Signatures of quantum effects in late time cosmology

Ankit Dhanuka

A thesis submitted for the partial fulfillment of the degree of Doctor of Philosophy

Department of Physical Sciences

Indian Institute of Science Education & Research (IISER) Mohali,
Knowledge City, Sector 81, SAS Nagar, Manauli PO 140306 Punjab, India

April 2023

dedicated to my family

Declaration

The work presented in this thesis has been carried out by me under the guidance of Dr. Kinjalk Lochan at the Indian Institute of Science Education and Research Mohali. This work has not been submitted in part or in full for a degree, a diploma, or a fellowship to any other university or institute. Whenever contributions of others are involved, every effort is made to indicate this clearly, with due acknowledgment of collaborative research and discussions. This thesis is a bona fide record of original work done by me and all sources listed within have been detailed in the bibliography.

Ankit Dhanuka

Place: ________________

Date: ________________

In my capacity as the supervisor of the candidate's thesis work, I certify that the above statements by the candidate are true to the best of my knowledge.

Dr. Kinjalk Lochan
Assistant professor
Department of Physical Sciences
IISER Mohali

Place: ________________

Date: ________________

List of Publications

This thesis is based on the following papers.

1. **In Journals:**

   - Stress energy correlator in de Sitter spacetime: Its conformal masking or growth in connected Friedmann universes.

     Ankit Dhanuka and Kinjalk Lochan.

     Physical Review D 102, 085009 (2020).

   - Fluctuations in the stress energy tensor of spinor fields evolving in general FRW spacetimes

     Ankit Dhanuka.

     Physical Review D 106, 023518 (2022).

   - Unruh deWitt probe of late time revival of quantum correlations in Friedmann spacetimes.

     Ankit Dhanuka and Kinjalk Lochan.

     Physical Review D 106, 125006 (2022).

2. **In Proceedings of Conferences:**

   - Behaviour of noise kernel in de Sitter and FRW space-times.

     Ankit Dhanuka and Kinjalk Lochan.

     In the Proceedings of Sixteenth Marcel Grossmann Meeting, Virtual Meeting - July 5-10, 2021.

Abstract

Quantum field theory in curved spacetime is an important framework not only from the point of view of understanding conceptual notions like particle creation, Hawking radiation, etc., but it has turned out to be useful in explaining many cosmological observations. Particularly, the quantization of the metric perturbations during the inflationary phase of the Universe seems to provide a good explanation for the observed temperature anisotropies in the cosmic microwave background. This thesis explores different aspects of quantum field theories in cosmologically important FRW spacetimes. In the semiclassical gravity approach, one is mainly concerned with the expectation value of the stress-energy operator and ignores the quantum fluctuations in it assuming them to be insignificant. However, these considerations, based solely on the first-order effects, are bound to fail in case the quantum fluctuations are significant. The stochastic gravity paradigm considers these fluctuations and quantifies them by the noise kernel. We show that, for scalar fields in de Sitter spacetime, the late time limit of the noise kernel shows a transition from vanishing to divergent behavior as the ratio, m/H, is changed in the range $[0, 3/2]$. Similarly, the noise kernel is found to diverge for massless scalar fields in certain FRW spacetimes. In those cases in which the noise kernel is non-vanishing (and comparable to the expectation values) or divergent, the first-order semiclassical analyses are expected to break down and must be supplemented with the second-order effects in order to make any robust predictions. For massive spinor fields in de Sitter spacetime, the late time limit of the noise kernel vanishes irrespective of the mass of the spinor field. As far as massless spinor fields in FRW spacetimes are concerned, the late time limit of the noise kernel vanishes for expanding FRW spacetimes whereas it diverges for contracting FRW spacetimes. In addition to the noise kernel behavior, one can also study the dynamics of quantum fields in FRW spacetimes by coupling them with Unruh deWitt (UdW) detectors. This thesis includes analysis for the case of both conventional as well as derivatively coupled UdW detectors with a particular focus on studying the infrared divergences in FRW spacetimes. We find that the infrared divergence of massless scalar fields in de Sitter spacetime contributes to the response rate of conventional UdW detector whereas it does not for derivatively coupled UdW detector. However, for massless scalar fields in nearly matter-dominated spacetimes, the infrared divergences contribute to the rates of both types of UdW detectors. Applying these results to the coupling of gravitational waves (GWs) with a hydrogen atom, which takes the form of a generalized derivative UdW coupling, it is argued that the quantized GWs lead to very rapid transitions within the states of a hydrogen atom while the Universe passes through matter-dominated phase in the expansion history. These conclusions provide an opportunity to witness quantum effects in relatively later phases of the Universe as opposed to the quantum effects studied mostly for the early inflationary

phase of the Universe. These investigations suggest that the cosmological observations corresponding to the later phases of the Universe may also contain potential quantum signatures.

