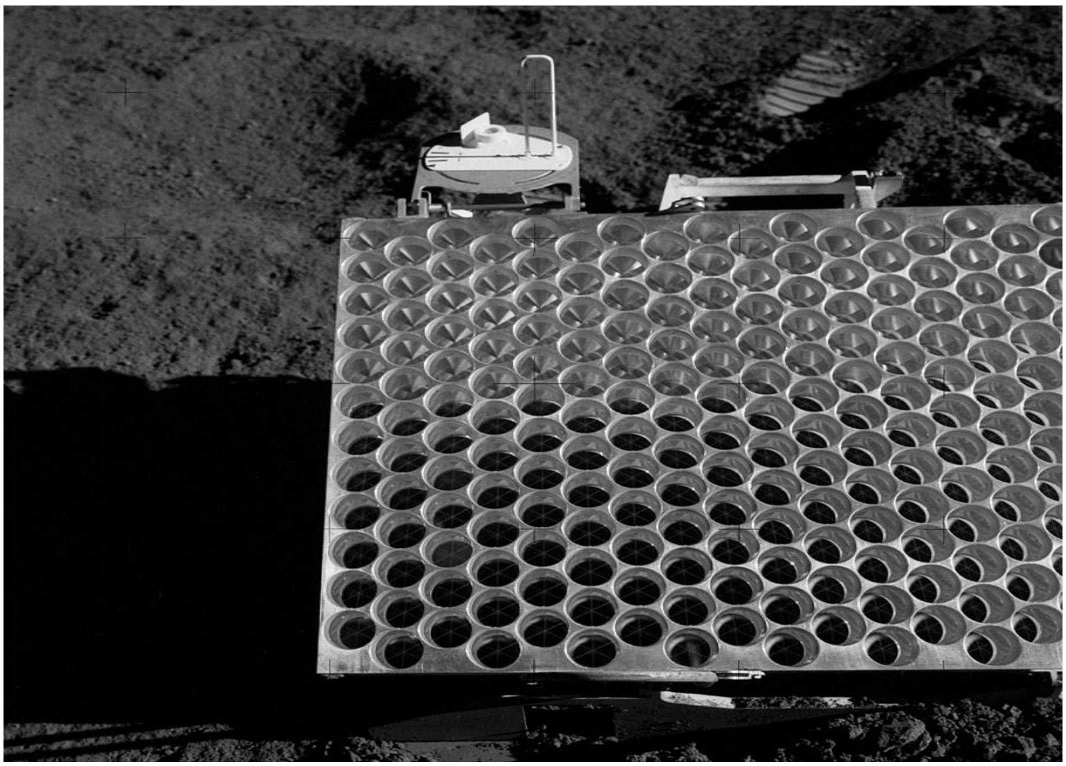


Figure 2.3: A portion of the Apollo 15 lunar laser ranging retroreflector array, as placed on the Moon and photographed by D. Scott.(Credits: NASA/D. Scott)

Chapter 3

Image processing tools: OpenCV and GIMP

3.1 OpenCV

3.1.1 Brief Introduction to OpenCV

OpenCV is a well known and multifunctional open source computer vision and machine learning software library. It was developed to provide a common infrastructure for computer vision applications and to accelerate the use of machine perception. OpenCV makes it easy for any developer to utilize and modify the code as it is a BSD-licensed(Berkeley Software Distribution) product.

OpenCV has more than 2500 optimized algorithms, comprises of both classic and state-of-the-art computer vision and machine learning algorithms. These algorithms have a wide range of uses in research in science and engineering with practical applications. Some of its applications can be listed as following: detecting and recognising faces, identifying objects, classifying human actions in videos and images, tracking camera and object movements, extracting 3D models of objects, producing 3D point clouds from stereo cameras, stitching images together to create a high resolution image of an entire scene, finding similar images from an image database, removing red eyes from images taken using flash, following eye/bodyparts movements, recognising scenery and establishing markers to overlay it with augmented reality, etc.

It has C++, Python, Java and MATLAB interfaces and supports Windows, Linux, Android, and Mac OS.

