The Nonlocality in Bipartite Quantum states

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Certificate of Examination

This is to certify that the dissertation titled **The Nonlocality in Bipartite Quantum states** submitted by **Mr.Amol Ratnaparkhe** (Reg.No.MS07005) for the partial fulfilment of BS-MS dual degree programme of the Institute, has been examined by the thesis committee duly appointed by the Institute. The committee finds the work done by the candidate satisfactory and recommends that the report be accepted.

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Declaration

The work presented in this dissertation has been carried out by me under the guidance of Dr. Pranaw Rungta at the Indian Institute of Science Education and Research Mohali.This work has not been submitted in part or in full for a degree, a diploma, or a fellowship to any other university or institute. Whenever contributions of others are involved, every effort is made to indicate this clearly, with due acknowledgement of collaborative research and discussions. This thesis is a bonafide record of original work done by me and all sources listed within have been detailed in the bibliography.

Amol Ratnaparkhe
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In my capacity as the supervisor of the candidate's project work, I certify that the above statements by the candidate are true to the best of my knowledge.

Dr. Pranaw Rungta
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Abstract

Quantum theory shows many interesting features like the uncertainty principle, entanglement or nonlocality. In order to understand these features, several attempts have been made to formulate quantum theory within a more general framework of probabilistic theories. Such a framework allows to formulate postulates and study their consequences in a general setting. In the past, generalized probabilistic theories have mostly been studied to understand the nonlocality of quantum theory.

In this thesis, we quantify the nonlocality of bipartite quantum states. More precisely, when a set of measurements is performed on a bipartite quantum state, it results in a joint probability distribution which characterizes quantum correlations. We study the nature of the correlations in terms of Bell inequalities and the Genuine inequalities that quantifies the nonlocality of the quantum correlations.

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Chapter 1

Introduction

The introduction covers mainly the basic material, that can be easily found in many sources. I have provided proper references for the material taken wherever I have used them. For the quantum physics from the foundational perspective, the books are Peres[Per95], Nielson and Chuang[NC00], Bohm[Boh51] and Bell[Bel87].

1.1 Review of Quantum Mechanics

Quantum theory can be understood as the the theoretical basis or pillar of modern physics that explains the nature and behavior of matter and energy on the atomic and subatomic level. The physical systems at these levels are known as quantum systems. So far a huge amount of theoretical predictions deriving from this theory have been confirmed by very accurate experimental data. Still, even after so many decades since its birth, many problems related to the interpretation of this theory persist: non-local effects of entangled states, wave function reduction and the concept of measurement in quantum mechanics and so on. The debate over whether quantum mechanics is a complete theory or whether it is just a statistical approximation of a deterministic theory dates to the beginning of the theory itself. Nevertheless, despite all these arguments quantum mechanics has proved itself to be a correct theory and no doubts can be raised on the validity of this theory.

It is a mathematical model of the physical world that describes the behavior of quantum systems. A physical model is characterized by how it represents *physical states*, *observables*,*measurements* and *dynamics* of the system under consideration. A quantum description of a physical model is based on the following concepts:

1.1.1 Fundamental concepts

A state is a complete description of a physical system. Quantum mechanics associates a ray in Hilbert space to the physical state of a system. What is Hilbert space?

- Hilbert space is a complex linear vector space associated with *inner product*. In Diracs ket-bra notation states are denoted by *ket vectors* $|\psi\rangle$ in Hilbert space.
- Corresponding to a *ket vector* $|\psi\rangle$ there is another kind of state vector called *bra vector*, which is denoted by $\langle\psi|$. The *inner product* of a bra $\langle\psi|$ and ket $|\psi\rangle$ is defined as follows:

$$\langle\psi|\{|\phi_1\rangle + |\phi_2\rangle\} = \langle\psi|\phi_1\rangle + \langle\psi|\phi_1\rangle \tag{1.1}$$
$$\langle\psi|\{c|\phi_1\rangle\} = c\langle\psi|\phi_1\rangle \tag{1.2}$$

  for any c $\in \mathbb{C}$, the set of complex numbers. There is a one-to-one correspondence between the bras and the kets.
- The state vectors in Hilbert space are normalized which means that the inner product of a state vector with itself gives unity, i.e.,

$$\langle\psi|\psi\rangle = 1. \tag{1.3}$$

Postulates of quantum mechanics

For an isolated quantum system, quantum theory is based on the following postulates:

- A ket vector $|\psi\rangle$ in Hilbert space gives a *complete description* of the state of the physical system.

- Dynamics are specified by *Hermitian operators* and time evolution is given by *Schrodingers equation*:

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = \hat{H}|\psi\rangle \ , \tag{1.4}$$

  where $\hat{H}$ is the *Hamiltonian operator*.

1.1.2 Qubits

In two-dimensional Hilbert space an orthonormal basis can be written as $\{|0\rangle, |1\rangle\}$. A general qubit state is then

$$|\psi\rangle = a|0\rangle + b|1\rangle \ , \tag{1.5}$$

where a, $b \in \mathbb{C}$ satisfying $|a|^2+|b|^2 = 1$. In other words, $|\psi\rangle$ is a unit vector in two-dimensional complex vector space for which a particular basis has been fixed. One of the simplest physical examples of a qubit is the spin -$\frac{1}{2}$ of an electron. The spin-up and spin-down states of an electron can be taken as the states $|0\rangle, |1\rangle$ of a qubit.

1.1.3 Quantum Measurement

The concept of measurement of a quantum state of many qubits is subtle and lies at the heart of quantum theory. The measurement postulate of quantum mechanics states:

- Mutually exclusive measurement outcomes correspond to *orthogonal projection operators* $\{P_0, P_1, ...\}$ and the probability of a particular outcome i is $\langle\psi|\hat{P}_i|\psi\rangle$. If the outcome i is attained the (normalized) quantum state after the measurement becomes

$$\frac{\hat{P}_i|\psi\rangle}{\sqrt{\langle\psi|\hat{P}_i|\psi\rangle}} \ . \tag{1.6}$$

Consider a measurement made on a qubit whose state vector resides in two-dimensional Hilbert space. A measuring device has associated an *orthonormal basis* with respect to which the quantum measurement takes place. Measurement transforms the state of the qubit into one of measuring devices associated basis vectors. Assume the measurement is performed on the qubit that has the state eq.(1.5). The measurement projects the state in eq.(1.5) to the basis $\{|0\rangle, |1\rangle\}$. Now in this case the measurement postulate says that the outcome $|0\rangle$ will happen with probability $|a|^2$ and the outcome $|1\rangle$ with probability $|a|^2$.

Furthermore, measurement of a quantum state changes the state according to the result of the measurement. That is, if the measurement of $|\psi\rangle = a|0\rangle + b|1\rangle$ results in $|0\rangle$, then the state $|\psi\rangle$ changes to $|0\rangle$ and a second measurement, with respect to the same basis, will return $|0\rangle$ with probability 1. Thus, unless the original state happened to be one of the basis vectors, measurement will change that state, and it is not possible to determine what the original state was.

Measurement made with orthogonal projection operators $\{P_0, P_1, ...\}$ is also called *projective measurement*.

1.1.4 Pure and mixed states

In quantum mechanics a pure state is defined as a quantum state that can be described by a ket vector:

$$|\psi\rangle = \sum_{k=1} c_k |\psi_k\rangle \ . \tag{1.7}$$

Such a state evolves in time according to the time-dependent Schrodinger equation. A *mixed quantum state* is a statistical mixture of pure states. In such a state the exact quantum- mechanical state of the system is not known and only the probability of the system being in a certain state can be given, which is accomplished by the *density matrix*.

1.1.5 Quantum Entanglement

Entanglement is possibly the most intriguing element of quantum theory. It plays a crucial role in quantum algorithms, quantum cryptography and the understanding of quantum mechanics itself. It enables us to perform quantum teleportation, as well as superdense coding.

Entanglement is a property shared by the quantum states for which Werner gave the explicit principle of quantum inseparability: "*If two systems interacted in the past it is possible to find the whole system in the state that cannot be written as a mixture of product states*". This principle leads to the following definition of general (pure and mixed) entangled states.

A state ρ is entangled or inseparable iff it cannot be written as a convex combination of direct-product states:

A state ρ is entangled or inseparable iff it cannot be written as a convex combination of direct-product states:

$$\rho \neq p_i \rho_A^i \otimes \rho_B^i \ , \tag{1.8}$$

with

$$\sum_i p_i = 1 \ . \tag{1.9}$$

Conversely, bi-partite states which do allow a decomposition in terms of a convex combination of product states are *separable*. The most simple examples of separable states are the direct-product states, i.e. $\rho = \rho^A \otimes \rho^B$.The convex sum of such direct-product states is the set of separable states.

