Department of Physics

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To Cyclotron Maser Instability

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Declaration

I, Dhruv Pandya, declare that this thesis titled as "To Cyclotron Maser Instability" and work presented in it are my own and I take full scientific responsibility of the claims asserted in the thesis. The work was done at Indian Institute of Science Education and Research Mohali under the guidance of Prof. Jasjeet Singh Bagla.

The work has been submitted to fulfill partial requirements of the BS-MS degree offered by Indian Institute of Science Education and Research Mohali. The work presented here is my original and effort has been made to give credits to relevant people whose contributions are involved.

signed:

(Dhruv Pandya , Candidate)

Certificate of Examination

This is to certify that the dissertation titled **To Cyclotron Maser Instability** submitted by **Dhruv Pandya (MS16095)** for the partial fullfilment of BS-MS dual degree programme of Indian Institute of Science Education and Research Mohali, has been examined by the committee duly appointed by the institute. The committee finds the work done by the candidate satisfactory and recommends that the report be accepted.

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Abstract

The present thesis work is motivated by the detection of MCVs in 2018. Radiation detected from them was circularly polarized, had narrow bandwidth, had very high brightness temperature. Such radiation profile can be explained using the model of cyclotron maser instability. Present thesis work is an attempt to understand cyclotron maser in the context of MCVs. A novel Approach to obtain angular and spectral profile of the radiation from an accelerated particle using Maxwell's equations on curved space time is discussed. Vlasov equation and a variant of H theorem for collisionless plasma are independently derived.

Contents

1 Introduction

The Magnetic Cataclysmic Variables (MCVs) are binary star systems which contain a white dwarf with magnetic field strong enough to control the accretion flow from a late type secondary which is called the donor star. An accretion disk is formed (see Figure 1) as matter is flowing towards the white dwarf. Magnetohydrodynamic equations govern the physics of plasma found in the accretion disk. The Plasma found here is dense and collisional. Plasma in the corona near the pole of the donor star is dilute as it has been shown in [11]. The plasma that predominantly emits in the optical, ultraviolet and X-ray emission can also emit radio emission through various processes. Most of them are relativistic processes. Antenna mechanisms, reactive instabilities and maser instabilities drive plasma emission processes ([13] , [14]). Observations of radio emission from 33 MCVs carried out with the Karl G. Jansky Very Large Array (VLA) at (C-, X-, and K-bands; 4–6, 8–10, and 20–22 GHz, respectively) at full polarization have recently been reported by Barrett et al.(see [1]), They reported that 24 of these MCVs with radio fluxes in the range of 6-8031Jy showed highly circularly polarized emission.

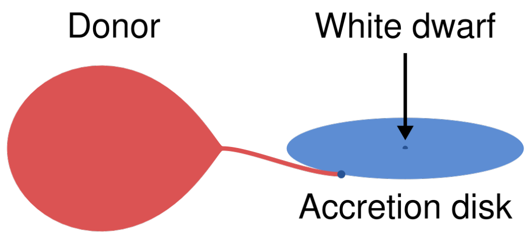

Figure 1: Schematic diagram of an MCV by Philip D. Hall

Using various arguments, cyclotron maser emission was found to be a suitable candidate which is supposed to be the cause of this radiation as emission was of short duration, had narrow band, had extremely high degree of circular polarization. The authors proposed Loss Cone mechanism but in the thesis work, it was subsequently found that loss cone mechanism may not be a suitable candidate as it is possible that coronal currents are sustained through potential difference along field lines. It was pointed out in [17]. The authors of [1] proposed that the lower corona of the star was the region from where the radiation was coming. The thesis work is an attempt to understand cyclotron maser emission process and relevant Physics behind this phenomenon.

Work on cyclotron Maser instability has its origin in the remarkable paper published by Twiss in 1958 (see [8]). Cyclotron Maser instability mechanism had been very successful in explaining decametric radiation from jovian moons. My attempt to explain radiation from MCVs is by constructing an analogy to this situation and using it as a leading line of attack to predict characteristics of radio emission from magnetic cataclysmic variables. The relevant background is like this. Requirements for electron-cyclotron masers are (i) a population inversion in the electron distribution and (ii) a magnetized plasma in which the electron-cyclotron frequency exceeds the plasma frequency (e.g. [4]). The first condition can be achieved when the magnetic field geometry in the source region allows the development of anisotropy in the electron distribution. Loss cone distribution (e.g. [19]), where electron pitch-angle(pitch angle is angle between electron velocity and magnetic field) anisotropy develops within a magnetic flux tube with converging field lines at each foot print is very popular. Large-pitch-angle electrons are magnetically reflected, This happens because there are adiabatic constants and magnetic moment of an electron generated through its gyrating motion is an adiabatic constant. At the footprint, Small-pitch-angle electrons are lost from the bottle and merge with high-density plasma at the foot of the flux tube. The second condition is satisfied in magnetized plasmas with a relatively low electron density and/or high magnetic field strength.

Loss cone anisotropy drives the emission from IO-Jupiter system(e.g.[6]). The Auroral Kilometric radiation from Earth's atmosphere is produced via cyclotron maser instability but it is not driven by loss cone anisotropy. Anisotorpic shell distributions generated through precipitation of electrons are the one triggering cyclotron maser instability. NASA satellites have successfully measured shell distribution function of electrons in the radiation belt. Wu and Lee tried to model AKR emission through loss cone distribution in 1979 (see [19]).

Cyclotron Maser is a misleading name. It does not have anything to do with energy levels and any quantum effect. Melrose and Dulk (1982) have formulated the theory in terms of the absorption coefficient and the resultant conclusions predict spectral similarity with quantum Masers. So, the name maser is retained.

Melrose narrates, "the theory of cyclotron instabilities in plasmas due to anisotropic velocity-space distribution was initiated in the early 1960's (e.g. Harris 1959, 1961, Sagdeev Shafranov 1961); it was discussed in some detail in Stix' book (Stix 1962). The main emphasis in this context has been the growth of waves with refractive index > 1 due to a temperature anisotropy or to a loss-cone anisotropy. The non relativistic approximation is made, specifically the Doppler condition is approximated as $\omega - s\Omega_e - k_{\parallel}v_{\parallel} = 0$ with $\Omega_e = \frac{qB}{m}$. ". [12]

Twiss(1958) used the above relation and found very low growth of cyclotron maser for physically plausible distributions. In the last section of the thesis, relativistic corrections made to this resonance condition are discussed as they are extremely necessary to obtain a good value of the growth rate.

Major blow to cyclotron maser theory came, when in the early 1980s, it was found that the loss-cone maser theory that was advocated by Wu and Lee in 1978 was not supported by observation of the AKR. Also as Treumann mentioned, two-dimensional particle simulations based on measured electron loss-cone distributions (Pritchett 1984a,b; Pritchett and Strangeway 1985) had failed as well([17]). Though later researchers like Dulk adopted shell distributions and in situ measurements taken by Swedish viking satellite and other satellites confirmed the anisotropic conditions. This was a mini renaissance in the field of cyclotron maser emission and by early 2000, researchers again carefully started studying cyclotron maser emission. In situ measurements in the radiation belt of Earth and in the region near Jupiter in late 20th and 21st century have confirmed that cyclotron maser instability is driving the coherent radiation process. The emission mechanism for the extremely bright pulsar radio emission is not known, but given the history and prospects of cyclotron maser emission, it is a strong candidate to explain radiation from pulsars.

This thesis work explains physics behind cyclotron maser instability and discusses potential applications of concepts of plasma physics in the context of magnetic cataclysmic variables. The new proofs of a modified Boltzmann H theorem for plasma, Vlasov equation and novel approaches to obtain Electromagnetic fields of a radiating particle are discussed.

2 Classical Radiation

2.1 Balance sheet of a periodically radiating classical system

In Classical theory of radiation, which we will adopt to explain cyclotron Maser instability, radiation is a direct consequence of Maxwell's equations and relaxation comes due to Lorentz Force law. In fact, radiation should be understood as a transfer of material energy and material momentum into field energy and field momentum and, subsequent escape. Without some external back up, the process cannot go on for eternity. It has been understood that just like particles, electromagnetic fields carry momentum and energy. The conservation laws for momentum and energy take field momentum and energy into account. It can be shown that for a finite volume, total energy in that volume can never be conserved if charged particles are in motion and the system is not in equilibrium.

$$\int_V \mathbf{J}.\mathbf{E}dV + \int_V (\mathbf{E}.\frac{\partial \mathbf{D}}{\partial t} + \mathbf{B}.\frac{\partial \mathbf{H}}{\partial t})dV = -\int_S (\mathbf{E} \times \mathbf{H}).\mathbf{n}da$$

Here, the first term is interpreted as the rate of change of kinetic energy as it can be directly shown from Lorentz force law. The second is field energy in the given volume. The right hand side term is known as Poynting vector which denotes energy flux out of the volume surface. This is the term that tells us about radiation outflow from any system confined in a finite volume.

Now, I am making an important remark regarding Poynting theorem that it doesn't actually portray the whole picture and it has limited applicability. It can be effectively used to analyse a plasma system if the effects of thermal velocities cancel each other and hence only the bulk motion has any significance. Nevertheless, it gives us a rough idea as to how a classically radiating system behaves. Astronomical signals that we receive on the Earth show very consistent periodic profile. An isolated radiating system without any external backup must never be able to show consistent radiation profile as motion of radiating particles will experience damping. That leads us to the conclusion that classically radiating astrophysical system is powered by power sources which are not electromagnetic in nature. Thermonuclear fusion processes which happen inside the star produce enormous amount of energy which lead to uneven heating of the star and that gives birth to convective currents.