3.1.2 Edge detection:The Canny Edge Detector

The Canny edge detection algorithm was developed way by John F. Canny in 1986. The algorithm can be broken down into the following steps:

1. Smoothen the image to remove high-frequency noise using a Gaussian

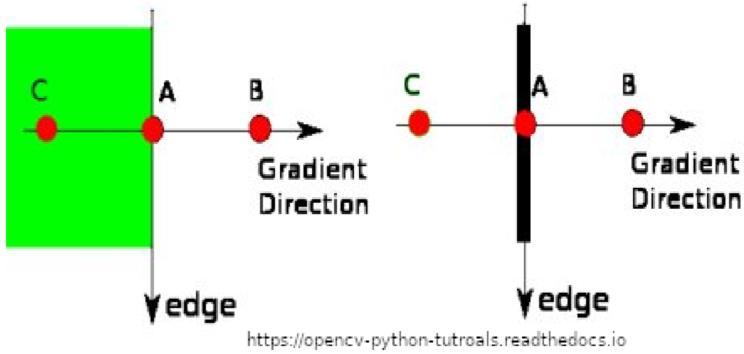


Figure 3.1: Non-maximum Suppression

filter: Edge detection is susceptible to noise in the picture. Therefore the first step is to remove the noise in the image with a Gaussian filter.

2. Computing the gradient intensity representations of the image: The smoothened image is filtered with a Sobel kernel in both vertical and horizontal direction to get the first derivative in both directions. From these two images, we can find edge gradient and direction for each pixel.

3. Applying non-maximum suppression to remove "false" responses to edge detection: After getting gradient magnitude and direction, a full scan of the image needs to be done to remove any unwanted pixels which may not constitute the edge. For this, at every pixel, pixel is checked if it is a local maximum in its neighborhood in the direction of gradient

   Point A is on edge in the vertical direction. Gradient direction is normal to the edge. Point B and C are in gradient directions. So point A is checked with point B and C to see if it forms a local maximum. If so, it is considered for the next stage. Otherwise, it is suppressed and put to zero. In Simple words, the result we get is a binary image with "thin edges".

4. Applying thresholding using a lower and upper boundary on the gradient values and Track edges using hysteresis by suppressing weak edges that are not connected to sharp edges: This stage decides which all edges are edges and which are not. For this, we need two threshold values, minVal and maxVal. Any edges with intensity gradient more than maxVal are sure to be edges, and those below minVal are sure to be non-edges, so discarded. Those who lie between these two thresholds are classified edges or non-edges based on their connectivity. If they are connected to "sure-edge" pixels, they are considered to be part of edges. Otherwise, they are also discarded.

Here is the code for the required output:

```
#Code Courtesy: https://www.pyimagesearch.com/
# import the necessary packages
import numpy as np
import argparse
import glob
import cv2

def auto_canny(image, sigma=0.33):
#A lower value of sigma  indicates a tighter
#threshold, whereas a larger value of
#sigma  gives a wider threshold.

#compute the median of the single channel pixel intensities

        v = np.median(image)

# apply automatic Canny edge detection
#using the computed median
        lower = int(max(0, (1.0 - sigma) * v))
        upper = int(min(255, (1.0 + sigma) * v))
        edged = cv2.Canny(image, lower, upper)

# return the edged image
        return edged

# construct the argument parse and parse the arguments
ap = argparse.ArgumentParser()
ap.add_argument("-i", "--images", required=True,
        help="path to input dataset of images")
args = vars(ap.parse_args())

# loop over the images
for imagePath in glob.glob(args["images"] + "/*.JPG"):
# load the image, convert it to grayscale,
#and blur it slightly
        image = cv2.imread(imagePath)
        gray = cv2.cvtColor(image, cv2.COLOR_BGR2GRAY)
        blurred = cv2.GaussianBlur(gray, (3, 3), 0)

# apply Canny edge detection using a wide threshold
        wide = cv2.Canny(blurred, 10, 200)

# show the images
        cv2.namedWindow('Edges', cv2.WINDOW_NORMAL)
        cv2.resizeWindow('Edges', 600,50)
```

```
        cv2.imshow("Edges", np.hstack([wide]))
#saving the processed image
        cv2.imwrite("new.jpg",np.hstack([wide]))
        cv2.waitKey(0)
# USAGE python auto_canny.py --images images
```