Consider a pure two qubit entangled state, $|\psi\rangle_{AB}$. is said to be separated iff it can be written as $|\psi\rangle_{AB} = |\psi\rangle_A \otimes |\psi\rangle_B$ with $|\psi\rangle_A \in \mathcal{H}_A$ and $|\psi\rangle_B \in \mathcal{H}_B$. But, if a state can not be written in this fashion then it is entangled. This is a unique feature of quantum states. It is important to note that for a pure entangled state say, $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ the local states of the two subsystems, ρ_A and ρ_A are completely mixed since,

$$\rho_A = Tr[|\psi\rangle_{AB}\langle\psi|] = \frac{1}{2}(|0\rangle_A\langle 0| + |1\rangle_A\langle 1|) = \frac{\mathbb{I}_A}{2}, \tag{1.10}$$

and, similarly for ρ_B. Therefore, the two local states cannot describe the joint pure state.

1.2 The EPR and Bell's inequalities

1.2.1 EPR Argument

The existence of entanglement follows naturally from the quantum mechanical formalism. This was first made explicit in the famous paper [AER91] by Einstein, Podolsky and Rosen (EPR), where it was used to argue that quantum mechanics as a physical theory is incomplete. Their argument runs as follows. Consider a particle with known position decaying into two equal particles. Without measurement, all we know is that the particles will drift apart with opposite momenta and that their centre of mass re- mains constant throughout. Assuming that both particles are well separated, there is no way a measurement on one of the particles can affect the other particle. This is the famous *local realism* assumption, which dictates that well separated systems can be completely and independently described. Now, measuring the momentum or position of the first particle enables one to predict either the momentum or position of the second system, without disturbing it. In quantum mechanics position and momentum are non- commuting observables and the theory cannot predict precise values for both, hence EPR are led to conclude that quantum mechanics is incomplete[Boh35].

In 1951 David Bohm formulated a version of the EPR argument with an entangled spin system. Here both well separated parties (A and B) share one half of the singlet state

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)\ . \tag{1.11}$$

Measurement of the spin of the first half in any direction reveals the spin of the other particle in that direction. By an EPR argument the quantum mechanical description of the second half cannot be complete. It is for such a system Bell derived his famous inequality [AAR82].

1.2.2 Local hidden variable(LHV) model

Consider a bipartite quantum system AB. Two observers Alice and Bob are performing measurements at distant spatially separated locations A and B, respectively . The state of the system is ρ_{AB}. From the measurement postulate of quantum mechanics, the probability distribution function $P(a, b|A, B)$ is given by:

$$P(a, b|A, B) = Tr_{AB}[M_A \otimes M_B \rho_{AB}] \ , \tag{1.12}$$

where the projection operators M_A and M_B constitute POVMs with $\Sigma M_A = I$ and $\Sigma M_B = I$, respectively. The outcomes a and b are dichotomous, i.e. they can only take values ± 1. The probability distribution function $P(a, b|A, B)$ for all the possible values of the outcomes a and b constitute a set. This set is called **correlation**.

1.2.3 No-signaling principle:

Consider a measurement is performed by two observers Alice and Bob on the observables A and B at spacelike separated locations with outcomes a and b. A no-signaling correlation for two parties is a correlation such that observer 1 cannot signal to the observer 2 by the choice of what observable is measured by party 1 and vice versa. This means that the marginals $P(a|A.B)$ and $P(b|A.B)$ are independent of B and A, respectively.

$$\Sigma_b P(a, b|A, B) = P(a|A, B) = P(a|A) \ , \tag{1.13}$$

and similarly,

$$\Sigma_a P(a,b|A,B) = P(b|A,B) = P(b|B)\ . \tag{1.14}$$

The joint probability distribution $P(a,b|A.B)$ is written as

$$P(a,b|A,B) = P(a|A)P(b|B)\ . \tag{1.15}$$

Local correlations are those that can be obtained if the parties are non-communicating and share classical information, i.e., they only have local operations and local hidden variables (also called shared randomness) as a resource. We take this to mean that these correlations can be written as

$$P(a,b|A,B) = \int d\lambda p(\lambda)P(a|A,\lambda)P(b|B,\lambda). \tag{1.16}$$

where λ is the shared local hidden variable. Condition (1.16) is supposed to capture the idea of locality in a hidden-variable framework and it is called Factorisability, and models that give only local correlations are called local hidden-variable (LHV) models.

1.2.4 Bell's inequality

This idea came from the brilliant mind of Bell [Bel64]. In 1964, he proposed conditions that any *classical* theory, i.e. any theory based on local hidden variables, has to satisfy, and which can be verified experimentally. These conditions are known as *Bell inequalities*. Intuitively, Bell inequalities measure the strength of non- local correlations attainable in any classical theory. Non-local correlations arise as the result of measurements performed on a quantum system shared between two spatially separated parties. Imagine two parties, Alice and Bob, who are given access to a shared quantum state $|\psi\rangle$, but cannot communicate. In the simplest case, each of them is able to perform one of two possible measurements. Every measurement has two possible outcomes labeled ± 1. Alice and Bob now measure $|\psi\rangle$ using an independently chosen measurement setting and record their outcomes. In order to obtain an accurate estimate for the correlation between their measurement settings and the measurement outcomes, they perform this experiment independently many times using an identically prepared state ± 1 in each round.

Both classical and quantum theories impose limits on the strength of non-local correlations. In particular, both should not violate the non-signaling condition of special relativity as put forward by EPR above. That is, the local choice of the measurement setting does not allow Alice and Bob to transmit information. Limits on the strength of correlations which are possible in the framework of any classical theory are the Bell inequalities. The best known Bell inequality is the Clauser, Horne, Shimony and Holt (CHSH) inequality [JFCH69]

$$\langle CHSH\rangle_c = |\langle X_1Y_1\rangle + \langle X_1Y_2\rangle + \langle X_2Y_1\rangle - \langle X_2Y_2\rangle| \leq 2 \ , \tag{1.17}$$

where X_1 , X_2 and Y_1 , Y_2 are the observables representing the measurement settings of Alice and Bob respectively. Quantum mechanics allows for a violation of the CHSH inequality, and is thus indeed non-classical: Quantum states violate this inequality for specific measurement directions. For an appropriate setting, the CHSH expression gives

$$\langle CHSH\rangle_q = |\langle X_1Y_1\rangle + \langle X_1Y_2\rangle + \langle X_2Y_1\rangle - \langle X_2Y_2\rangle| = 2\sqrt{2}. \tag{1.18}$$

In the following chapter, I have discussed extensively about the violation of Bell-CHSH inequality by quantum states. It makes use of several different measurement settings incorporating the noncommutativity between the measured observables. Most importantly, this violation can be experimentally verified allowing us to test the validity of the theory. The first such tests were performed by Clauser and Shimony[CS78]. and Aspect, Dalibard and Roger [AAR82].

1.2.5 Tsirelson's bound

Curiously, even quantum mechanics itself still limits the strength of non-local correlations. Tsirelsons bound [Tsi80] says that for quantum mechanics

$$\langle CHSH\rangle_q = |\langle X_1Y_1\rangle + \langle X_1Y_2\rangle + \langle X_2Y_1\rangle - \langle X_2Y_2\rangle| \leq 2\sqrt{2}\,. \tag{1.19}$$

Looking at the uncertainty relations, which rest at the heart of the EPR paradox, we might suspect that the violation of the CHSH inequality of Alice and Bob. Indeed, it has been shown by Landau [Lan87], and Khalfin and Tsirelson [KT87], there exists a state $|\psi\rangle$ such that depends on the commutation relations between the local measurements

$$\begin{aligned}\langle CHSH\rangle_q &= |\langle X_1Y_1\rangle + \langle X_1Y_2\rangle + \langle X_2Y_1\rangle - \langle X_2Y_2\rangle| \qquad (1.20)\\ &= \sqrt{4 + |\langle [A_1, A_2] \otimes [B_1, B_2]\rangle|}\,.\end{aligned}$$

So, we observe that the commutation relations limit the violation of the Bell inequality for the quantum states.