If we integrate both sides of Poynting's Equation and also introduce non electromagnetic work done on the system, Energy transactions for a classically radiating system powered by a thermal battery will be,

$$W_{nonelectromagnetic} - \Delta E_{radiation} = \Delta K.E + \Delta F.E$$

For a complete cycle, change in kinetic energy and field energy is zero.So, all energy that has escaped through radiation comes from non electromagnetic sources.

$$W_{nonelectromagnetic} = E_{radiation}$$

The right hand side has its origin in thermonuclear processes. So, the main conclusion is that a radiating plasma system is just a conduit. All the various ways in which Plasma can radiate are working as just intermediaries.

Now, the assumption of periodic condition is not a completely valid one. Even for a magnetic cataclysmic variable, we see that there is steady transfer of material. For polar stars, this transfer is magnetically channelized. But even when scale of non periodicity is taken into account, the fluctuations arising from non periodicity won't be able to explain the enormous amount of radiation that has escaped. The reservoir of energy has to be at the core of star which is continuously providing the energy.

The steady value of magnetic field that will be assumed throughout the thesis is very difficult to maintain. The famous Dynamo problem is not in the scope of this thesis.

2.2 Radiation and Classical Inverse Square Law

Plasma is a collection of roughly free charged particles and it is not always possible to assume linear continuum. Radiation emitted from each electron needs to be understood. The formal way to approach this is through Maxwell's equations.

$$\nabla.\mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla.\mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

It is difficult not to overemphasize the importance of these equations. Fields produced by a single charged particle can be thought of as something similar to Green's function but not the Green's function itself. Green's function for Maxwell's equations when they are decoupled, physically corresponds to a situation where the charge momentarily came into existence and then vanished again. The standard procedure to approach Maxwell's equation is to introduce vector and scalar potentials. The wave equation is obtained by introducing Lorentz guage.

$$\mathbf{B} = \nabla \times A$$

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

It is well known that these scalar and vector potentials aren't unique, different potentials can correspond to the same physical situation and they are related by guage transformations. In Coloumb guage, which is more appropriate for electrostatics and also in magnetohydrodynamics, we set $\nabla.\mathbf{A} = 0$ and in Lorentz guage,

$$\nabla.\mathbf{A} = \frac{-1}{c^2}\frac{\partial V}{\partial t}$$

There is a short proof that can show that we can actually set divergence of vector potential to these terms. When we put expressions of electric and magnetic fields into the Maxwell's equations, we get,

$$\nabla^2 V - \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0}$$
$$\nabla^2 \mathbf{A} - \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J}$$

These are 4 scalar wave equations. Green's function for it is known. Its form has been deduced by several authors(e.g [9]). Both advanced and retarded Green's functions are valid solutions but we will choose the retarded one to save causality. It is to be mentioned that Maxwell's equations don't imply causality. In the systems with dispersion(Which plasma is) Causality has to be saved using additional assumptions. Kramer Kronig relations put severe restrictions on permittivity that a dispersive medium can have.

$$G^{(+-)}(\mathbf{r}, \mathbf{r}^{'}, t, t^{'}) = \frac{\delta(t^{'} - [t + -\frac{|\mathbf{r}-\mathbf{r}^{'}|}{c}]}{|\mathbf{r} - \mathbf{r}^{'}|}$$

$$4\pi\epsilon_0 V(\mathbf{r}, t) = \int\int G^{(+-)}(\mathbf{r}, \mathbf{r}^{'}, t, t^{'})\rho(\mathbf{r}^{'}, t^{'})d^3x^{'}dt^{'}$$

Here, few very crucial and interesting physical conditions have to be specified if we want to obtain physically plausible potentials from Green's function. First of all, we are selecting retarded Green's function, then we need to assume that the source is localized in time and space. Only then we can deduce the form of potential. This means that source is in existence only for a finite amount of time - No matter how long but finite. This assumption is valid in astrophysics too if we think of stars as systems which exist for a finite amount of time. It may look like that we need to know about charge and current distributions that will be formed in the future to evaluate the present value of potentials but the form of argument under dirac delta prevents that.

The retarded potentials and fields can be obtained by carefully taking curl and gradient of retarded potentials.

$$V(\mathbf{r}, t) = \int \frac{\rho(\mathbf{r}^{'}, \tau)}{|\mathbf{r} - \mathbf{r}^{'}|} d^3 r^{'}$$

$$\mathbf{A}(\mathbf{r}, t) = \int \frac{\mathbf{J}(\mathbf{r}^{'}, \tau)}{|\mathbf{r} - \mathbf{r}^{'}|} d^3 r^{'}$$

Where, $\tau = t - \frac{|\mathbf{r}-\mathbf{r}^{'}|}{c}$. The straightforward interpretation is that reaction of the fields to changes in the source is not instantaneous and there is a lag. Information travels at the speed of light.

When we are receiving radiation from the objects which are light years away, not all terms have significance. Only those terms in electric and magnetic field which are falling by 1/r will matter. Rest of the terms have insignificant contribution to the intensity observed at large distances from the source. Careful algebra is needed to obtain electric and magnetic fields. Jefimenko obtained the expressions for fields. They are,

$$\mathbf{E} = \frac{1}{4\pi\epsilon}\int[\frac{\mathbf{r}-\mathbf{r}^{'}}{|\mathbf{r}-\mathbf{r}^{'}|^{3}}\rho(\mathbf{r}^{'},\tau) + \frac{\mathbf{r}-\mathbf{r}^{'}}{|\mathbf{r}-\mathbf{r}^{'}|^{2}}\frac{\partial\rho(\mathbf{r}^{'},\tau)}{\partial t} - \frac{1}{|\mathbf{r}-\mathbf{r}^{'}|c^{2}}\frac{\mathbf{J}(\mathbf{r}^{'},\tau)}{\partial t}]d^{3}r'$$

$$\mathbf{B} = \frac{1}{4\pi\epsilon}\int[\frac{\mathbf{r}-\mathbf{r}^{'}}{|\mathbf{r}-\mathbf{r}^{'}|^{3}}\times\mathbf{J}(\mathbf{r}^{'},\tau) - \frac{\mathbf{r}-\mathbf{r}^{'}}{|\mathbf{r}-\mathbf{r}^{'}|^{2}c}\times\frac{\mathbf{J}(\mathbf{r}^{'},\tau)}{\partial t}]d^{3}r'$$

When we are talking about the radiation that we are receiving on the Earth, we can safely ignore terms which are falling faster than the growth of the area of the sphere. If we look at the terms which are falling by 1/r we can easily see that in radiation zone, Electric and magnetic fields are always perpendicular. Poynting vector ($\mathbf{E}\times\mathbf{H}$) which denotes energy flux falls by inverse square of the radius. Polarization of the radiation can be deduced too. If charge density varies harmonically with a certain frequency, we will receive radiation of that frequency. The claim that frequency is always source dependent stands vindicated. Angular characteristics of radiation can be obtained by looking at the surfaces of constant flux.

Another point that I will make is that the charge and current distributions are localized in the space. So, multipole expansions are often very useful if we want to get some idea about the radiation profile.

If we somehow estimate the current in coronal loops and its time dependent growth and fall, obtaining the radiation profile should not be difficult if the above approach worked without any modification but the fact is that above approach is valid if individual character of particles is lost and only the bulk motion is significant. In the next section, it will be discussed in more detail.

2.3 Limitations and Scope of Macroscopic approach and Origin of Cyclotron Maser Instability

The world is made of discrete particles which are in motion. Classical macroscopic electrodynamics is applicable on continuous media. It is not always the case that averaged over charge density and current at a point are sufficient representatives of the source condition. Especially when the source particles are facing back reaction. When particles are evolving under the electromagnetic field that the whole configuration itself is producing, the current density at t_2 may not be directly obtained from the current density at t_1. Something more than just averaged over picture is needed. Actually the burden of justification lies on the shoulders of the claim that averaged over behaviour is sufficient to explain the radiative processes.

When there is isotropy in the velocity distribution around mean velocity, we can use local average velocity as the representative velocity. When full local equilibrium is there, distribution is Maxwellian. This can be proven. When there is local equilibrium in Plasma we can ignore individual character of the particles. At sufficiently high temperatures, a dense plasma cannot be considered collisionless. There is a deeper claim that suggests that sufficient amount of collisions is extremely necessary if local equilibrium is to be achieved. Analysis of radiation profile directly from Maxwell equations as described in the previous section is sufficient for such dense radiating Plasma. Plasma near the accretion disk in an MCV is of this nature.

The Magnetized Plasma systems which are dilute enough show the kind of behavior where continuum mechanics fails. Geometry of Magnetic field crucially affects how particles move. Exotic magnetic field geometries cause many anisotropies in velocity distribution functions. Very infrequent collisions in a collisionless plasma prevent the plasma to go into local equilibrium. It is to be stressed that Simple macroscopic radiation is generally not powerful enough to generate sharp radio bursts. We are receiving radio bursts of short duration from MCVs. In magnetized plasma, charged particles gyrate around magnetic field lines and gyration frequency is $\frac{qB}{m}$. The gyrating charged particles emit radiation and they also experience electromagnetic force from the EM waves generated by other particles. The frequency of EM force experienced by charged particles matches with the cyclotron frequency. This reinforces the cyclotron motion of charged particles. This is called resonance. Doppler shift in the EM wave experienced by each particle plays very crucial role in the spectral spread of cyclotron emission. This will be discussed in detail in later sections.