An example of output from the above code is as follows:

Figure 3.2: Original Photo

Figure 3.3: Photo after Canny edge detection

3.2 GIMP

3.2.1 Brief Introduction of GIMP

GNU Image Manipulation Program is popularly known as GIMP. It is a freely distributed program for tasks such as photo retouching, image composition, and image authoring.

3.2.2 Overlay of Photos in GIMP

Overlaying two photos is an easy task in GIMP. To find Parallax shift we need to overlay the two pictures with an overlapping background. For this purpose, we need to rotate and translate the overlaying photo. Here is the method to execute this task:

1. Open a photo in GIMP.
2. Select any two stars and find their pixel location.
3. Now find the slope of the straight line going through the two stars.
4. open the second photo in the GIMP and select the same pair of stars and find the slope
5. Find angles θ_1 and θ_2 from the above two values of slope and find the difference $\theta = \theta_1 - \theta_2$
6. Open First Picture in the Gimp and then open another photo in GIMP as a new layer
7. Set transparency around 50%, now we are able to see objects in both the images.
8. Now Rotate(Rotate Tool) the second photo by the angle to make both the straight lines(joining the selected two stars in both photos) of the same slope.
9. After the rotation we can overlap the background stars by translating(Move Tool) the overlaying photo.

3.2.3 Edge Detection Using GIMP

As we have performed edge detection to an image using OpenCV, We can also perform it using GIMP. It has an inbuilt filter namely edge detection. It comes under the filter section and has these edge detection utilities- Difference of Gaussians, Edge, Laplace, Neon, Sobel, and Image Gradient. It also has a noise reduction filter with multiple alternatives.

Chapter 4

Measuring Eath-Moon distance by Parallax shift using Digital Photos

4.1 Finding distance to a distant object with help of an object of known distance

When we have two objects in a frame and distance to one object is known, then we can calculate the distance to the unknown object.For this task to be executed we need two photos having both the concerned objects in both the photos and the photos need to be taken from two different locations. The two locations are separated by a distance which is too less in comparison to the distances we want to calculate. This separation is named as baseline.

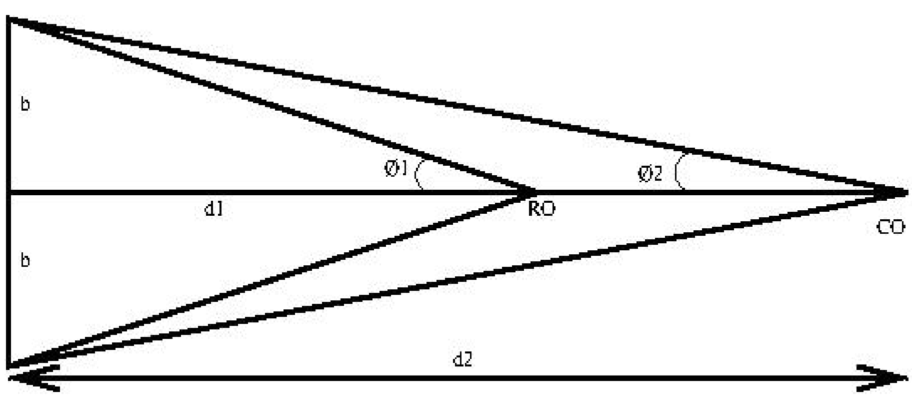


Figure 4.1: Finding Distance using Reference Object

RO= Reference Object; CO= Concerned Object.

$$\tan\theta = \frac{b}{d}$$
$$\implies \frac{b}{d} = \theta \quad \text{for } \theta \text{ being too small} \tag{4.1}$$

Hence

$$d_1 = \frac{b}{\theta_1} \quad \text{and} \quad d_2 = \frac{b}{\theta_2} \tag{4.2}$$