Chapter 2

Monogamy of Correlations

2.1 Kinds of correlations:

The concept of correlations originates from the historic debate over the incompleteness of Quantum Mechanics started by EPR. They made a critical statement about the incompleteness of the quantum theory which was eventually disproved by John Bell. John Bell used the idea of a Local hidden variable(LHV) model and showed that the quantum correlations cannot be reproduced by any such theory. The probability distribution $P(a, b)$ can be written as

$$P(a, b|A, B) = \int d\lambda p(\lambda) A(a|A, \lambda) B(b|B, \lambda) \ . \tag{2.1}$$

2.1.1 Classical correlations:

Classical correlations are those correlations which can be reproduced by a local hidden variable theory. From the definition of the correlation, the joint probability distribution $P(a, b|A, B)$ is written as

$$P(a, b|A, B) = \int d\lambda p(\lambda) A(a|A, \lambda) B(b|B, \lambda). \tag{2.2}$$

If all the elements of the set can be reproduced in this fashion, then such correlations are called **classical** correlations. Bell inequality gives an upper bound on the correlation outcomes which can be reproduced by any local hidden variable theory.

2.1.2 No-signaling nonlocal correlations:

Any theory for which the absolute value of the correlations overshoots the upper bound of 2, is called a generalized no-signaling nonlocal theory. Such kind of correlations are called **no-signaling nonlocal** correlations. Looking at the Bell-CHSH inequality, we see that there are 4 terms and they can algebraically add up to give a maximum value of 4. So, the correlations ranging between 2 and 4 have to be no-signaling and nonlocal. An example of such correlations is in the theory of PR box proposed by Popescu and Rohlich where the correlations upto the value 4.

2.1.3 Quantum correlations:

Quantum theory is an example of a generalized no-signaling nonlocal theory since the quantum correlations form a subset of the bigger set of no-signaling nonlocal correaltions. **Quantum** correlations are the correlations between the interating systems as per the postulates of the quantum theory. The commutation relations which lie at the heart of quantum theory allow the quantum correlations to reach their maximum value of $2\sqrt{2}$ instead of 4. An example would be a Bell state, which gives the maximum violation of $2\sqrt{2}$.

2.2 Monogamy of correlations:

The term 'monogamy' in its 'actual' sense would mean that a person is allowed to marry only one woman at a time. The same concept can be applied for the no-signaling nonlocal correlations. It was coined by Ben Toner in context of the non-local correlations associated with the quantum states. It says that in a composite system $A-B-C$, if any two subsystems are non-locally correlated with each other, then the combined system has to be locally correlated with the third subsystem.

$$P(a,b,c|A,B,C) = P(a,b|A,B)P(c|C) \ . \tag{2.3}$$

Here, subsystem A is non-locally correlated with subsystem B. The third subsystem C is locally correlated with the combined system AB. Mathematically, all the no-signaling correlations that violate the Bell inequality follow a certain monogamy which follows from the inequality for a tripartite system,

$$|\langle \mathcal{B}_{AB} \rangle| + |\langle \mathcal{B}_{AC} \rangle| \leq 4 \; , \tag{2.4}$$

where $\mathcal{B}_{AB}$ refers to the Bell-CHSH inequality for the bipartite system AB and similarly, $\mathcal{B}_{AB}$ for the bipartite system AC.

So, with the monogamy of an generalized no-signaling nonlocal correlations, Toner and Verstraete came up with an even tighter monogamy relation for the quantum correlations and that is

$$\langle \mathcal{B}_{AB} \rangle_{qm}^2 + \langle \mathcal{B}_{AC} \rangle_{qm}^2 \leq 8 \; . \tag{2.5}$$

This is a tighter relation and this we can understand from the fact that the quantum correlations can reach as far as $2\sqrt{2}$ as compared to any generalized no-signaling nonlocal theory where the correlations can reach upto 4[See09].

2.2.1 Other kinds of monogamy:

The monogamy relations disscussed in this section is a part of the original work done by me and the PhD scholar, C. Jebarathinam under the supervision of Dr. Pranaw Rungta[**not published yet**].

Till now, it must have been believed that the quantum nonlocality is an intrinsic property of the two(or more) entangled subsystems under consideration. That is, it does not distinguish between the two subsystems in the sense whether subsystem 1 and 2 are in their original positions or they have been interchanged. Actually, it is true. But, it also has some constraints. Let us take a bipartite system consisting of two spin-$\frac{1}{2}$ particles as an example. It is known that quantum nonlocality can be detected only by using some specific directions which mostly makes use of the non-commutativity of the observables. This includes the local vertical mesurements(observables are anti-commuting) as well. So, it is quite reasonable to assume that the nonlocality can be detected in two different ways at the same time. One of the ways is when the particle 1 is at location A and particle 2 is at the location B. The other way is flip the two particles i.e. particle 1 goes to location B and vice versa. Then , it has been found that these two particles can never be simultanuously nonlocal in both the ways. So, this gives rise to a new monogamy relation which can be mathematically stated as

$$\langle \mathcal{B}_{AB} \rangle_{qm}^2 + \langle \mathcal{B'}_{AB} \rangle_{qm}^2 \leq 8 \ , \tag{2.6}$$

where $\mathcal{B'}_{AB}$ refers to the Bell-CHSH inequality when the particles have been interchanged.

For example, cosider a pure two qubit entangled state $|\psi\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle$. On performing a measurement on this state with the following measurement setting:

$\hat{a}_1 = (\sin\beta, 0, \cos\beta)$; $\hat{a}_2 = (-\sin\beta, 0, \cos\beta)$;

$\hat{b}_1 = (\sin\gamma, 0, \cos\gamma)$; $\hat{b}_2 = (-\cos\gamma, 0, \sin\gamma)$.

The result is $\mathcal{B} = 2\sqrt{2}\cos\gamma$ and, $\mathcal{B}' = 2\sqrt{2}\sin\gamma$ It is clear that the result we got here satisfies the monogamy relation.

There is a new momogamy relation again for the bipartite systems. The above mentioned relation was analogous to the original monogamy relation given by Toner and Verstraete for a tripartite system. The monogamy relation of local correlations, comes from the 'genuine' nonlocality which has been defined in the next chapter. It is followed by only by those states which can be reproduced classically. Therefore, the states which do not follow this monogamy relation have to be non-locally correlated. It can be mathematically stated as

$$|\langle \mathcal{S}_{AB} \rangle| + |\langle \mathcal{S'}_{AB} \rangle| \leq 2 \ , \tag{2.7}$$

where $|\langle \mathcal{S}_{AB} \rangle|$ and $|\langle \mathcal{S'}_{AB} \rangle|$ stand for $|\langle A_1 B_2 \rangle| + |\langle A_2 B_1 \rangle|$ and $|\langle A_1 B_1 \rangle| - |\langle A_2 B_2 \rangle|$, respectively.

Chapter 3

Bell Nonlocality and Quantum Theory

3.1 Bell Nonlocality

Bell-CHSH inequality has been the only inequality to detect the quantum nonlocality in the quantum entangled states as far as the bipartite systems are concerned. The Bell- CHSH inequality reads as

$$\mathcal{B}_{AB} = |\langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle| \leq 2 \ . \tag{3.1}$$

Now, regarding the Stern-Gerlach settings for the record, the settings used by Bell in the SG apparatus for the systems A and B were along the directions $\vec{a}$,$\vec{b}$ and $\vec{b}$,$\vec{c}$. We, however would follow the general settings used by CHSH where all the four measurement directions are different i.e. along $\vec{a}_1$,$\vec{a}_2$ and $\vec{b}_1$,$\vec{b}_2$. And, thus the observables can be given as,

Observables on A :

$$A_1, A_2 \qquad \rightarrow \qquad \vec{a}_1 \cdot \vec{\sigma}, \vec{a}_2 \cdot \vec{\sigma} \qquad \text{(for Alice)} \tag{3.2}$$

Observables on B :

$$B_1, B_2 \qquad \rightarrow \qquad \vec{b}_1 \cdot \vec{\sigma}, \vec{b}_2 \cdot \vec{\sigma} \qquad \text{(for Bob)} \tag{3.3}$$

CHSH expression,

$$|\langle \mathcal{B} \rangle| = |\langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle|. \tag{3.4}$$

Cirel'son expression,

$$|\langle W \rangle_\rho| = \sqrt{4 + |\langle [A_1, A_2] \otimes [B_1, B_2] \rangle_\rho|} \tag{3.5}$$

To detect quantum nonlocality in the bipartite quantum states, there are different categories of states:

- **Classically correlated states** : $\rho_{CC} = \cos^2\theta|00\rangle\langle 00| + \sin^2\theta|11\rangle\langle 11|$

  with the parameter θ varying from $\theta = 0$ to $\theta = \pi/4$.

