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Markov Chain Monte Carlo Methods in Quantum Field Theories

A Modern Primer

Springer

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To my parents Ouseppachan and Elyakutty Velliyamkandathil with gratitude

Foreword

This book may well be the most concise and pedagogical introduction to Monte Carlo methods in Quantum Field Theory available in the market today. Beginning with simple examples of how to numerically integrate ordinary functions in a small number of dimensions, the book gradually warms up the reader to the idea of using random numbers to estimate integrals. These 'Monte Carlo' ideas are easily tested in low-dimensional cases, but they stand without competition when the dimensionality becomes large.

Quantum Field Theory in its path-integral formulation is an example of where even infinitely-dimensional integrals can be needed in order to compute. The trick is to return to the very definition of the path integral, chop up space and time in small discretized intervals and enclose everything in a finite volume. The result is very large-dimensional integral that can be evaluated approximately by Monte Carlo techniques. This sounds simple, but the reader is warned along the way regarding potential difficulties or even misleading estimates. Methods are tested in great detail on analytically solvable cases, drawing on modern examples that also include supersymmetry. The book ends with the most important example for relativistic field theories: that of gauge fields defined on a space-time lattice. In an outlook, the reader is given a peek at what may lie ahead of new developments based on machine learning. Finally, a full set of C++ programs are included so that no student will be lost.

Anosh Joseph has the advantage of coming from the main analytical approach to Quantum Field Theory while having continually worked hand in hand with numerical techniques. This gives him a fresh look on the matter and it enables him to draw on example that standard practitioners of Monte Carlo methods in, say, lattice gauge theory may not be aware of. The book is delightfully well written and is bound to please students who would like to learn the subject from the scratch.

Copenhagen, Denmark
March 2020

Poul H. Damgaard
Professor Theoretical Physics, Niels Bohr Institute, Director, Niels Bohr International Academy

Preface

Quantum field theory is a tool to understand a vast array of perturbative and non-perturbative phenomena found in physical systems. Some of the most interesting features of quantum field theories, such as spontaneous symmetry breaking, phase transitions, and bound states of particles, demand computational tools beyond the machinery of ordinary perturbation theory. Monte Carlo methods using Markov chain based sampling algorithms provide powerful tools for carrying out such explorations.

We can use lattice regularized quantum field theories and simulation algorithms based on Monte Carlo methods to reveal the non-perturbative structure of many interesting quantum field theories, including Quantum Chromodynamics (QCD), the theory of strong interactions. The rapidly developing field of Machine Learning could provide novel tools to find phase structures and order parameters of systems where they are hard to identify.

This book contains 9 chapters. In Chap. 1 we discuss various simple methods of numerical integration, including the rectangle rule, midpoint rule, trapezoidal rule, and Simpson's rule. Random numbers are introduced next. We discuss pseudo-random numbers and how they can be generated on a computer using a seed number. After that, we move on to discuss the Monte Carlo method for numerical integration. We also discuss how to compute the error in Monte Carlo integration, the questions on when Monte Carlo is useful for integration and when it can fail. In Chap. 2 we discuss Monte Carlo with importance sampling and how it reduces the variance of the Monte Carlo estimate of the given integral. In Chap. 3 we introduce Markov chains and discuss their properties and convergence to the unique equilibrium distribution when the chain is irreducible and aperiodic. In Chap. 4 we introduce Markov chain Monte Carlo. Concepts such as Metropolis algorithm and thermalization of Markov chains are introduced. In Chap. 5 we discuss the connection between Markov chain Monte Carlo and Feynman path integrals of Euclidean quantum field theories. We also numerically study a zero-dimensional quantum field theory that undergoes dynamical supersymmetry breaking, one-dimensional simple harmonic oscillator, and a unitary matrix model that undergoes Gross-Witten-Wadia phase transition. In Chap. 6 we discuss the reliability of Monte Carlo simulations and introduce the idea of auto-correlation time in the observables. The method of Hybrid (Hamiltonian) Monte Carlo is discussed next in Chap. 7. There, we look at the properties of Hamiltonian dynamics and how the Leapfrog integration method can be used to evolve the system in simulation time. We then apply Hamiltonian Monte Carlo to a Gaussian model and a zero-dimensional supersymmetric model. In Chap. 8 we briefly discuss how Markov chain Monte Carlo can be used to extract physics from quantum field theories formulated on a spacetime lattice. In Chap. 9 we discuss how Machine Learning and quantum field theory can work together to further our understanding of the nature of the physical systems we are interested in. This book ends with several appendices containing various C++ programs that were used to generate data and numerical results provided in this book.

Mohali, India
February 2020

Anosh Joseph

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