This is purely a microscopic phenomenon so radiation emitted by a single particle needs a close inspection as each gyrating electron is radiating. Feynmann, Heaviside and Liénard–Wiechert obtained formulas for the fields produced by a single particle. There are subtle differences in the formulae. I will discuss some issues with them and then I will suggest a novel way to obtain EM fields produced by an accelerated charged particle. It is claimed by several authors that expressions obtained by Jefimenko and Feynmann are equivalent but they not.

2.4 Radiation from an Accelerated Particle and Issues with Jefimenko Equations

In the previous section, we obtained Jefimenko equations. An error in Jefimenko's derivation was pointed out(see [5]). J.H Field wrote, the fields of an accelerated charge given by the Feynman are the same as those derived from the Lienard-Wiechert potentials but not those given by the Jefimenko formulae. The author comprehensively analyses calculation done by Heavisde, Feynmann and Jefimenko. In Jefimenko equations, When we put charge density as dirac delta with particle's trajectory embedded in the argument of it, we can directly get EM fields produced by a charged particle executing an arbitrary motion. J.H argued that the equivalence between Jefimenko's formulae and Heaviside-Feynmann is erroneously claimed by several authors.

The main problem that authors spot in Jefimenko's derivation has to do with subtle definitions of partial derivatives which are very delicate to deal with when we are introducing retarded time which depe nds on position too. Jefimenko uses this relation.

$$\nabla[\rho] = \frac{-1}{c}\frac{\partial[\rho]}{\partial t}\hat{\mathbf{r}'}$$

Here, quantities under square brackets mean that they have retarded time as their time argument. This relation is spurious and the author shows how in the paper [5]. x_q is the qth coordinate of particle position which are function of time.

$$(\frac{\partial \tau}{\partial x_q})_t = \frac{-1}{c}(\frac{\partial r'}{\partial x_q})_t$$

$$(\frac{\partial r'}{\partial x_q})_t = \frac{x_q - x_Q(\tau)}{r'(1 - \hat{r'}.\mathbf{v})}$$

Here, **v** is velocity scaled by the speed of light, We will call this term K. If we combine these two equations we get,

$$(\frac{\partial \tau}{\partial x_q})_t = \frac{-1}{c}\frac{x_q - x_Q(\tau)}{r'K}$$

Thus,

$$\nabla[\rho] = \frac{\hat{i}}{c}\frac{x_q - x_Q(\tau)}{r'K}\frac{d[\rho]}{d\tau} + ...$$

Here, it looks tempting to substitute $\frac{1}{K}\frac{d}{d\tau} \rightarrow \frac{\partial}{\partial t}$ and we get the expression for divergence of ρ that was used by Jefimenko (e.g.[10]). But when we evaluate electric and magnetic fields from taking spatial derivatives of potentials, we are evaluating them at constant t while the above operator relation is valid at constant x_q. The faulty relation was used by Jefimenko in his original derivation and also by several other authors. Feynmann's approach was the correct one. In the next section, a new approach will be proposed that is theoretically sound but may produce other difficulties while approaching a class of problems.

2.5 A Novel Approach for Obtaining Radiating Fields Produced by an Accelerated Charged Particle

I am proposing that in the rest frame of the particle, the situation is electrostatic and one can obtain EM fields in the rest frame, and use generalized Lorentz transformation to get fields in the original inertial frame. This approach is not as straightforward as it is when we evaluate fields produced by a charged particle moving with a constant velocity. Maxwell's equations are Lorentz invariant and coordinates in the rest frame of a particle moving with constant velocity are directly related to those in lab frame via simple and linear Lorentz transformation. Maxwell's equations have the same form in both frames. But that won't hold true for an accelerated particle because in flat space-time, frame of an accelerated particle is not inertial. Corollary of this observation is that forms of Maxwell's equations will change.

Accelerated frames in flat space time can be dealt by using Fermi Walker Transport and generalized Lorentz transformation. It is to be mentioned that constancy of 4-acceleration does not imply constancy of 3-acceleration. This happens because 4-acceleration is obtained by differentiating 4-momentum by proper time and 4-momentum itself is obtained by differentiating 4-position in space-time. Constant 4-acceleration condition is satisfied by a class of motions. Uniform circular motion has constant 4-acceleration. So, an electron gyrating around a magnetic field is said to be executing a motion with uniform acceleration. Born coordinates are used for Langevin observers. A gyrating electron can be considered as a Langevin observer and We can write Maxwell's equations in that frame.

Electric and magnetic fields don't transform separately.The transformations are best described by the transformation of an anti symmetric tensor quantity $F_{\mu\nu}$. If A is transformation matrix, $F_{\mu\nu}$ transforms as $F_{lp} = A_l^{\mu} A_p^{\nu} F_{\mu\nu}$. For a given accelerated motion, A will be a particular type of generalized Lorentz transformation. Fields in rest frame and accelerated frames will be related in this way. Several authors have found the form of A for various types of accelerated frames.

Transformation of Maxwell's equations in accelerated frames is trickier part. Elaborate processes have been described by physicists. Here, dwelling into each case falls out of scope of the thesis. Though the case with Rindler coordinates will be described. Maxwell equations in Rindler coordinates where a particle is in constant acceleration in x direction will be,

$$\nabla.(\frac{E}{\sqrt{h}}) = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}(\frac{B}{\sqrt{h}})$$

$$\nabla.(\frac{\mathbf{B}}{\sqrt{h}}) = 0$$

$$\nabla \times \mathbf{B} = \epsilon_0 \mu_0 \frac{\partial}{\partial t}(\frac{\mathbf{E}}{\sqrt{h}}) + \mu_0 \mathbf{J}$$

Here, $h = g_{00}$, where g_{00} is the first coordinate of Rindler metric which is $(1 + \frac{ax}{c^2})$. Here, current $\mathbf{J}$ will be zero and charge density ρ is just dirac delta centered at origin. Clearly, $\frac{\partial \rho}{\partial t} = 0$. This is what makes the problem electrostatic but E and B may have time dependence if h has time dependence but that is not the case. So, all time derivatives of electric and magnetic fields are zero. So, the equations become,

$$\nabla.(\frac{E}{\sqrt{h}}) = \frac{q\delta(0)}{\epsilon_0}$$

$$\nabla \times \mathbf{E} = 0$$

$$\nabla.(\frac{\mathbf{B}}{\sqrt{h}}) = 0$$

$$\nabla \times \mathbf{B} = 0$$

Here, if we put the boundary condition that fields are zero at infinity, we can directly obtain B which will be 0. Now, we can say that, $\mathbf{E} = \nabla V$ as curl of electric field is 0. Once we put this substitution, we will get an elliptic differential equation. Which will be,

$$\nabla^2 V - \frac{a}{2c^2}\frac{\partial V}{\partial x} = -\frac{\rho\sqrt{h}}{\epsilon_0}$$

Potential is just a Green's function of this elliptic equation. Once, field is obtained from potential, we can obtain electric and magnetic field in the rest frame, hence, we obtain field produced by an accelerated particle. Case by case analysis of each case will be discussed by me in future work.

I will say that I am not providing a general expression but I am providing a general procedure to obtain EM fields which may become more handy in certain cases. Above example was for a particle moving with constant acceleration in x direction. For a particle executing circular motion, cylindrical coordinate system will be more natural. In more general way, we can write Maxwell's equations in curved space time as,

$$F_{\mu\nu} = \frac{\partial A\nu}{\partial x_\mu} - \frac{\partial A\mu}{\partial x_\nu}$$

$$D^{\alpha\beta} = \frac{1}{\mu_0} g^{\alpha\mu} F_{\mu\nu} g^{\nu\beta} \frac{\sqrt{-g}}{c}$$

$$J^\alpha = \frac{\partial}{\partial x_\beta} D^{\alpha\beta}$$

metric g will be Minkowsky metric expressed in suitable coordinates appropriate for given arbitrary motion. J^α will have dirac delta as its zeroth component and rest of the components will be zero. A^μ can be obtained as a solution of a differential equation and fields in lab frame produced by a charge executing arbitrary motion will be obtained by transforming $F_{\mu\nu}$.

References

[1] Classical Electrodynamics - JD Jackson

[2] Retarded electric and magnetic fields of a moving charge: Feynman's derivation of Lienard-Wiechert potentials revisited
J.H.Field
arXiv:0704.1574v3 [physics.class-ph]

3 Basic Approaches Towards Plasma

Systematic study of Plasma began with Langmuir's groundbreaking papers in 1920s. Plasma was identified with high degree of ionization in gas. The most basic line of attack is to ignore all collective properties of charged particles. What do I mean by this is that charged particles are not creating fields that significantly affect motion of other particles. All of them are moving as independent particles which are subjected to external fields. Obviously, this is not true but this approach still works in few conditions and it gives us some rough idea about the behavior of Plasma. The conclusions that we get by adopting this approach are relevant under following conditions,

- Plasma is very dilute.
- The dominant field in the plasma is produced by external source.
- The trajectories of particles don't converge.

Above assumptions hold true up-to some degree on the outskirts of the lower corona of the donor star, dilute plasma is needed for cyclotron Maser instability as waves with $< \omega_{plasma}$ become evanescent in plasma. So, cyclotron frequency should be greater than plasma frequency which depends on the density of plasma if radiation from cyclotron maser instability is escaping the plasma. For radio bursts governed by cyclotron maser instability, magnetic field is not too strong. If the above condition for the propagation of EM waves holds, plasma must be dilute.