- **Werner states**: $\rho_W = p|\Phi^+\rangle\langle\Phi| + \frac{(1-p)}{4}\mathbb{I}$

- **Schmidt state:** $|\psi\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle$

  with the parameter θ varying from $\theta = 0$ to $\theta = \pi/4$.

I have used various measurement settings to detect the nonlocality associated with these states. The results for the Schmidt states follow:

3.2 Results

3.2.1 Trivial Cases: 1-3

1. $[A_1, A_2] = 0$ (commuting) and, $[A_1, A_2] = 0$ (commuting),

   CHSH bound, $|\langle \mathcal{B} \rangle| \leq 2$

   Cirel'son bound, $|\langle W \rangle_\rho| \leq 2$

2. $[A_1, A_2] = 0$ (commuting) and, $[A_1, A_2] \neq 0$ (non-commuting),

   CHSH bound, $|\langle \mathcal{B} \rangle| \leq 2$

   Cirel'son bound, $|\langle W \rangle_\rho| \leq 2$

3. $[A_1, A_2] \neq 0$ (non-commuting) and, $[A_1, A_2] = 0$ (commuting),

   CHSH bound, $|\langle \mathcal{B} \rangle| \leq 2$

   Cirel'son bound, $|\langle W \rangle_\rho| \leq 2$

3.2.2 Non-Trivial Cases: 4-7

4. $[A_1, A_2] \neq 0$ (vertical/anti-commuting) and,

   $[B_1, B_2] \neq 0$ (vertical/anti-commuting),

   $\hat{a}_1 = (0, 0, 1)$; $\hat{a}_2 = (-1, 0, 0)$;

   $\hat{b}_1 = (-\cos\gamma, 0, \sin\gamma)$; $\hat{b}_2 = (\sin\gamma, 0, \cos\gamma)$.

   CHSH bound,

$$\begin{aligned} |\langle \mathcal{B} \rangle| &= [\cos\gamma + \sin\gamma](1 + \sin 2\theta) &\quad (3.6)\\ &\leq \sqrt{2}(1 + \sin 2\theta) \quad (\text{for } \gamma = \frac{\pi}{4}) &\quad (3.7)\\ &\leq 2\sqrt{2} \quad (\text{for } 2\theta = \frac{\pi}{2}). &\quad (3.8)\end{aligned}$$

   Cirel'son bound,

$$\begin{aligned} |\langle W \rangle_\rho| &\leq \sqrt{4 + 4\sin 2\theta} &\quad (3.9)\\ &\leq 2\sqrt{2}. &\quad (3.10)\end{aligned}$$

   It is clear from Fig.(3.1), that for this measurement setting, we get the violation of the nonlocality for all possible values of θ[PR92].

5. $[A_1, A_2] \neq 0$(vertical/anti-commuting) and,

   $[B_1, B_2] \neq 0$(non-vertical/non-commuting),

   $\hat{a}_1 = (\sin\beta, 0, \cos\beta)$; $\hat{a}_2 = (-\cos\beta, 0, \sin\beta)$;

   $\hat{b}_1 = (\sin\gamma, 0, \cos\gamma)$; $\hat{b}_2 = (-\sin\gamma, 0, \cos\gamma)$.

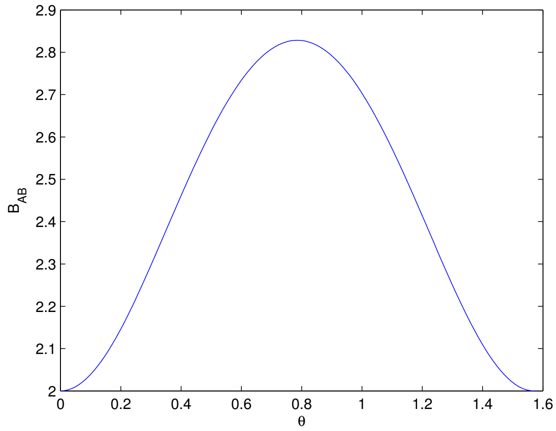

Figure 3.1: $|\mathcal{B}|$ (3.8) vs. θ

CHSH bound,

$$\begin{aligned} |\langle\mathcal{B}\rangle| &= 2\cos\beta(\cos\gamma - \sin 2\theta \sin\gamma) && (3.11)\\ &\leq 2(\cos\gamma - \sin 2\theta\sin\gamma) \quad (\text{maximum for } \beta = 0) && (3.12)\\ &\leq 2\sqrt{1+\sin^2 2\theta} \quad (\text{for } \gamma = \tan^{-1}(\sin 2\theta)). && (3.13)\end{aligned}$$

Cirel'son bound,

$$\begin{aligned} |\langle W\rangle_\rho| &\leq \sqrt{4 + 4\sin 2\gamma \sin 2\theta} && (3.14)\\ &\leq 2\sqrt{1+\sin 2\theta} \quad (\text{for } 2\gamma = \frac{\pi}{2}) && (3.15)\\ &\leq 2\sqrt{2}. && (3.16)\end{aligned}$$

6. $[A_1, A_2] \neq 0$ (non-vertical/non-commuting) and,

   $[B_1, B_2] \neq 0$ (vertical/anti-commuting),

   $\hat{a}_1 = (\sin\beta, 0, \cos\beta)$; $\hat{a}_2 = (-\sin\beta, 0, \cos\beta)$;

   $\hat{b}_1 = (\sin\gamma, 0, \cos\gamma)$; $\hat{b}_2 = (-\cos\gamma, 0, \sin\gamma)$.

CHSH bound,

$$
\begin{aligned}
|\langle \mathcal{B} \rangle| &= 2\cos\gamma(\cos\beta - \sin 2\theta \sin\beta) && (3.17)\\
&\leq 2(\cos\gamma - \sin 2\theta \sin\beta) \quad (\text{maximum for } \gamma = 0) && (3.18)\\
&\leq 2\sqrt{1+\sin^2 2\theta} \quad (\text{for } \beta = \tan^{-1}(\sin 2\theta)). && (3.19)
\end{aligned}
$$

Cirel'son bound,

$$
\begin{aligned}
|\langle W \rangle_\rho| &\leq \sqrt{4 + 4\sin 2\beta \sin 2\theta} && (3.20)\\
&\leq 2\sqrt{1+\sin 2\theta} \quad (\text{for } 2\beta = \frac{\pi}{2}) && (3.21)\\
&\leq 2\sqrt{2}. && (3.22)
\end{aligned}
$$

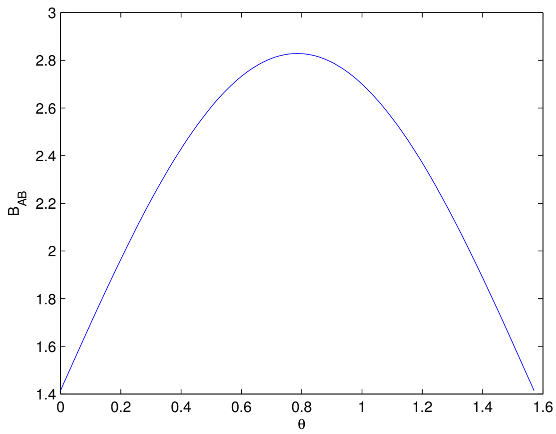

Figure 3.2: $|\mathcal{B}|$ (3.13) vs. θ

From Fig(3.2), it is observed that the local vertical measurements show some kind of hidden nonlocality. For $\sin\theta < \sqrt{2} - 1$, the nonlocality is not observed[ea08].

7. $[A_1, A_2] \neq 0$ (non-vertical/non-commuting) and,

   $[B_1, B_2] \neq 0$ (non-vertical/non-commuting),

   $\hat{a}_1 = (\sin\beta, 0, \cos\beta)$; $\hat{a}_2 = (-\sin\beta, 0, \cos\beta)$;

$\hat{b}_1 = (\sin\gamma, 0, \cos\gamma)$; $\hat{b}_2 = (-\sin\gamma, 0, \cos\gamma)$.

CHSH bound,

$$
\begin{aligned}
|\langle \mathcal{B} \rangle| &= 2\cos\beta\cos\gamma - 2\sin 2\theta \sin\beta\sin\gamma && (3.23)\\
&= 2(\cos\gamma - \sin 2\theta\sin\beta) \quad (\text{maximum for } \gamma = 0) && (3.24)\\
&\leq 2\sqrt{\cos^2\beta + \sin^2 2\theta \sin^2\beta} \quad (\text{for } \tan\beta = -(\sin 2\theta \tan\beta)). && (3.25)
\end{aligned}
$$

No violation.