So we get

$$\boxed{d_1\theta_1 = d_2\theta_2} \tag{4.3}$$

But, we do not have parallax angle, here we have is apparent parallax shift in pixels. The farther the object is, the lesser is the parallax angle and the lesser is the apparent parallax shift, Means:-

$$p \propto \theta \implies p = k\theta \,\{k \text{ is some constant}\} \tag{4.4}$$

where p is the parallax shift in pixels
For two objects in consideration we will have the following:-

$$p_1 = k\theta_1 \, and \, p_2 = k\theta_2$$

where p_1 and p_2 are parallax shifts(in pixels) of reference object and concerned object respectively
using the equation-4.1 we get the following realtions:-

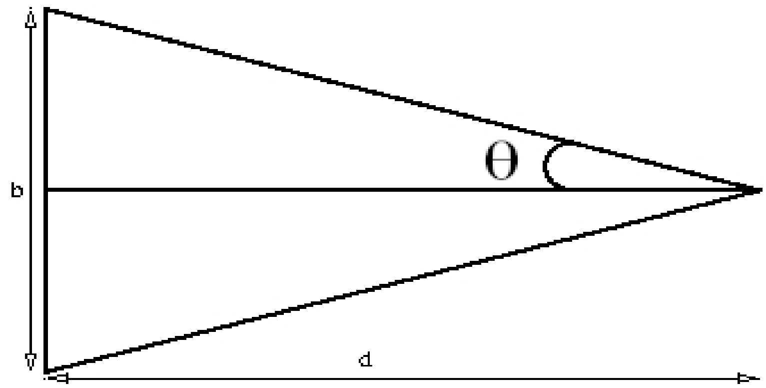
$$p_1 = k\frac{b}{d_1} \quad and \quad p_2 = k\frac{b}{d_2}$$

Hence we get the following relation:-

$$p_1 d_1 = p_2 d_2$$
$$\implies \boxed{d_2 = \tfrac{p_1 d_1}{p_2}}$$

4.2 Finding Distance to Distant object without reference object

We do not always find nearby reference objects which we can use to find distance to distant objects. to remove this bar we can find a parameter for a camera. this parameter is a constant and may vary with camera model. let us call this constant k.



$$\tan\theta = \frac{b/2}{d}$$
$$\implies \frac{b}{2d} = \theta \quad \text{for } \theta \text{ being too small} \tag{4.5}$$

The farther the object is, the lesser is the parallax angle and the lesser is the apparent parallax shift, Means:

$$p \propto \theta \implies p = k\theta \,\{k \text{ is some constant}\} \tag{4.6}$$

where p is the parallax shift in pixels thus

$$\boxed{k = \frac{2pd}{b}} \tag{4.7}$$

Let us now have a figurative visualisation of baseline and distance to the Moon:

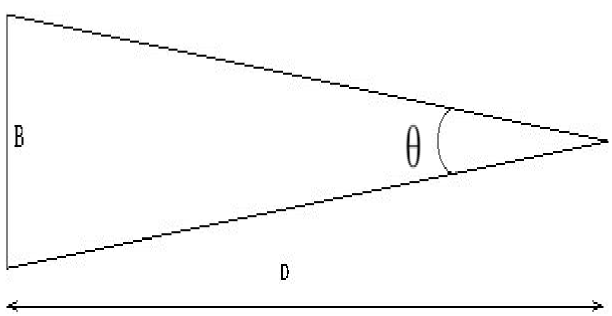


Figure 4.2:

Here B is baseline and D is the distance to the Moon. The baseline is too small comparative to the D. so we can write

$$B = D\theta$$

using equation 4.6 we have

$$B = D\frac{p}{k}$$

So we have for the distance D

$$\boxed{D = k\frac{B}{p}} \tag{4.8}$$

Here, D is distance to Moon, B is the baseline or separation between two observers, and p is the parallax shift in pixels. We need baseline and parallax shift to find distance using this constant k.