In [1], strong argument was made that proposed that relevant magnetic field in the lower corona of the donor star is produced by the inner core of the donor star. The centrifugal forces produced by intense rotations of donor star conjugated with forces produced by dipolar magnetic produced by the inner core don't let the trajectories of charged particles converge. That is why intuitions gained by adopting above approach are valuable to rule out many physical conditions. In fact, pre instability conditions can be very close to ideal conditions but small fluctuations don't damp out and instability is generated. The crucial task is to obtain an estimate of pre instability number density, magnetic field etc. IO-Jupiter circuit is one such model to obtain reliable estimates of these quantities. Even in IO-Jupiter circuit model, many conclusions from basic approach hold true(see [6]).

Even if we adopt basic model where we ignore all collective properties, obtaining trajectories of particles under magnetic and electric fields is not easy. We do have to make suitable assumptions. The main assumption is that of weak gradient assumption and it is reported in [3] that plasmas cannot sustain parallel electric and magnetic fields for a long time. So, we can always assume that electric and magnetic fields are perpendicular in very dilute plasmas but in AKR model, electric fields parallel to the magnetic field lines play a crucial role in the elimination of loss cone anisotropy scenario as reported in [17].

3.1 Guiding Centre Theory

When a very dilute plasma is subjected to magnetic field, the force experienced by each particle is,

$$\mathbf{F} = q\mathbf{V} \times \mathbf{B}$$

By taking dot product with magnetic field and velocity on both sides, we immediately see that acceleration is always perpendicular to magnetic field and velocity both. The resultant motion is always helical. Velocity component which is parallel to velocity remains unaffected and magnetic field lines work as axes of helices, Very dilute solar winds under the influence of interstellar magnetic field lines travel in this way. Here, guiding center is at field lines and it is travelling in the straight line. Now, z axis is along magnetic field line,

$$\dot{v_z} = 0$$

$$\dot{v_x} = \Omega v_y$$

$$\dot{v_y} = -\Omega v_x$$

$\Omega = \frac{qB}{m}$ is signed cyclotron frequency of gyration frequency of a charged particle. We can decouple the equations and we get 2 simple harmonic oscillators. But above equations must hold so they must differ by a phase of $\frac{\pi}{2}$. So, the resultant motion is circular in xy plane if we ignore z component of velocity,

$$x - x_0 = R \sin \Omega t$$

$$y - y_0 = R \cos \Omega t$$

$$R = \frac{v_\perp}{|\Omega|}$$

R is called gyro radius. Throughout the thesis we will assume that gyro radius is very small. If magnetic field is having gradient, we will assume that magnetic field remains fairly constant over the distances of order of gyro radius. (x_0, y_0) is travelling guiding centre. The signed nature of cyclotron frequency implies clockwise and anti clockwise gyration of particles having opposite signs.

Cyclotron maser emission has peak at cyclotron frequency. AKR(Auroral Kilometric radiation) radiation is a type of cyclotron emission. Decametric Jovian emission is also cyclotron maser emission. If a site is proven to be emitting cyclotron radiation we can directly estimate the value of magnetic field at that site. We now know a lot about Jovian magnetic field than we did before.

Guiding center approach is useful when fields slowly change in space and time. Suppose, constant electric field is applied to a very dilute plasma which is perpendicular to magnetic field then,

$$\dot{v_z} = 0$$

$$\dot{v_x} = \Omega v_y + \frac{q}{m} E_x$$

$$\dot{v_y} = -\Omega v_x$$

$$F = q(\mathbf{E} + \mathbf{V} \times \mathbf{B})$$

Without any loss of generality, We can assume that electric field is in x direction. Equation of motion will be,

$$\dot{v_z} = 0$$

$$\dot{v_x} = \Omega v_y + \frac{q}{m} E_x$$

$$\dot{v_y} = -\Omega v_x$$

If, we replace v_y by $v_y + \frac{E}{B}$, we get the same equation as we got before when only magnetic field was applied. This means that particle is still executing gyration around magnetic field but it's guiding centre is also moving in y direction with constant speed. $\frac{E}{B}$ is guiding centre drift. There is simple relativistic explanation of this phenomenon. If we look at the situation in a frame where electric field vanishes we should see only helical motion in that frame. Field transformation when we go into a frame moving with v velocity is,

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}$$

If we put $E' = 0$ we get the the drift of $\frac{E}{B}$. In general form, drift of guiding centre can be written as,

$$\mathbf{v}_d = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

This will also fail in fast moving plasma. Proper relativistic approach is needed in fast moving plasma. Guiding centre often loses its meaning in fast moving plasma. The most striking result here is that positive and negative charges experience same drift in same direction. This happens because they have opposite sense of gyration. Plasma when it is subjected to electric field won't show any drift in the direction of electric field. Several authors(e.g [2]) have obtained expressions for various other types of drift. All such drifts can be super imposed. The general form of the drift when force **F** is applied is,

$$\mathbf{v}_F = \frac{1}{\Omega} (\frac{\mathbf{F}}{m} \times \frac{\mathbf{B}}{B})$$

When electric field slowly varies in space and time, the charged particles with opposite signs react differently. This leads to current. Such drift is called polarization drift. When straight magnetic field lines show change in value we also see drift. When there is curvature in magnetic field lines, we also see drift. All such drifts have their effective s. They are as follows,

- $$\mathbf{F}_\nabla = -\mu \nabla B$$
- $$\mathbf{F}_P = -m \frac{d\mathbf{E}}{dt}$$
- $$\mathbf{F}_G = -m\mathbf{g}$$

Here, $\mu = \frac{mv_\perp^2}{2B}$ and $\frac{d}{dt}$ is convective derivative which is $\frac{\partial}{\partial t} + \mathbf{v}.\nabla$. All of these drifts give birth to currents drift speeds will be in opposite direction for opposite charges.

drift of the guiding centre is inversely proportional to cyclotron frequency. Out of MCVs which were detected, some of them are strong candidates for Polar stars. They have very strong magnetic fields and hence very high cyclotron frequency. The drift from gravity and other effects are very low. So, the guiding centre always follows magnetic field lines. This is a rough argument that predicts that material transfer in a polar MCV is magnetically channelized at least in the region where plasma is very dilute. It is known that polar stars emit near infrared cyclotron radiation.

IO driven jovian cyclotron emission can be described using unipolar inductor model. A circuit is formed and magnetic field lines work as wires that guide electrons. The plasma in the interstellar place between the Jupiter and IO is dilute enough and magnetic field is strong enough. That is why guiding centre theory works well to predict that particles will be following magnetic field lines with nominal drift. But magnetic field lines start converging near auroral footprints. When magnetic field lines start converging particle trajectories will also start converging and basic approach fails in that region.Cyclotron Maser emission comes from the site where magnetic field lines rapidly converge. Ring current in radiation belt can be satisfactorily studied using guiding centre theory. Another important concept which is of relevance when cyclotron maser instability is operating is adiabatic invariants.

3.2 Relevant Adiabatic Invariants

Constants of motion do not change at all in time. Energy of charged particles when they are only under the influence of magnetic field is a constant of motion. Adiabatic invariants are not absolute constants but they remain approximately constant. Plasma has few such important adiabatic invariants. $\mu = \frac{W_\perp}{B}$ is an adiabatic invariant. W is kinetic energy. For a purely magnetic plasma, total kinetic energy W is an absolute constant.

$$W = W_\parallel + W_\perp$$

$$\frac{dW}{dt} = \frac{dW_\parallel}{dt} + \frac{dW_\perp}{dt} = 0$$

$$\frac{dW_\perp}{dt} = \mu \frac{dB}{dt} + B \frac{d\mu}{dt}$$

Now, More significant motion is happening along magnetic field lines. Curvature of magnetic field lines is negligible. $\frac{d}{dt} = \frac{v_\parallel}{ds}$ where s denotes distance travelled along field lines. Effective force provided by magnetic field gradient is $-\mu \nabla B$.So, rate of change of parallel velocity can be found.

$$m \frac{v_\parallel}{dt} = -\mu \nabla_\parallel B = -\mu \frac{dB}{dt}$$

we can multiply both sides by $v_\parallel$, left hand side is derivative of parallel kinetic energy.

$$-\frac{dW_\perp}{dt} = \frac{dW_\parallel}{dt} = -\mu \frac{dB}{dt}$$

This implies that μ remains constant.

The above derivation adopted from [2] does not seem to say anything about the approximately constant nature of μ. But when we brought the concept of effective force from magnetic field gradient, the assumption of negligible cyclotron radius and coarse graining up-to time scales much larger than $\frac{1}{\Omega}$ was implicit. This makes μ approximately constant.

Magnetic mirror is formed due to approximately adiabatic invariance of magnetic moment of gyrating electrons. If α is the angle between velocity and magnetic field which is called pitch.Then $\mu = \frac{mv^2 \sin^2 \alpha}{2B}$.If we know the pitch at one location we can calculate pitch at other location as total kinetic energy is conserved.

$$\frac{\sin^2 \alpha_2}{\sin^2 \alpha_1} = \frac{B_1}{B_2}$$

For converging magnetic field geometry, for every particle we can find B_0 where pitch angle becomes 90^0. Here, parallel component of velocity vanishes. The particle is reflected from that point. Particles can oscillate back and forth. We can also define l as length of magnetic field line between 2 mirror points. If magnetic field attains maximum at some point, particles with $B_0 > B_{max}$ will not get reflected.They will be lost. Particles with the pitch under a cone will be lost. This is loss cone. This is a severe form of anisotropy in pitch distribution. This creates a ripe condition for cyclotron maser instability.