Cirel'son bound,

$$
\begin{aligned}
|\langle W \rangle_\rho| &\leq \sqrt{4 + 4\sin 2\beta \sin 2\gamma \sin 2\theta} && (3.26)\\
&\leq 2\sqrt{1 + \sin 2\theta} \quad (\text{for } 2\beta = 2\gamma = \frac{\pi}{2}) && (3.27)\\
&\leq 2\sqrt{2}. && (3.28)
\end{aligned}
$$

3.3 Conclusions:

1. While Cirel'son bound can show(mathematically) violation of Bell's inequalities for every possible measurement settings for the two pairs of observables A and B, except for any one of the pairs to be commuting(or both), from repeated observations, it has been found that the violation strongly depends upon some specific measurement directions chosen.

2. The Cirel'son bound $|\langle W \rangle_\rho|$, always posseses the same form irrespective of the measurement settings, which is given by,

$$|\langle W \rangle_\rho| \leq \sqrt{4 + 4 \sin 2\beta \sin 2\gamma \sin 2\theta} \tag{3.29}$$

   where the parameters β and γ correspond to the measurement settings of the observables A and B.

3. For local vertical measurement settings, $|\langle \mathcal{B} \rangle|$ has the form,

$$|\langle \mathcal{B} \rangle| \leq \sqrt{2}(1 + \sin 2\theta), \tag{3.30}$$

   which gives the violation of the Bell's inequalities we've been looking for. For $(\sin 2\theta < \sqrt{2} - 1)$, thus, it reveals the fact that if the degree of entanglement is not enough(< 0.266), then local vertical measurements will not violate the Bell's inequalities.

4. For exactly one of the observables(either one of them) to be locally vertical and the other non-vertical(but not parallel), we have the form,

$$|\langle \mathcal{B} \rangle| \leq 2\sqrt{1 + \sin^2 2\theta}, \tag{3.31}$$

   which suggests that even a hint of entanglement will also violate the Bell's inequalities for these measurement settings unlike the case we had earlier where the degree of entanglement mattered so much so as to decide whether the settings would violate the Bell's inequalities or not.

Chapter 4

Genuine Nonlocality and Quantum theory

The genuine nonlocality disscussed in this section is a part of the original work done by me and the PhD scholar, C. Jebarathinam under the supervision of Dr. Pranaw Rungta[**not published yet**].

4.1 Limitations of Bell Nonlocality:

Although the defined Bell-CHSH inequality arising from the local hidden variable model gives nice results at least for pure entangled states, but turns out to be little bit stronger conditions as far as the mixed entangled states are concerned. The possible reason for this discrepancy could be extracted from the definition of the hidden variable model taken under consideration. The inequality arises from the bound on the local correlations governed by the joint probability distribution of the two observables. It does not take into account the marginals, whether they have been produced correctly or not. Mathematically, every element of the 'correlation' set can be written as

$$P(a,b) = \int d\lambda p(\lambda) A(a|A,\lambda) B(b|B,\lambda) \ , \tag{4.1}$$

where the dichotomous outcomes a and b correspond to the observables A and B. This is the joint probability distribution for the two observables, and the marginals for them individually can be calculated as

$$P(a) = \int d\lambda p(\lambda) A(a|A, \lambda) \tag{4.2}$$

and,

$$P(b) = \int d\lambda p(\lambda) B(b|B, \lambda) \ . \tag{4.3}$$

4.1.1 Motivation for a Genuine Nonlocality:

The whole point behind the motivation for a genuine nonlocality arised because of the marginals not being considered in the derivation of the Bell-CHSH inequality. In fact, the concept of realism seems incomplete if we leave the marginals since alongwith the joint probability distribution, the local correlations would account for the correct expectation values of the observables as well. Only then we can talk of the elements of reality associated with the system as a whole.

4.2 Genuine Nonlocality

4.2.1 The role of marginals:

Let me discuss a little bit about the hidden variable model for one more time. The joint probability distribution function $P(a, b)$ which forms the whole 'Correlation' set can be written as

$$P(a, b) = \int d\lambda p(\lambda) A(a, \lambda) B(b, \lambda) \ . \tag{4.4}$$

The marginals can individually be calculated as

$$P(a) = \int d\lambda p(\lambda) A(a, \lambda) \tag{4.5}$$

and,

$$P(b) = \int d\lambda p(\lambda) B(b, \lambda) \ . \tag{4.6}$$

Now onwards whenever we talk of correlations, the set includes the marginals as well in addition to the ususal joint probability distribution. Thus, it can be written collectively as $\{P(a,b)\}$,$\{P(a)\}$ and $\{P(b)\}$. From the above mentioned model, we can extract the required things.

$$P(a,b) = \sum_{\lambda} P(a|A,\lambda) P(b|B,\lambda) \ , \tag{4.7}$$

$$\langle AB \rangle = \sum_{\lambda} \langle A \rangle_{\lambda} \langle B \rangle_{\lambda} \ , \tag{4.8}$$

$$\langle A \rangle = \sum_{\lambda} \langle A \rangle_{\lambda} \ , \tag{4.9}$$

and,

$$\langle B \rangle = \sum_{\lambda} \langle B \rangle_{\lambda} \ . \tag{4.10}$$

The inequality is written as

$$\begin{aligned} |\langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle| &+ |\langle A_1 B_1 \rangle - \langle A_2 B_2 \rangle| && (4.11) \\ &\leq \sum_{\lambda} p_{\lambda} \{ |\langle A_1 \rangle_{\lambda} \langle B_2 \rangle_{\lambda} + \langle A_2 \rangle_{\lambda} \langle B_1 \rangle_{\lambda}| \\ &+ |\langle A_1 \rangle_{\lambda} \langle B_1 \rangle_{\lambda} - \langle A_2 \rangle_{\lambda} \langle B_2 \rangle_{\lambda}| \} && (4.12) \end{aligned}$$

For the density matrix,

$$\rho = \sum_{i,j} p_{i,j} |i\rangle\langle i| \otimes |j\rangle\langle j| \tag{4.13}$$

we have the genuine inequality as

$$
\begin{aligned}
\mathcal{G}_{AB} &=: |\langle \mathcal{S}_{AB}\rangle| + |\langle \mathcal{S'}_{AB}\rangle| - \{|\langle \mathcal{M}_{AB}\rangle| + |\langle \mathcal{M'}_{AB}\rangle|\} && (4.14)\\
&= |\langle A_1B_2\rangle + \langle A_2B_1\rangle| + |\langle A_1B_1\rangle - \langle A_2B_2\rangle| \\
&- \sum_{i,j} p_{i,j}\{|\langle A_1\rangle_i\langle B_2\rangle_j + \langle A_2\rangle_i\langle B_1\rangle_j| \\
&+ |\langle A_1\rangle_i\langle B_1\rangle_j - \langle A_2\rangle_i\langle B_2\rangle_j|\} && (4.15)\\
&\leq 0 && (4.16)
\end{aligned}
$$

where $|\langle \mathcal{S}_{AB}\rangle|$ and $|\langle \mathcal{S'}_{AB}\rangle|$ stand for $|\langle A_1B_2\rangle + \langle A_2B_1\rangle|$ and $|\langle A_1B_1\rangle - \langle A_2B_2\rangle|$ repectively.

Similarly, $|\langle \mathcal{M}_{AB}\rangle|$ and $|\langle \mathcal{M'}_{AB}\rangle|$ correspond to $\sum_{i,j} p_{i,j}|\langle A_1\rangle_i\langle B_2\rangle_j + \langle A_2\rangle_i\langle B_1\rangle_j|$ and $\sum_{i,j} p_{i,j}|\langle A_1\rangle_i\langle B_1\rangle_j - \langle A_2\rangle_i\langle B_2\rangle_j|$ repectively.