4.2.1 Finding Value of the constant k

The two Camera We used to photograph Moon are Nikon D5200 and Nikon D5300. We chose focal length to be 18mm for both the camera while clicking the photos. The camera used for Base picture was D5200 and for overlaying picture it was D5300. We found the constant k for the D5200 at 18mm. To find the k, Three objects were placed at three locations which were 20, 50, 100 meters away from the camera respectively. Now with a baseline (shift in camera's position) of 5cm two picture of three objects were clicked, overlapped and the pixel shift was measured, afterwards using equation 4.3, k was calculated. The same task was executed with a baseline of 10cm. Thus we found the value **k=12000**.

4.3 Finding Earth-Moon Distance

to find earth moon distance we need two cameras separating each other by a distance of few hundred kilometres. Moon is too far away, so we do need the baseline a few hundred kilometres long, so that we can see a shift in its position. For calculations we can use formula derived in the previous section.

following is an example where the baseline is 466 kilometres long. after overlaying the pictures we can clearly see a shift in position of the Moon..

Figure 4.3: Photo from Visitors hostel IISER Mohali

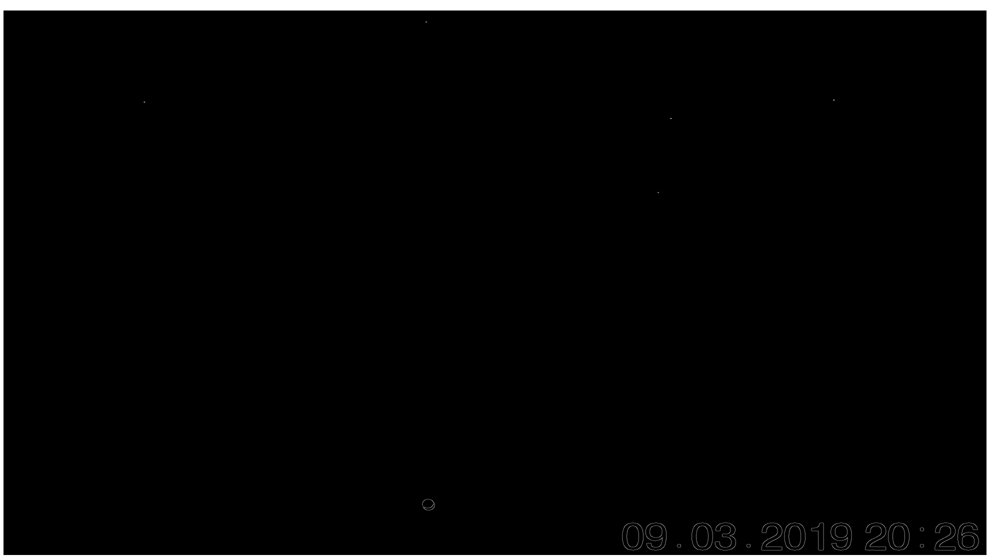


Figure 4.4: Photo from Visitors hostel IISER Mohali-after canny edge detection

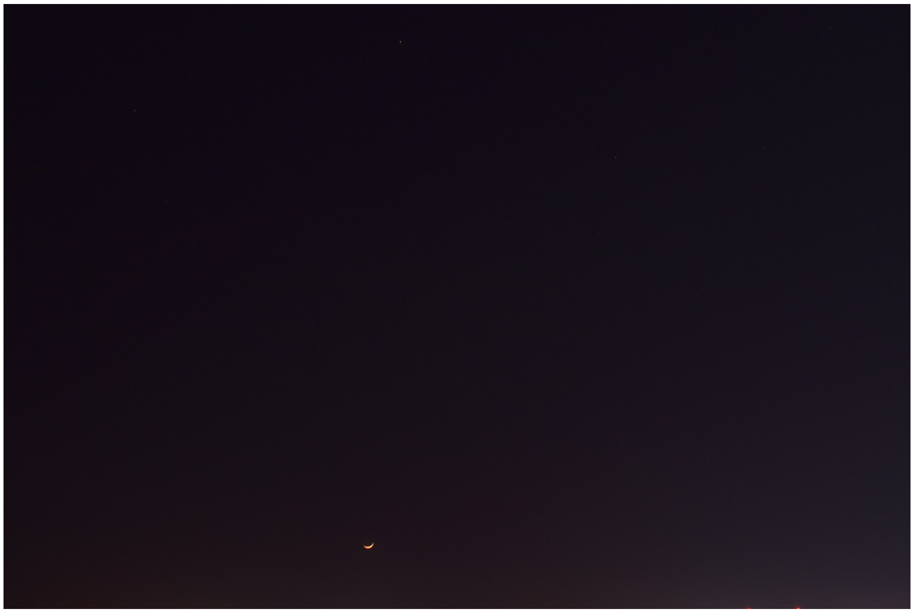


Figure 4.5: Photo from Somra, Nagaur, Rajasthan

Figure 4.6: Photo from Somra, Nagaur, Rajasthan-after canny edge detection

In the following photo we can see the shift, this shift is exactly in the direction of the baseline on the earth.

Figure 4.7: overlay - after canny edge detection(Cropped to show only useful details)

4.4 Data and Results

Here is data and their corresponding results

Table 4.1:

Sr.	Date	Time	Pix Loc-1	Pix Loc-2	Shift (in Pixels)	Distance (lakh kms)
1	9 Mar,19	20:26:01	3398,3550	3406,3538	14.42	3.88
2	9 Mar,19	20:27:28	3240,3664	3251,3655	14.21	3.94
3	9 Mar,19	20:28:15	3265,3706	3277,3697.5	14.71	3.80
4	9 Mar,19	20:28:53	3287,3731	3295,3718	15.26	3.66
5	9 Mar,19	20:30:19	3131.5,3591	3136.5,3577	14.87	3.76
6	10 Mar,19	19:50:28	3189,2617	3198.5,2607.5	13.43	4.16