Loss cone anisotropy that drives jovian emission has its origin in approximately constant nature of μ. Other adiabatic invariant called longitudinal invariant plays a crucial role in generation of Kinetic energy anisotropy. For approximately periodic motion,

$J_i = \oint p_i dq_i$

remains approximately constant. It is an action variable. If a particle is experiencing drift due to gradient in magnetic field, this Ji will remain constant for each magnetic field line if that line contains mirror points for the particle as the state of motion will remain same if the particle returns to the given magnetic line after wandering around. This happens because total kinetic energy is constant and magnetic moment is an adiabatic invariant.

Using Hamilton Jacobi theory, we can show why J_i will be constant. Various proofs are available(e.g. [7]). We can show how energy anisotropy can be generated when particle is experiencing drift and is changing magnetic field lines. For each line,

$$J = \oint mv_{\parallel} ds = 2ml\overline{v_{\parallel}}$$

Where bar over velocity denotes average and l denotes length of the magnetic field line between 2 mirror points. We can obtain relation between average parallel kinetic energy of 2 lines.

$$\frac{\overline{W_{1\parallel}}}{\overline{W_{2\parallel}}} = \frac{l_1^2}{l_2^2}$$

The bounce path decreases and average parallel speed increases. For each line, magnetic moment is also conserved. We can define degree of anisotropy for a line. That is,

$$A_W = \frac{\overline{W_\parallel}}{\overline{W_\perp}}$$

We can combine both adiabatic invariance and get a relation for degree of anisotropy.

$$\frac{A_{1W}}{A_{2W}} = \frac{B_2}{B_1}\frac{l_2^2}{l_1^2}$$

This relation gives us the idea as to how anisotropy gets created due to drift currents. Auroral Kilometric Radiation that we get from polar region is due to cyclotron maser instability. Drift induced anisotropy plays a major role in producing highly anisotropic cell distributions.

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4 Kinetic Plasma Theory

The most sophisticated approach to study plasma is by using kinetic theory. In hamiltonian dynamics, evolution of a point in 6N dimensional phase space is studied. In kinetic theory, we look at the evolution of probability density in the phase space. Boltzmann adopted this approach and derived his famous Boltzmann transport equation.

It must be stressed that plasma is generally composed of 2 entities and it exists in quasi neutral state. The results will be proved for a system composed of one entity and they will be carried forward with few modifications when we will discuss why we don't need complete 2-fluid model to explain cyclotron maser Instability. However, for electron driven cyclotron maser instability, calculations obtained by ignoring protons altogether yield accurate results. This happens due to low mobility of protons. Cyclotron maser instability works on much Smaller time scales. For those time scales, protons can be considered as particles at rest. What makes plasmas very different from gases is that long range forces dominate in plasma. Collisions don't play that big of a role. Particles in Neutral gases behave as free particles within their mean free paths and keep experiencing collisions which suddenly change their velocities. In later sections, it will be shown that local equilibrium that gives meaning to temperature is very difficult to achieve in plasma. Equations which govern the evolution of phase space density of plasma can be obtained from celebrated Liouville theorem.

4.1 A Fresh Derivation of Vlasov Equation

Liouville theorem describes evolution of full phase space density. It contains much more information than it is necessary. $\rho(\mathbf{p}_i, \mathbf{q}_i, t)$ represents the probability that the system is in a state in which particles have $\mathbf{q}_i$ positions and $\mathbf{p}_i$ momenta. H is total hamiltonian of the system. Liouville theorem asserts that phase space density behaves like an incompressible fluid under hamiltonian flows in the phase space. That means $\frac{d\rho}{dt} = 0$.

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \sum_{i=1}^{i=3N} (\frac{\rho}{\partial p_i}\frac{dp_i}{dt} + (\frac{\partial \rho}{\partial q_i}\frac{dq_i}{dt}) = 0$$

From, Hamilton's equations,

$$\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$$

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}$$

. We can combine the above equations and we will get,

$$\frac{\partial \rho}{\partial t} + \sum_{i=1}^{i=3N} -(\frac{\partial \rho}{\partial p_i}\frac{\partial H}{\partial q_i}) + (\frac{\partial \rho}{\partial q_i}\frac{\partial H}{\partial p_i}) = 0$$

Poisson bracket can be recognized.

$$\frac{\partial \rho}{\partial t} = -\{\rho, H\}$$

This is the standard form of Liouville theorem for the full phase space density. We are more interested in the standard unconditional probability distribution for n particles. It describes probability of finding n particles in particular places having particular velocities.

$$\rho_s(\mathbf{p}_i, \mathbf{q}_i, t) = \int \prod_{j=s+1}^{j=N} d^3p_i d^3q_i \rho_(\mathbf{p}_i, \mathbf{q}_i, t)$$

We want to normalize the distribution in such a way that integrating over whole phase space yields total number of particles. So we have to introduce a normalization constant.

$$f_n = \frac{N!}{(N-n)!}\rho_s$$

f_n is called n particle phase space density. It is enough to know 1- particle phase space density to compute local number density and all other important quantities. All particles are identical and we don't need to keep track of all particles. We need to study the effect produced by a particle at given position having given velocity. It doesn't matter which one is there. In order to find evolution of n particle density we need to do grouping of hamiltonian.

$$H_((\mathbf{p}_i, \mathbf{q}_i, t)) = \sum_{i=1}^{N} [\frac{\mathbf{p}_i^2}{2m} + U(\mathbf{q}_i, \mathbf{p}_i)]$$

Here, we have not included 1-1 particle interaction. We are assuming that coulombic field produced by 1 particle gets shielded by other particles of opposite charges which are momentarily passing by. This is called Debye shielding.$\lambda_D < L$ should hold if plasma is to be maintained in quasi neutral state. L is the physical dimension of the system. $U(\mathbf{q}_i, \mathbf{p}_i)$ is generalized velocity dependent macroscopic potential. It can be written in more neat form as,

$$H = \sum_{i=1}^{N} [\frac{(\mathbf{p} - q\mathbf{A})^2}{2m} + qU(\mathbf{q}_i)]$$

Where, particles are experiencing only macroscopic electromagnetic forces which are governed by Maxwell's equations. And entries in Maxwell's equations will be determined via moments of phase space density. We can plug this hamiltonian into Liouville theorem and we will integrate over all coordinates except for one particle.

$$\frac{\partial \rho}{\partial t} = -\{\rho, H\}$$

$$\int \frac{\partial \rho}{\partial t} \prod_{i=2}^{N} d^3q_i d^3p_i = -\int \{\rho, H\} \prod_{i=2}^{N} d^3q_i d^3p_i$$

We can see that hamiltonian can be neatly separated into separate components which depend on only 1 particle's variables.

$$\frac{\partial f}{\partial t} = N\sum_{i=1}^{i=N}\sum_{j=1}^{j=3}(\frac{\partial \rho}{\partial q_{ij}}\frac{\partial H}{\partial p_{ij}}) - (\frac{\partial H}{\partial q_{ij}}\frac{\partial \rho}{\partial p_{ij}})\prod_{i=2}^{N} d^3q_i d^3p_i$$

We know that ρ falls rapidly as it goes to infinity in both position and momentum coordinates. We can perform integration by parts.

$$\frac{\partial H}{\partial p_{ij}} = \frac{p_{ij} - eA_j(\mathbf{q}_j, t))}{m}$$

$$\frac{\partial H}{\partial q_{ij}} = \sum_{l=1}^{l=3}(\frac{p_{il} - eA_l(\mathbf{q}_i, t)}{m})\frac{\partial eA_l}{\partial q_{ij}} + e\frac{\partial V}{\partial q_{ij}}$$

We can see that if ψ is some function then

$$\int \psi\frac{\partial \rho}{\partial x} = -\int \frac{\partial \psi}{\partial x}\rho$$

This is true if phase space density is falling sufficiently rapidly in x coordinate. We can apply this to the equation obtained above and we will get,

$$\frac{\partial f}{\partial t} = -N\sum_{i=1}^{i=N}\sum_{j=1}^{j=3}(\rho\frac{\partial H}{\partial p_{ij}\partial q_{ij}}) - (\frac{\partial H}{\partial q_{ij}p_{ij}})\ \rho\prod_{i=2}^{N} d^3q_i d^3p_i$$

Due to symmetry of 2nd derivatives, all will be cut and only the terms with derivatives with respect to the coordinates of the first particle will survive.

$$\frac{\partial f}{\partial t} = -N\int\sum_{j=1}^{j=3}(\frac{\partial \rho}{\partial q_j}\frac{\partial H}{\partial p_j}) - (\frac{\partial H}{\partial q_j}\frac{\partial \rho}{\partial p_j})\prod_{i=2}^{N} d^3q_i d^3p_i$$

$$\frac{\partial f}{\partial t} = -N\int\sum_{j=1}^{j=3}(\frac{\partial \rho}{\partial q_j}[\frac{p_i - eA_i(\mathbf{q}, t)}{m}] - [\sum_{l=1}^{l=3}\frac{(p_j - eA_j(\mathbf{q}, t))}{m}\frac{\partial A_j}{\partial q_i}]\frac{\partial \rho}{\partial p_j})\prod_{i=2}^{N} d^3q_i d^3p_i$$

We identify $v_i = \frac{p_i - eA_i(\mathbf{q},t)}{m}$ where v stands for velocity. Also,

$$\frac{\partial \rho}{\partial p_i} = \frac{1}{m}\frac{\partial \rho}{\partial v_i}$$

We also identify fields from the potentials. By using those we can write,

$$\frac{\partial f}{\partial t} + \sum_i \frac{\partial f}{\partial q_i}v_i + \sum_i \frac{e}{m}[\sum_j v_j\frac{\partial A_j}{\partial q_i} + E_i + \frac{\partial A_i}{\partial t}\frac{\partial f}{\partial v_i}]$$

In index free form we can finally write,

$$\frac{\partial f}{\partial t} + \nabla f.\mathbf{v} + \frac{e}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B})\nabla_{\mathbf{v}}.f = 0$$

This is Vlasov equation. Here, I must stress that I have pre assumed that the particles are not coming too close to each other and thus intra particle interactions are not there. All particles are moving under the influence of macroscopic electromagnetic field which may be produced by the collective effect of the particles. This is why we could decompose hamiltonian into terms involving coordinates of only one particle. Vlasov equation here is linear in nature if we assume that electric and magnetic fields are external. The particles are moving in such a way that they are not producing the fields of their own. This happens if no effective currents are there due to parallel motion of particles with opposite charge. When the motion of charged particle is significantly altering the fields then we cannot solve Vlasov equation in isolation. We have to combine it with Maxwell's equations. The resultant system is a non linear system. In the next section, I will describe how we can use linear Vlasov equation to model Magnetic Cataclysmic Variables.