Here, the positive terms correpond to the total(local + nonlocal) correlations and the negative terms account only for the local correlations. From this inequality, we can infer that the first part comprises of the local + nonlocal content of the correlations. From that, we are subtracting the local content of the correlations and thus we are left with what is called the nonlocal contribution to the total correlations. Till the inequality is satisfied, the correlations can be reproduced by a local hidden variable model but, as it goes beyond the mark of 0, the nonlocal correlations start coming into the picture. We shall show how exactly the inequality works using various kinds of entangled states

4.2.2 Classically correlated states

The density matrix for the classically correlated states is

$$\rho_{CC} = \cos^2\theta|00\rangle\langle 00| + \sin^2\theta|11\rangle\langle 11| \;, \tag{4.17}$$

with

$$\langle AB \rangle = \cos^2\theta \, \langle A \rangle_{|0\rangle} \, \langle B \rangle_{|0\rangle} + \sin^2\theta \, \langle A \rangle_{|1\rangle} \, \langle B \rangle_{|1\rangle} \ . \tag{4.18}$$

The product density matrix which would contribute solely to the local correlations is given by

$$\begin{aligned} \rho &= \rho_A \otimes \rho_B \qquad (4.19) \\ &= \cos^4\theta|00\rangle\langle 00| + \cos^2\theta\sin^2\theta\{|01\rangle\langle 01| + |10\rangle\langle 10|\} + \sin^4\theta|11\rangle\langle 11| \ . \end{aligned}$$

For a suitable measurement setting
$\hat{a}_1 = \hat{z}$, $\hat{a}_2 = \hat{z}$, $\hat{b}_1 = \hat{z}$ and $\hat{b}_2 = \hat{z}$ we have,

$$\mathcal{G}_{AB} = 0 \ . \tag{4.20}$$

So, the inequality obviously does not get violated as we expected from a classically correlated state.

4.2.3 Schmidt states

The state can be written as

$$|\phi^+\rangle = \cos\theta \, |00\rangle + \sin\theta \, |11\rangle \ . \tag{4.21}$$

The product density matrix which would contribute solely to the local correlations is given by

$$\begin{aligned} \rho_{prod} &= \rho_A \otimes \rho_B \\ &= \cos^4\theta|00\rangle\langle 00| + \cos^2\theta\sin^2\theta\{|01\rangle\langle 01| + |10\rangle\langle 10|\} \\ &\quad + \sin^4\theta|11\rangle\langle 11| \ . \end{aligned} \tag{4.22}$$

These are some of the results for various measurement settings

1. For $\hat{a}_1 = \hat{z}$, $\hat{a}_2 = \hat{x}$, $\hat{b}_1 = \cos t\hat{z} + \sin t\hat{x}$ and $\hat{b}_2 = \cos t\hat{z} - \sin t\hat{x}$

$$\mathcal{G}_{AB} = |2\sqrt{1+\sin^2 2\theta}| - |\frac{2}{\sqrt{1+\sin^2 2\theta}}| \ . \tag{4.23}$$

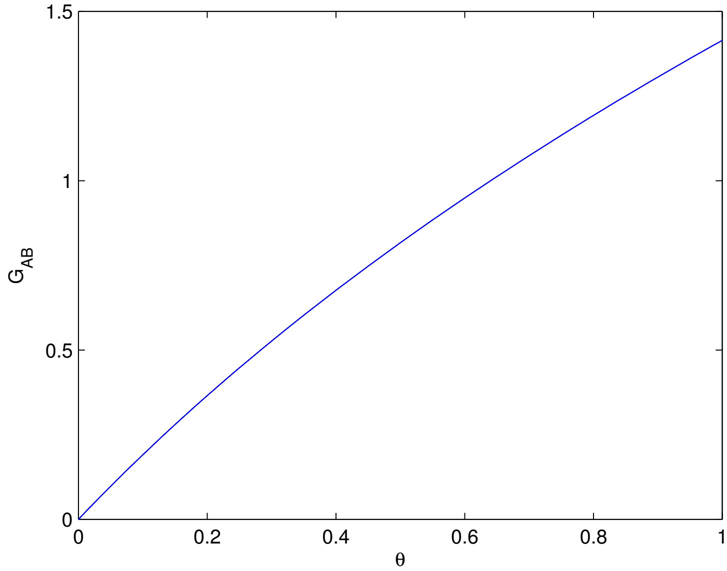

Figure 4.1: $\mathcal{G}_{AB}$.(4.23) vs τ

Here, the genuine inequality $\mathcal{G}_{\mathcal{AB}}$ has been plotted against τ where $\tau = \sin^2 2\theta$ is an entanglement measure.

2. For $\hat{a}_1 = \hat{x}$, $\hat{a}_2 = \hat{y}$, $\hat{b}_1 = \frac{1}{\sqrt{2}}(\hat{x} - \hat{y})$ and $\hat{b}_2 = \frac{1}{\sqrt{2}}(\hat{x} + \hat{y})$

$$\mathcal{G}_{AB} \ = \ 2\sqrt{2}\sin 2\theta \ . \tag{4.24}$$

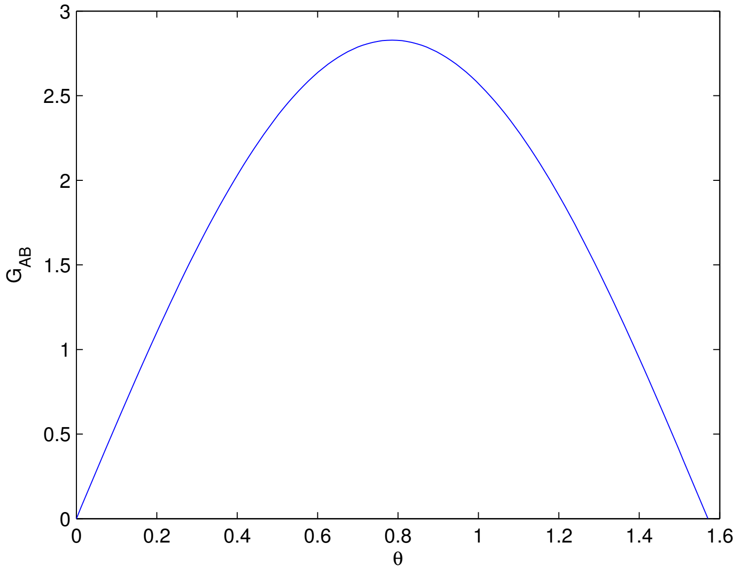

Figure 4.2: $\mathcal{G}_{AB}$ (4.24) vs τ

3. For $\hat{a}_1 = \hat{x}$, $\hat{a}_2 = \hat{y}$, $\hat{b}_1 = \hat{x}$ and $\hat{b}_2 = \hat{y}$

$$\mathcal{G}_{AB} = 2 \sin 2\theta \ . \tag{4.25}$$

4. For $\hat{a}_1 = \cos\theta \hat{x} + \sin\theta \hat{y}$, $\hat{a}_2 = -\cos\theta \hat{y} + \sin\theta \hat{x}$, $\hat{b}_1 = \cos\theta \hat{x} + \sin\theta \hat{y}$ and $\hat{b}_2 = -\cos\theta \hat{y} + \sin\theta \hat{x}$

$$\mathcal{G}_{AB} = 2 \sin 2\theta (\sin 2\theta + \cos 2\theta) \ . \tag{4.26}$$

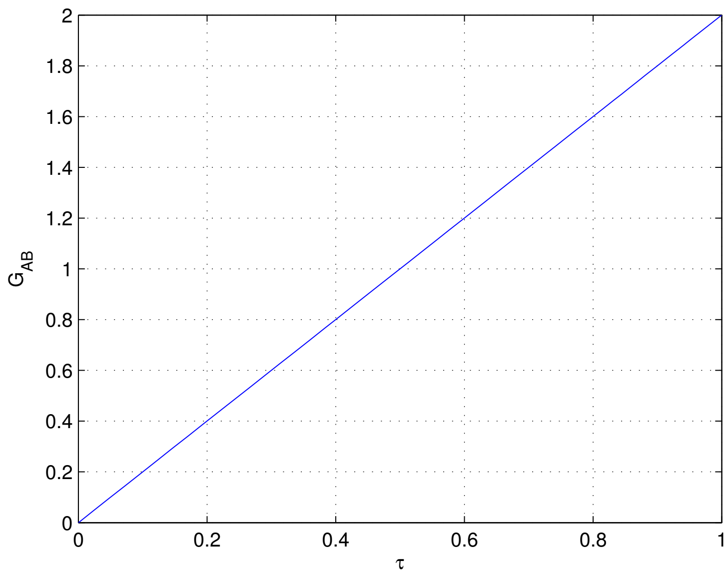

Figure 4.3: $\mathcal{G}_{AB}$ (4.25) vs τ

4.2.4 Werner states

The density matrix is given by

$$\rho_W = p|\Phi^+\rangle\langle\Phi^+| + (1-p)\frac{\mathbb{I}}{4}\ . \tag{4.27}$$

The measure of entanglement, concurrence is

$$C(\rho_W) = \max\{0, \frac{3p-1}{2}\}\ . \tag{4.28}$$

At $p = 1/2$,

$$C(\rho_W) = \frac{1}{4}\ .$$

The product density matrix comes out to be

$$\rho_{prod} = \frac{\mathbb{I}}{4} \tag{4.29}$$

- $\forall\ A_1, A_2, B_1 \& B_2$, $\mathcal{G}_{AB} > 0$ iff $C(\rho_W) > 1/4$.