7	10 Mar,19	19:53:42	3001.5,2329.5	3012.5,2320.5	14.21	3.94
8	10 Mar,19	20:07:20	3151,2558	3155.5,2547	14.53	3.85
9	10 Mar,19	20:11:43	3055.5,2051	3062,2039	13.65	4.10
10	10 Mar,19	20:15:57	3057.5,2669	3069,2662	13.46	4.15

From the above data we get an average distance of 3,92,386.68 Km with standard deviation 16,049.96 Km. So the Distance D to the Moon from the Parallax method using Digital Photography comes out to be-

$$\boxed{D = 3.92 \pm 0.16 \, \text{lakh kilometers}}$$

4.5 Sources of error

- The image contains millions of pixels, in this case 24MP, so it is difficult to find the same pixel for both the images to measure the exact pixel shift
- The moon orbits the earth at a speed of 3683km per hour, around 1km per second. It is better to have both the camera at same exposure settings(preferably at faster shutter speed) and the photos have to be clicked at a same instant strictly for a better result.
- We can reduce the error margin by increasing baseline; the farther apart the camera positions are, the better the results are.

Chapter 5

Results and Discussion

The moon is not always situated at the same distance from Earth because the orbit is not a perfect circle; it is elliptic. When the moon is the farthest away(at apogee), it is 4,05,696 kilometers (2,52,088 miles) away, and when it is closest(at perigee), the distance to it is 363105.02131 kilometers(2,25,623 miles). On average, the distance from Earth to the moon is about 3,84,400 km.[1]

There are some websites which tell us real-time Earth-Moon distance, and they also provide us the concerned distance at a particular day and time through simulations. The samples were taken on 9th March and 10th March 2019. On 9th March the actual distance to the moon from earth was 3,97,379 kilometers, and on 10th March it was 3,94,018 kilometers on average while taking samples.[2]

On 9th March the calculated Earth-Moon distance is 380795 kilometers, the error is 4.17% from the simulated value. Similarly, for the 10th March, the calculated average distance is 403978 kilometers, and the error is 2.53%.

[1] data taken from *https://spaceplace.nasa.gov/moon-distance/en/*

[2] According to simulations on *http://time.unitarium.com/moon/where.html*

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