4.2 Magnetic Cataclysmic Variables and Jeans' theorem

Most of stable astronomical systems are able to sustain stable magnetic fields. White dwarves are sources of great magnetic field. Plasma in corona of the donor star is dilute enough to be considered collisionless. The dominant magnetic field is provided by the violent white dwarf or by inner dynamo of the donar star. Later assumption should hold true if the radiation falls in radio range. This becomes more and more true if the separation between the donor and white dwarf increases. It is very reasonable to assume that magnetic field in the lower corona of the donor star is external in nature. So, phase space density in the corona is a solution of linear Vlasov equation. A natural question arises, how is cyclotron maser instability possible if the fields produced by the gyrating particles are ignored ? It is because the phase space density obtained as a solution to linear Vlasov equation is background phase space density. Small electromagnetic perturbations in the region where suitable anisotropy conditions are met will grow. The growth cannot go for eternity.The timescales for which the growth happens are small. Once radiation escapes, relaxation comes. Now, Phase space density after relaxation is again a solution of linear Vlasov equation where magnetic field provided by the inner dynamo of the donor star. We can obtain a large variety of solutions of Vlasov equation by using Jeans' theorem. We can construct various types of anisotropic phase space densities which are solutions of linear Vlasov equation and are physically plausible.

Jeans' theorem states that if phase space density is purely a function of 'constants of motion' of a particle put under same electromagnetic field then it is solution of Vlasov equation.

The trajectory of particle experiencing Lorentz force has 5 independent constants of motion. A more general claim is that a classical system having n degrees of freedom has 2n-1 independent constants of motion. Let c_i be constants of motion. $\frac{dc_i}{dt}$ for a trajectory where, $\frac{dx_i}{dt} = v_i$ and $\frac{dv_i}{dt} = \frac{e}{m}((\mathbf{v} \times \mathbf{B})_i + E_i)$. Let phase space density be a function of c_i which are functions of velocity and position.

$$f = g((c_i(\mathbf{v}, \mathbf{x}))$$

If a f satisfies Vlasov equation then the following equations should hold.

$$\frac{df}{dt} = 0 = \frac{\partial f}{\partial t} + \nabla f.\mathbf{v} + \frac{e}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B})\nabla_{\mathbf{v}}.f = 0$$

Here, $\frac{df}{dt}$ is convective derivative.

c_i will satisfy,

$$\frac{dc_i}{dt} = \frac{\partial c_i}{\partial t} + \nabla c_i.\mathbf{v} + \frac{e}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B})\nabla_{\mathbf{v}}.f = 0$$

$$\frac{df}{dt} = \sum_i \frac{\partial g}{\partial c_i}\frac{dc_i}{dt}$$

Now, as $\frac{dc_i}{dt} = 0$ f will satisfy Vlasov equation. The most basic example of a distribution function which is a function of constant of motion is Maxwellian distribution. It is solely a function of energy. It won't satisfy Vlasov equation if plasma is electrified. Though if we replace $\frac{1}{2}mv^2$ with $\frac{1}{2}mv^2+eV$ in the argument of Maxwellian then the resultant distribution will satisfy Vlasov equation. In fact, we can deduce Debye length parameter of plasma if introduce one extra electron in Plasma and use this modified Maxwellian. V will be deduced to be a potential that falls exponentially after Debye length.

Pure Maxwellian also satisfies Vlasov equation for a magnetized plasma as kinetic energy is a constant of motion but it is very uncommon. Maxwellian is very common for unmagnetized plasmas. Landau Damping of electrostatic wave in unmagnetized plasma is driven by Maxwellian distributions. (discussed in detail in [16]). The most basic loss cone distribution is just Maxwellian with a removed cone in the velocity space where the probability of finding a particle is zero. It must be noted that spatial variation of distribution function is often negligible in the regions where field lines don't have wild geometries. But at the auroral footprints from where cyclotron maser emission is generated, it is not the case that distribution function has no spatial variation. But we only know how to obtain growth of cyclotron maser instability for a region where magnetic field is constant and distribution function has no spatial variation. But we also know that cyclotron maser instability does not grow forever. We will define R_{cyclo} to be trapping radius. It shows how much of a region is covered under the growth of instability. Now, we if the spatial distribution function $f(r)$ does not vary significantly in this scale then the calculation is justified. The same will be the case with lower corona of the donor star in a magnetic cataclysmic variable. Magnetic field is constant for a region where cyclotron maser instability grows but on large scale there is variation in magnetic field geometry and in spatial distribution function of electrons.

When $|R_{cyclo}\nabla B| << 1$ we can calculate all extensive quantities of radiation for a local region and then integrate it over the whole region to obtain total radiation profile. Form of spatial distribution can be estimated by adopting proper model. For IO driven Jovian emissions where IO-Jupiter EMF circuit is established, it falls by R^{-3} where R is the distance from the footprint.

Operation of cyclotron maser is possible if there is a source of free energy. It is a process in which thermal energy is being transformed into usable radiation energy. That free energy is reflected in anisotropic distribution. Entropy of the system is defined using distribution function. Collisionless nature of plasma has unique consequences. In the next section, how entropy evolves in collisionless plasma and balance sheet of information during cyclotron maser instability are discussed. Up-to now, fields were completely external. A radiating plasma is a non linear system. We can linearize the system by using perturbation theory. This perturbative approach remains valid for a small time scale.

4.3 A Variant of Boltzmann H theorem for Collisionless Plasma

Hamiltonian equations of motion are time reversible but most of the statistical systems show irreversibility. Clearly, some additional assumption is working in background. Ludwig Boltzmann constructed a collision model in which BBGKY hierarchy was terminated after 2 equations. That means only 2 particles interact at a time. simultaneous interaction between 3 particles are very less probable. Before collision, particles have independent probability distributions but after the collision takes place, their distribution functions become correlated and these collisions are frequent. This is the assumption of molecular chaos and this assumption is time irreverssible in nature. For all the systems for which assumption of molecular chaos holds, irreverisible evolution is observed. This is manifested in Boltzmann H theorem.

$$H = \int f(\mathbf{p}, \mathbf{q}, t) \log(f(\mathbf{p}, \mathbf{q}, t)) d^3p_i d^3q_i$$

Boltzmann showed that for Boltzmann equation, $\frac{dH}{dt} < 0$ holds until the equilibrium is reached. This is the justification of the second law of thermodynamics. Entropy is defined as $-k_b H$. Entropy will always increase. At equilibrium, distribution remains steady with time. If we put $f(\mathbf{p}, \mathbf{q}, t) = \delta(H(\mathbf{p}, \mathbf{q}) - E)$ we recover the classic formula $S = K_B \log \Omega$. Ω is number of microstates or scaled area of the phase space surface for which the energy and volume are fixed. The origin of irreversibiliy lies in the adoption of collision model. For collisionless plasma, we are ignoring collisions altogether so we should not expect these to hold for collisionless plasma whose distribution follows Vlasov equation.

$$H = \int f(\mathbf{p}, \mathbf{q}, t) \log(f(\mathbf{p}, \mathbf{q}, t)) d^3p_i d^3q_i$$

$$\frac{dH}{dt} = \frac{d \int f(\mathbf{p}, \mathbf{q}, t) \log(f(\mathbf{p}, \mathbf{q}, t)) d^3p_i d^3q_i}{dt}$$

$$\frac{dH}{dt} = \int \frac{\partial [f(\mathbf{p}, \mathbf{q}, t) \log(f(\mathbf{p}, \mathbf{q}, t))]}{\partial t} d^3p_i d^3q_i$$

$$\frac{dH}{dt} = \int \frac{\partial f}{\partial t} + \log f \frac{\partial f}{\partial t} d^3p_i d^3q_i$$

$$\frac{dH}{dt} = \int \frac{\partial f}{\partial t} (\log f + 1) d^3p_i d^3q_i$$

We can now put expression for partial time derivative of f from Vlasov equation. It must be noted that we don't need it to be linear. Electric and magnetic field may depend on the integrals of p and

q. The subsequent proof is perfectly valid for a plasma experiencing cyclotron maser instability.

$$\frac{dH}{dt} = -\int (\nabla f.\mathbf{v} + \frac{e}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}).\nabla_{\mathbf{v}} f)(1 + \log f) d^3p_i d^3q_i$$

$$\frac{dH}{dt} = -\int \nabla f.\mathbf{v} + \frac{e}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}).\nabla_{\mathbf{v}} f + \log f(\nabla f.\mathbf{v} + \frac{e}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}).\nabla_{\mathbf{v}} f) d^3p_i d^3q_i$$

Now, for both velocity and position space, we can use vector identities. By using Fubini's theorem, we can interchange the order of integrals and treat the other variable as constant. We will separate all these terms and use Gauss's theorem for each term. We can exploit the fact that $f \to 0$ when velocity or position approaches infinity. Electric and magnetic fields are solely in function of position.