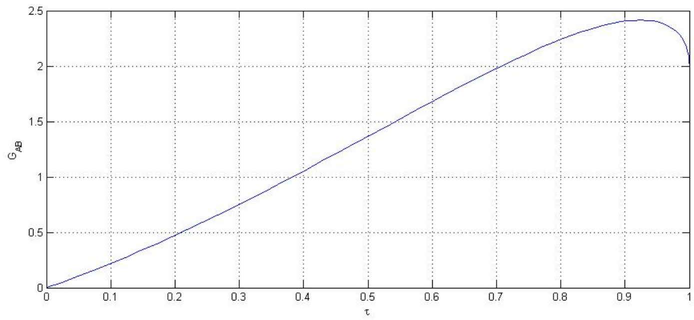

Figure 4.4: $\mathcal{G}_{AB}$ (4.26) vs τ

- $\forall\ A_1, A_2, B_1 \& B_2$, $\mathcal{G}_{AB} + 1/3 > 0$ iff $p > 1/3$.

Here are some nice results for various measurement settings to detect the genuine nonlocality.

1. For $\hat{a}_1 = \hat{x}$, $\hat{a}_2 = \hat{z}$, $\hat{b}_1 = \hat{x}$ and $\hat{b}_2 = -\hat{z}$

$$\mathcal{G}_{AB} \quad = \quad 2p - 1 \ . \tag{4.30}$$

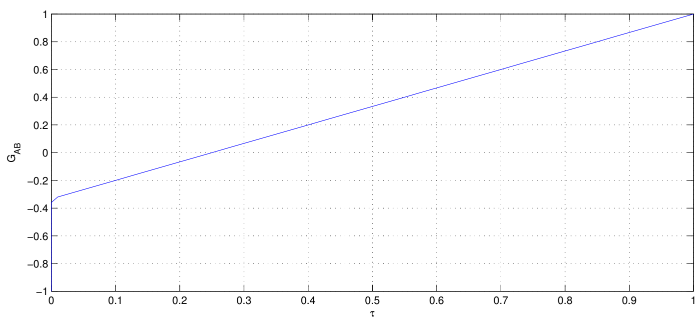

Figure 4.5: $\mathcal{G}_{AB}$ (4.30) vs C_{AB}

2. For $\hat{a}_1 = \hat{z}$, $\hat{a}_2 = \hat{x}$, $\hat{c}_1 = \sqrt{p}\hat{z} + \sqrt{1-p}\hat{x}$ and $\hat{c}_2 = \sqrt{1-p}\hat{z} - \sqrt{p}\hat{x}$

$$\mathcal{G}_{AC} = (\sqrt{p} + \sqrt{1-p})(2p-1) \ . \tag{4.31}$$

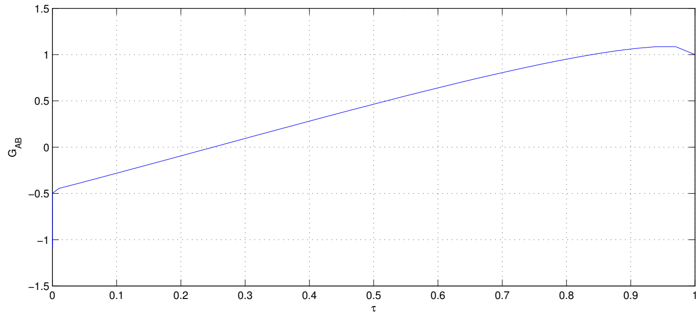

Figure 4.6: $\mathcal{G}_{AB}$ (4.31) vs. C_{AB}

3. For $\hat{a}_1 = \sqrt{p}\hat{z} + \sqrt{1-p}\hat{y}$, $\hat{a}_2 = -\sqrt{p}\hat{y} + \sqrt{1-p}\hat{z}$, $\hat{b}_1 = \sqrt{p}\hat{z} + \sqrt{1-p}\hat{y}$ and $\hat{b}_2 = -\sqrt{p}\hat{y} + \sqrt{1-p}\hat{z}$

$$\mathcal{G}_{AB} \quad = \quad (2p-1)(|2\sqrt{p(1-p)}| + |(2p-1)|) \ . \tag{4.32}$$

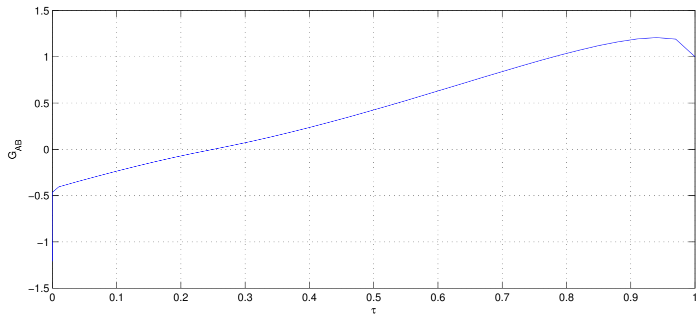

Figure 4.7: $\mathcal{G}_{AB}$ (4.32) vs. C_{AB}

4. For $\hat{a}_1 = \hat{z}$, $\hat{a}_2 = \hat{x}$, $\hat{b}_1 = \frac{1}{\sqrt{2}}(\hat{z} - \hat{x})$ and $\hat{b}_2 = \frac{1}{\sqrt{2}}(\hat{z} + \hat{x})$

$$\mathcal{G}_{AB} = \sqrt{2}(2p - 1) \ . \tag{4.33}$$

5. For $\hat{a}_1 = \hat{x}$, $\hat{a}_2 = \hat{y}$, $\hat{b}_1 = \frac{1}{\sqrt{2}}(\hat{x} - \hat{y})$ and $\hat{b}_2 = \frac{1}{\sqrt{2}}(\hat{x} + \hat{y})$

$$\mathcal{G}_{AB} = 2\sqrt{2}p \ . \tag{4.34}$$

6. For $\hat{a}_1 = \sqrt{p}\hat{x} + \sqrt{1-p}\hat{y}$, $\hat{a}_2 = -\sqrt{p}\hat{y} + \sqrt{1-p}\hat{x}$, $\hat{b}_1 = \sqrt{p}\hat{x} + \sqrt{1-p}\hat{y}$ and $\hat{b}_2 = -\sqrt{p}\hat{y} + \sqrt{1-p}\hat{x}$

$$\mathcal{G}_{AB} = 2p(|2\sqrt{p(1-p)}| + |(2p-1)|) \ . \tag{4.35}$$

4.2.5 Guhne *et al.* states

The density matrix for this state[ea06] would be given by ,

$$\rho = p \left|\psi^-\right\rangle \left\langle\psi^-\right| + (1-p)\rho_{sep} \tag{4.36}$$

where $\rho_{sep} = \frac{2}{3}|00\rangle\langle 00| + \frac{1}{3}|01\rangle\langle 01|$.

And, the product density matrix is given by

$$\rho_{prod} = \{\frac{p}{2}\mathbb{I} + (1-p)|0\rangle\langle 0|\} \otimes \{\frac{p}{2}\mathbb{I} + (1-p)(2/3|0\rangle\langle 0| + 1/3|1\rangle\langle 1|)\} \tag{4.37}$$

The results for few of the measurement settings are as follows:

1. For $\hat{a}_1 = \hat{x}$, $\hat{a}_2 = \hat{z}$, $\hat{b}_1 = \hat{x}$ and $\hat{b}_2 = -\hat{z}$

$$\mathcal{G}_{AB} = |\frac{1-7p}{3}| - 1 \ . \tag{4.38}$$

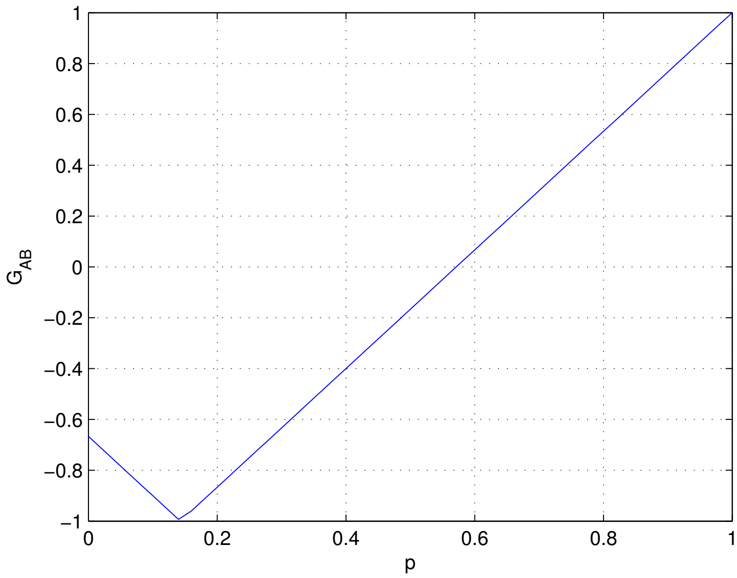

Figure 4.8: $\mathcal{G}_{AB}$ (4.38) vs p

2. For $\hat{a}_1 = \hat{z}$, $\hat{a}_2 = \hat{x}$, $\hat{b}_1 = \frac{1}{\sqrt{2}}(\hat{z} + \hat{x})$ and $\hat{b}_2 = \frac{1}{\sqrt{2}}(\hat{z} - \hat{x})$

$$\mathcal{G}_{AB} = \sqrt{2}(|\frac{1-7p}{3}| - 1) \; . \tag{4.39}$$

$\mathcal{G}_{AB} > 0$ if $p > 4/7$.