$$\int \nabla f.\mathbf{v} d^3q = \int \nabla.(f\mathbf{v}) d^3q$$

$$= \int_S f\mathbf{v}.\mathbf{n} da = 0$$

For this to hold, f should fall faster than $\frac{1}{r^2}$.

$$\int \nabla_{\mathbf{v}} f.\mathbf{E} d^3v = \int \nabla_{\mathbf{v}}.(f\mathbf{v}) d^3v$$

$$= \int_S f\mathbf{E}.\mathbf{n} da = 0$$

In both of these equations S is the surface at infinity in position and velocity space respectively. For radiating plasma electric field falls by $\frac{1}{r}$ and so does magnetic field.

$$\int (\mathbf{v} \times \mathbf{B}).\nabla_{\mathbf{v}} f d^3v$$

$$= -\int \mathbf{B}.(\mathbf{v} \times \nabla_{\mathbf{v}} f) d^3v$$

$$= -\mathbf{B} \int .(\mathbf{v} \times \nabla_{\mathbf{v}} f) d^3v$$

$$= -\mathbf{B}. \int (\nabla_{\mathbf{v}} \times (\mathbf{v}f)) d^3v$$

$$= \mathbf{B}. \int_S (\mathbf{v}f) \times \mathbf{n} da = 0$$

Now, we can use all of these results and can put them in the expression for $\frac{dH}{dt}$ and we can see the terms which don't contain log f will be all zero. Thus,

$$\frac{dH}{dt} = \int \log f(\nabla f.\mathbf{v} + \frac{e}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}).\nabla_{\mathbf{v}} f) d^3p_i d^3q_i$$

We can see that for both velocity and position gradient,

$$\log f \nabla f = -\nabla f + \nabla(f \log f)$$

Same relation will hold for velocity gradient and the expression will become,

$$\frac{dH}{dt} = \int (\nabla(f \log f).\mathbf{v} + \frac{e}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}).\nabla_{\mathbf{v}}(f \log f)) d^3p_i d^3q_i - \int \nabla f.\mathbf{v} + \frac{e}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}).\nabla_{\mathbf{v}} f d^3p_i d^3q_i$$

The streaming terms are zero as described in the previous steps so,

$$\frac{dH}{dt} = \int (\nabla(f \log f).\mathbf{v} + \frac{e}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}).\nabla_{\mathbf{v}}(f \log f)) d^3p_i d^3q_i$$

Now, just like in previous steps, we can put each term to zero if $f \log f$ falls faster than $\frac{1}{r^2}$ which is true for localized plasmas. If this holds then all of the terms will drop to zero as we can convert them into surface integrals of functions which fall faster than $\frac{1}{r^2}$ so,

$$\frac{dH}{dt} = 0$$

This result holds for collision plasma which is experiencing cyclotron maser instability and then experiencing subsequent relaxation. The conversion of thermal energy into radiation energy should decrease entropy but that is not happening. Entropy remains constant in the whole process while total energy in a finite ball containing plasma is decreasing. This shows that temperature loses all its meaning when cyclotron maser instability is under operation. In fact, for loss cone distribution we can define parallel and perpendicular temperature which has little to do with conventional notion of temperature.

The total energy of plasma when relaxation comes is lower but the information is not lost. Information is conserved and that is why plasma gets prepared again for the next round of emission. Similar process could not have taken place in collisional plasma where entropy would keep on increasing. It is very difficult for necessary Gibb's free energy to mount up without any external coherent support.

5 Perturbation and Cyclotron Maser in Collisionless Plasmas

5.1 Vlasov Maxwell System

We have seen that in collision less Plasma, particles move under the influence of macroscopic fields produced by the plasma particles themselves and external sources. f_s is distribution of sth entity. For simplicity, we will assume that only protons and electrons are there in Plasma. Macroscopic charge density and current density will be,

$$\rho = e\int f_p(\mathbf{v},\mathbf{x},t)d^3v - e\int f_e(\mathbf{v},\mathbf{x},t)d^3v$$

$$J = e\int \mathbf{v}_p f_p(\mathbf{v},\mathbf{x},t)d^3v - e\int \mathbf{v}_e f_e(\mathbf{v},\mathbf{x},t)d^3v$$

So, complete equation for Plasma will become,

$$\nabla.\mathbf{E} = \frac{\rho_{ext}}{\epsilon_0} + \frac{e\int f_p(\mathbf{v},\mathbf{x},t)d^3v - e\int f_p(\mathbf{v},\mathbf{x},t)d^3v}{\epsilon_0}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla.\mathbf{B} = 0$$

$$\nabla\times\mathbf{B} = \mu_0\mathbf{J}_{ext} + e\int \mathbf{v}_p f_p(\mathbf{v},\mathbf{x},t)d^3v - e\int \mathbf{v}_e f_e(\mathbf{v},\mathbf{x},t)d^3v + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$\frac{\partial f_e}{\partial t} + \nabla f_e.\mathbf{v} + \frac{e}{m}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\nabla_{\mathbf{v}}.f = 0$$

All of these equations are coupled. Derivatives of fields depend on the integrals of distribution and derivatives of distribution depend on the fields. The resultant system is non linear. We cannot solve it. What we can is that we can study the growth of perturbations made to the steady solution. This is how we can linearlize the system.

In a Magnetic Cataclysmic Variable, steady solution can be obtained if we put $E = 0$ and we can assume B to be constant in the the polar region of the donor star as it is supposed to be produced by the inner dynamo. We haven't included Vlasov equation for protons. It is assumed that in the time scales in which electron cyclotron maser operates, protons can be considered immovable. So, we have not adopted 2 fluid model here. Only constant magnetic field existed before EM perturbations grew. Perturbation in Electromagnetic field will perturb distribution function too and vice versa. If we replace all X by $X_0 + \delta X$ where X_0 is known quantity which is perturbed.Vlasov equation will then become,

$$(\frac{\partial}{\partial t} + \mathbf{v}.\nabla + \frac{e}{m}\mathbf{v}\times\mathbf{B}_0.\frac{\partial}{\partial\mathbf{v}})\delta f(\mathbf{v},\mathbf{x},t) = -\frac{e}{m}(\delta\mathbf{E}+\mathbf{v}\times\delta\mathbf{B}).\frac{\partial f_0}{\partial\mathbf{v}}$$

perturbed currents and perturbed charged density will be,

$$\delta\mathbf{J} = \sum_s e_s \int d^3v\mathbf{v}\delta f_s$$

$$\delta\rho = \sum_s \int d^3v\delta f_s$$

In the perturbed Vlasov equation, left hand side is a total time derivative and the right hand side describes change in it as a particle moves in phase space.So, formally the solution for δf will be,

$$\delta f(\mathbf{v}(t), \mathbf{x}(t), t) = -\frac{e}{m}\int_{-\infty}^{t} dt'\{\delta\mathbf{E}[\mathbf{x}(t'), t'] + \mathbf{v}(t') \times \delta\mathbf{B}[\mathbf{x}(t'), t']\}.\frac{\partial_0[\mathbf{v}(t')]}{\partial\mathbf{v}(t')}$$

Methods to obtain δf are developed by Melrose and Treumman which can be found in [12] and in [18]. Both assume that particle will be gyrating around the axis of B_0 and for a short time we can make an assumption that the motion of electron is still cyclotron motion. We are interested in harmonic solutions so harmonic time dependence of field is assumed. Treumann shows that,

$$\delta f(\mathbf{v}) = -\frac{e\delta\mathbf{E}(\mathbf{k}, \omega)}{m\omega}.\int_0^{\infty} d\tau e^{-\phi(\tau)}\{\mathbf{kv}(\tau) + \mathbf{I}[\omega - \mathbf{kv}(\tau)].\frac{f_0[\partial\mathbf{v}(\tau)]}{\partial\mathbf{v}(\tau)}$$

Where, $\tau = t' - t$, $\phi(\tau) = \omega\tau + \mathbf{k}.[\mathbf{x} - \mathbf{x}(\tau)]$ and $\mathbf{kv}$ denotes tensor with $k_i v_j$ as its components. ω is fourier counterpart of time. $\mathbf{k}$ is fourier counterpart of position. We can now use the above expression to compute δj. Perturbed Maxwell's equations are linear. We can eliminate magnetic field and obtain a linear relation between $\delta\mathbf{j}$ and $\delta\mathbf{E}$. Dielectric tensor for plasma is defined as a tensor which relates these two quantities.

$$\delta\mathbf{j}(\omega, \mathbf{k}) = -i\omega\epsilon_0[\mathbf{A}(\omega, \mathbf{k}) - \mathbf{I}]\delta\mathbf{E}(\omega, \mathbf{k})$$

For a wave, $\mathbf{k}$ denotes the direction in which the wave travels. We are interested in only those waves which travel parallel to the magnetic field and those which travel perpendicular to the magnetic field.

$$\delta\mathbf{j}(\omega, \mathbf{k}) = -sum_s\frac{\epsilon_0\omega_{ps}^2}{n_0\omega}\int_0^{\infty}\int_{-\infty}^{\infty}\int_0^{2\pi} v_\perp dv_\perp dv_\parallel d\psi \times \mathbf{E}(\omega, \mathbf{k})\int_0^{\infty} d\tau e^{-\phi(\tau)}\{\mathbf{kv}(\tau) + \mathbf{I}[\omega - \mathbf{kv}(\tau)].\frac{f_0[\partial\mathbf{v}(\tau)]}{\partial\mathbf{v}(\tau)}$$

ω_{ps} is plasma frequency of sth entity. We have defined dielectric tensor. By comparing the terms, Treumann computed the form of dielectric tensor.

- We need to perform integrations over velocity space. Integrals under analysis have singular points.
- $\mathbf{S}_{ls} = \begin{bmatrix} \frac{l^2\Omega_{gs}^2}{k_\perp^2}J_l^2 & \frac{ilv_\perp\Omega_{gs}}{k_\perp}J_l J_l' & \frac{lv_\parallel\Omega_{gs}}{k_\perp}J_l^2 \\ -\frac{ilv_\perp\Omega_{gs}}{k_\perp}J_l J_l' & v_\perp^2 J_l'^2 & -iv_\parallel v_\perp J_l J_l' \\ \frac{lv_\parallel\Omega gs}{k_\perp}J_l^2 & iv_\parallel v_\perp J_l J_l' & v_\parallel^2 J_l^2 \end{bmatrix}$ J_l and J_l' are respectively lth Bessel func-

tion and its derivative.

- The argument under bessel functions is $\frac{k_\perp}{\Omega_{gs}}$

$$\mathbf{A}(\omega,\mathbf{k}) = (1-\sum_s \frac{\omega_{ps}^2}{\omega})\mathbf{I}-\sum_s \sum_{l=-\infty}^{l=\infty} \frac{2\pi\omega_{ps}^2}{n_{0s}\omega^2}\int_0^\infty \int_{-\infty}^\infty v_\perp dv_\perp dv_\parallel (k_\parallel \frac{\partial f_{0s}}{\partial v_\parallel}+\frac{l\omega_{gs}}{v_\perp}\frac{\partial f_{0s}}{\partial v_\perp}\frac{\mathbf{S}_{ls}(v_\parallel, v_\perp)}{(k_\parallel v_\parallel - l\Omega_{gs}-\omega)}$$

This is macroscopic dielectric tensor. Now, we can consider plasma as a continuous media and apply Maxwell's equations in a continuous media where permittivity is replaced by this dielectric tensor. For an anisotropic dispersive media, Dispersion equation of a wave is,

$$Det[\frac{k^2c^2}{\omega^2}(\frac{\mathbf{kk}}{k^2}-\mathbf{A}(\omega,\mathbf{k})]=0$$

This relation is obtained by recoginizing $\delta\mathbf{D}(\omega,\mathbf{K}) = \mathbf{A}(\omega,\mathbf{K}).\delta\mathbf{E}(\omega,\mathbf{K})$ and by writing Maxwell's equations in fourier space.

All roots of this equations represent different waves. Plasma can sustain lots of types of waves. Cyclotron Maser Instability is a condition where few wave modes with frequency near cyclotron frequency exponential kind grow exponentially. This is not possible for every kind of background distribution. For Maxwellian background distribution, all waves experience damping. As discussed before, background distribution can be guessed by using Jeans' theorem. The integral when we compute dielectric tensor involves periodic singular points for each ω. This is the origin of cyclotron maser instability which will be discussed in the next section.

5.2 Growth of Cyclotron Maser Instability

When $\mathbf{A}(\omega,\mathbf{K})$ has particular form, The EM fluctuation may grow exponentially. This happens when absorption coefficient turns out to be negative. DB Melrose in [12], derived the formula for the absorption coefficient. We will focus only on perpendicular driven maser. It is also known that only -o and -x modes can escape plasma and both have circular polarization(see [4]). -o is left circularly polarized and -x is right circularly polarized. Out of 24 MCVs, some were emitting right circularly polarized radiation and some were emitting left circularly polarized radiation. We can use instability for -x mode and -o mode respectively. Dielecric tensor becomes scalar once we specify the mode. We will denote it by $\epsilon_{s,\sigma}$.

Resonance condition from the formula for the dielectric tensor is,

$$\omega - s\Omega - k_\parallel v_\parallel = 0$$

$$B(\mathbf{k}) = \int d^3\mathbf{p}\epsilon_{s,\sigma}\delta(\omega - \frac{s}{1}\Omega_e - k_\parallel v_\parallel)(\frac{s\Omega_e\partial}{v_\perp\partial p_\perp}+k_\parallel\frac{\partial}{\partial p_\parallel})f(\mathbf{p})$$

Where $B(\mathbf{k})$ is the absorption coefficient for the wave travelling in $\mathbf{k}$ direction. Distribution function is always positive. The value of B cannot have a great negative value under this condition especially for inverted-V distribution found in AKR zone. Cyclotron Maser instability becomes a failed model to explain any coherent radiation coming from a distant astrophysical object. This was noted by Twiss in 1958. The solution is the realization that cyclotron maser instability is intrinsically a relativistic phenomenon even though particles are not moving at relativistic speeds. The collective resonance effect is such that even very small relativistic correction to each electron matters and we have to make relativistic corrections. The relativistic resonance condition will be,

$$\omega - \frac{s}{\gamma}\Omega_e - k_\parallel v_\parallel = 0$$

This is an ellipse in velocity space and absorption coefficient will involve an integration around it.

$$B(\mathbf{k}) = \int d^3\mathbf{p}\epsilon_{s,\sigma}\delta(\omega - \frac{s}{\gamma}\Omega_e - k_\parallel v_\parallel)(\frac{s\Omega_e\partial}{\gamma v_\perp \partial p_\perp} + k_\parallel \frac{\partial}{\partial p_\parallel})f(\mathbf{p})$$

$$(\frac{s\Omega_e\partial}{\gamma v_\perp \partial p_\perp} + k_\parallel \frac{\partial}{\partial p_\parallel})f(\mathbf{p}) > 0$$

This should hold for points on the boundary of the resonance ellipse if we want growth of the wave. Melrose notes,"The relativistic effect cannot be ignored when the radius of the resonance circle is comparable with the speed of the electrons that drive the maser. The paradox that one must include the relativistic correction to treat perpendicular driven maser emission even for non relativistic electrons, is resolved by noting that the non relativistic approximation is formally $c \to \infty$, whereas c is necessarily finite when we are treating electromagnetic radiation."(see [15]). This comment of Melrose is important even for fundamental physics. Cyclotron Maser instability is an example of a physical process where Newtonian picture fails even for slow moving particles. Radiation itself is a relativistic phenomenon. Maxwell's equations are consistent with relativity and they are independent from Newtonian dynamics. Only interaction between fields and particle is through Lorentz force which is interpreted a bit differently in relativistic picture. It can be easily checked that loss Cone and inverted -V distributions satisfy this condition.

For a given distribution and given background magnetic field, we can directly infer the shape of the spectral profile. To obtain angular profile and absolute values of intensities at particular frequencies, further calculations have to be performed. They are excluded from this thesis.

The independent work done up-to now and the conclusions drawn from it are essential to model an MCV emitting cyclotron maser emission. I hope that the thesis will work as a stepping stone towards solving a much larger and specific problem.

References

[1] Paul Barrett, Christopher Dieck, Anthony J. Beasley, Paul A. Mason, and Kulinder P. Singh. Radio observations of magnetic cataclysmic variables. *Advances in Space Research*, 66(5):1226–1234, Sep 2020.

[2] Wolfgang Baumjohann and Rudolf Treumann. *Basic Space Plasma Physics (Revised Edition)*. World Scientific Publishing Company, 2012.

[3] Claudio Chiuderi and Marco Velli. *Basics of plasma astrophysics*. Springer, 2015.

[4] George A Dulk. Radio emission from the sun and stars. *Annual review of astronomy and astrophysics*, 23:169–224, 1985.

[5] JH Field. Retarded electric and magnetic fields of a moving charge: Feynman's derivation of li\'enard-wiechert potentials revisited. *arXiv preprint arXiv:0704.1574*, 2007.

[6] Peter Goldreich and Donald Lynden-Bell. Io, a jovian unipolar inductor. *The Astrophysical Journal*, 156:59–78, 1969.

[7] Herbert Goldstein, Charles Poole, and John Safko. Classical mechanics, 2002.

[8] J. L. Hirshfield. The electron cyclotron maser—an historical survey. In *1976 International Conference on Submillimeter Waves and Their Applications*, pages 90–92, 1976.

[9] John David Jackson. Classical electrodynamics, 1999.

[10] Oleg D Jefimenko. Direct calculation of the electric and magnetic fields of an electric point charge moving with constant velocity. *American journal of physics*, 62(1):79–85, 1994.

[11] Christian Knigge. The donor stars of cataclysmic variables. *Monthly Notices of the Royal Astronomical Society*, 373(2):484–502, 2006.

[12] D. B. Melrose and G. A. Dulk. Electron-cyclotron masers as the source of certain solar and stellar radio bursts. , 259:844–858, August 1982.

[13] DB Melrose. A plasma-emission mechanism for type i solar radio emission. *Solar Physics*, 67(2):357–375, 1980.

[14] DB Melrose and ME Gedalin. Relativistic plasma emission and pulsar radio emission: a critique. *The Astrophysical Journal*, 521(1):351, 1999.

[15] Donald B Melrose. *Instabilities in space and laboratory plasmas*. 1986.

[16] Clément Mouhot, Cédric Villani, et al. On landau damping. *Acta mathematica*, 207(1):29–201, 2011.

[17] Rudolf A. Treumann. The electron-cyclotron maser for astrophysical application. , 13(4):229–315, August 2006.

[18] Rudolf A Treumann and Wolfgang Baumjohann. *Advanced space plasma physics*, volume 30. Imperial College Press London, 1997.

[19] C. S. Wu and L. C. Lee. A theory of the terrestrial kilometric radiation. , 230:621–626, June 1979.