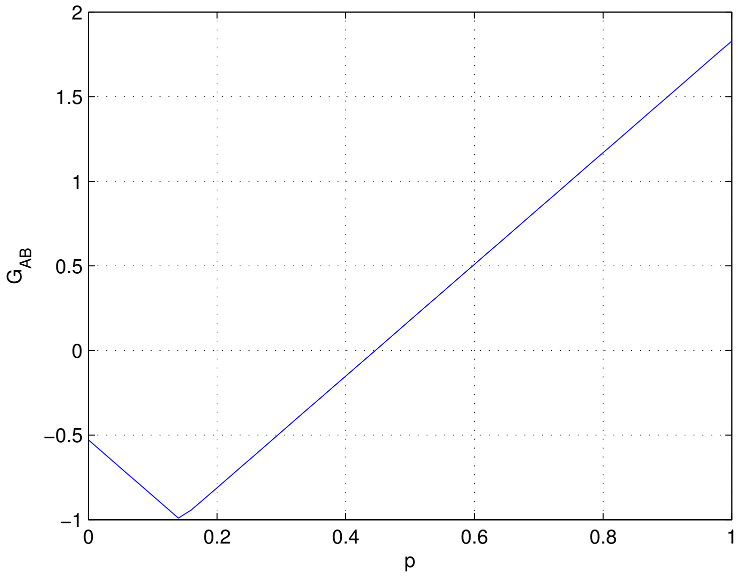

Figure 4.9: $\mathcal{G}_{AB}$ (4.39) vs. p

4.2.6 Discordant states

The density matrix is given by

$$\begin{aligned}\rho_{AB} &= \lambda_0|0\rangle\langle 0| \otimes |+\rangle\langle +| + \lambda_1|1\rangle\langle 1| \otimes |-\rangle\langle -| + \lambda_2|+\rangle\langle +| \otimes |1\rangle\langle 1| \\ &+ \lambda_3|-\rangle\langle -| \otimes |0\rangle\langle 0| \,. \end{aligned} \tag{4.40}$$

and, the product density matrix is given by

$$\begin{aligned}\rho_{prod} &= \lambda_0|0\rangle\langle 0| \otimes (\lambda_3|0\rangle\langle 0| + \lambda_2|1\rangle\langle 1| + \lambda_1|-\rangle\langle -| + \lambda_0|+\rangle\langle +|) \\ &+ \lambda_1|1\rangle\langle 1| \otimes (\lambda_3|0\rangle\langle 0| + \lambda_2|1\rangle\langle 1| + \lambda_1|-\rangle\langle -| + \lambda_0|+\rangle\langle +|) \\ &+ \lambda_2|+\rangle\langle +| \otimes (\lambda_3|0\rangle\langle 0| + \lambda_2|1\rangle\langle 1| + \lambda_1|-\rangle\langle -| + \lambda_0|+\rangle\langle +|) \\ &+ \lambda_3|-\rangle\langle -| \otimes (\lambda_3|0\rangle\langle 0| + \lambda_2|1\rangle\langle 1| + \lambda_1|-\rangle\langle -| + \lambda_0|+\rangle\langle +|) \end{aligned} \tag{4.41}$$

For $\hat{a}_1 = \hat{z}$, $\hat{a}_2 = \hat{x}$, $\hat{b}_1 = -\hat{z}$ and $\hat{b}_2 = \hat{x}$

$$
\begin{aligned}
\mathcal{G}_{AB} &= \lambda_0 + \lambda_1 + \lambda_2 + \lambda_3 \\
&- |\lambda_0^2 + \lambda_1^2 + \lambda_2^2 + \lambda_3^2 + 2\lambda_0\lambda_1 + 2\lambda_2\lambda_3| \\
&- |2(\lambda_0\lambda_2 + \lambda_0\lambda_3 + \lambda_1\lambda_2 + \lambda_1\lambda_3)| \,. \qquad (4.42)
\end{aligned}
$$

It is also a separable state and as expected, it also does not violate the genuine inequality like the classically correlated state.

Chapter 5

Conclusion

In this thesis, we have investigated various aspects of nonlocality, particularly the bipartite quantum systems. Violation of Bell inequality through performing various experiments on quantum system is a signature of nonlocality. This was the main subject of the first part of my thesis. In particular, we have mainly discussed about the no-signaling nonlocal correlations. More specifically, we have studied quantum correlations which are actually responsible for the nonlocality arising in the entangled states. Quantum Theory is an example of any generalized no-signaling nonlocal theory and the quantum correlations are a subset of the no-signaling nonlocal correlations. While the nonlocal correlations in the PR Box reach as far as 4, the postulates of quantum mechanics limit the reach of quantum correlations upto $2\sqrt{2}$. Apart from this, the historic debate over the issue of the incompleteness of the quantum theory proposed by EPR has also been discussed.

In the following chapter, first I have discussed the known results about the monogamy of correlations including the tighter bound on the quantum correlations. Apart from the usual monogamy relations, I have tried to incorporate two new monogamy relations. The first one among them, the monogamy relation for bipartite system deals with impossibility to achieve nonlocality in two different ways. In this relation, I have showed that it is impossible to violate both the Bell inequalities, $|\langle \mathcal{B}_{AB} \rangle|$ and the other Bell inequality, $|\langle \mathcal{B}'_{AB} \rangle|$ which we get by interchanging the two observers, simultaneously.

Either you can violate the first inequality or the other inequality, but noth both at the same time. Moreover, if one of the inequalities is maximally violated, then, the other inequality results in completely local correlations. The other monogamy relation is actually a monogamy relation for the local correlations. This relation has been generated from the genuine inequality. And, this relation is strictly followed by the classical or local correlations. Any state with a smallest amount of nonlocal correlations will not follow this monogamy relation.

The following chapter is fully devoted to the nonlocality and quantum theory. The first part of the chapter is dedicated to the understanding of the Bell nonlocality and the quantum theory. We studied many different classes of entangled states. From the pure Schmidt state to the different classes of mixed entangled states like the Werner state or the Guhne et.al state, we have tried working out various properties of the observables and the measurement settings, be it the non-commutativity or the local vertical measurements, we have analyzed every aspect of entanglement due to these properties and how does it actually work to detect the nonlocality.

The last part of the thesis is devoted to the topic of genuine nonlocality and quantum theory. The basic need to arive at this inequality comes from the principle of realism. A local hidden variable model is assumed to reproduce all the local correlations which include the joint probability distribution for any given quantum state. This joint probability distribution is collectively known as correlation. But, as a property of the hidden variable, which is also an element of reality, it should also be able to reproduce the correct marginals. Then only, we can surely say that the hidden variable shared among two observers is an element of reality. While formulating the Bell inequality, only joint probability distribution was taken into consideration and the marginals were left as it is. So, the Bell inequality is a little stronger condition in some sense. This statement gets easily verified when we consider two different mixed entangled states, Werner state and the Guhne *et al.* state. Bell inequality starts getting violated in both the cases at $p = \frac{1}{\sqrt{2}}$ while in case of the genuine inequality, Werner state violates it from $p = \frac{1}{\sqrt{2}}$ but the other state starts violating the genuine inequality at $p = \frac{4}{7}$. So, we can easily observe that the Bell inequality does not make any distinction between the two states in the range of $p = \frac{4}{7}$ to $p = \frac{1}{\sqrt{2}}$.

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