List of Figures

Contents

Chapter 1

Introduction

The proposal of the existence of an exponentially accelerating inflationary phase [1, 2] for the early Universe provides mechanisms to explain a number of cosmological observations, for example, the temperature anisotropies in the cosmic microwave background (CMB), the large-scale structure of the Universe, etc. [3, 4]. For the Universe to undergo an exponentially accelerating phase, it is required that the fluid (or matter) driving the Universe through such a phase has negative pressure. The most commonly used model for inflation assumes that the matter content of the Universe during inflation is carried by a scalar field (called inflaton) in the presence of a potential [5–8]. By imposing that the form of the potential satisfies certain 'slow roll' conditions, it can be ensured that the scalar field drives the Universe through a (near de Sitter) inflationary phase. Inflation provides resolutions to many issues associated with the hot big bang model of the Universe like the horizon problem, flatness problem, etc. [5, 8, 9]. However, the most remarkable predictions, that inflation provides, come from analyses which combine the notions of both quantum theory and general relativity. The inflaton field is taken to be independent of spatial coordinates, which gives rise to the average homogeneous and isotropic dynamics of the Universe. On the other hand, the large-scale structure of the Universe represents its deviations from a purely homogeneous and isotropic evolution. These deviations can be accounted for by considering perturbations over the otherwise homogeneous and isotropic spacetime and the corresponding matter content of the Universe [5–8, 10]. In fact, quantum mechanical treatment of these perturbations during inflation gives rise to predictions which fit well with the observations like the temperature anisotropies in the CMB, the large-scale structure of the Universe, etc [3, 4]. Hence, this success of the inflationary paradigm suggests a quantum mechanical origin of the present day Universe. Since these metric and matter perturbations or their combinations evolve over the (near de Sitter) inflationary phase of the early Universe, treating them quantum mechanically requires us to apply the concepts of quantum field theory in curved spacetime. Thus, we see that the early Universe provides us with a very rare situation where both quantum theory and relativity are important and one can treat the early universe as a test bed for theories which try to build a unified framework for gravity and quantum theory. The predictions that one obtains by considering quantum treatment of perturbations, depend upon the choice of initial state in which the perturbations are placed at the start of the inflation. The choice of initial state for perturbations amounts to assuming specific initial conditions for the Universe. For example, by placing the perturbations in the 'Bunch-Davies' vacuum [11–13], we obtain a nearly scale-invariant power spectra for these perturbations which matches well with many cosmological observations like CMB anisotropies, etc. [5, 7, 8]. Different choices [14–16] for the initial state like non Bunch-Davies vacua or non-vacuous states can also be made, provided that the chosen initial state gives an almost scale-invariant power spectrum with some small characteristic features of its own. One can expect to be able to distinguish between different initial states only with more detailed cosmological data which would put further constraints on the allowed initial states. Apart from the choice of initial state for perturbations, there also exists a plethora of different inflationary models which try to generate the observationally found near scale-invariant power spectrum for CMB anisotropies. These different models [17] also predict their own additional characteristic features for the cosmological data and hence, can be constrained with the availability of detailed data. Therefore, there are many directions in which the quantum origin of our Universe is being explored [5, 8, 17, 18]. In this chapter, we review some of these aspects of early Universe physics. We discuss how the early Universe provides a scenario where both general relativity and quantum mechanics play an important role and gives rise to results that can be observationally tested. Particularly, we recall the basics of cosmological perturbation theory and the decomposition of metric and matter perturbations into scalar, vector and tensor parts. By treating these perturbations upto leading order and taking the case of a single scalar field driven inflation, we discuss the dynamics of both scalar and tensor perturbations. We consider the quantization of both scalar and tensor perturbations and their corresponding scale-invariant power spectra. The resultant power spectra for scalar and tensor perturbations are related to the temperature and polarization maps, respectively, of the CMB data. In addition to these considerations, there are also other related directions where quantum effects in early Universe physics are expected to play an important role, for example, the transPlanckian issues in early Universe physics, primordial magnetogenesis, effects of non-vacuous states for the fluctuations, etc. We provide a brief overview of some of these avenues in early Universe physics where quantum effects are important before going on to discuss a potential revival of quantum correlations in late time cosmology.

1.1 Cosmological perturbation theory

Observations [3, 4, 19] tell us that the homogeneity and isotropy of the Universe is only an approximation. There exist perturbations over the otherwise homogeneous and isotropic FRW geometry of the Universe and these perturbations are believed to give rise to galaxies, stars, etc., that we observe in the Universe and also the temperature anisotropy in the cosmic microwave background (CMB). The magnitude of these perturbations is, however, very small. For example, the temperature variations relative to the average temperature in the CMB are of the order of 10^{-5} [19]. Therefore, one can treat these departures from the homogeneous and isotropic FRW geometry by employing perturbation theory and maintaining terms upto first order in the perturbations. The perturbed FRW geometry of the spacetime can be written as follows [5–8, 10]

$$\begin{aligned} ds^2 &= a^2\Big(\eta_{\mu\nu}+h_{\mu\nu}\Big)dx^\mu dx^\nu \\ &= a^2\Big\{-(1+2\phi)d\eta^2+2(\partial_i B-S_i)d\eta dx^i+[(1-2\psi)\delta_{ij}+2\partial_i\partial_j E+2\partial_{(i}F_{j)}+h_{ij}]dx^i dx^j\Big\}, \end{aligned} \tag{1.1}$$

where $\partial^i S_i=\partial^i F_i=\partial^i h_{ij}=\partial^j h_{ij}=0$, $h_{ij}\delta^{ij}=0$ and $h_{ij}=h_{ji}$.

In the above expression, the perturbations to the FRW metric have been decomposed into scalar, vector, and tensor perturbations. ϕ,ψ,B and E are scalar perturbations, S_i and F_i are vector perturbations and h_{ij} are tensor perturbations.

Using the Einstein equations and the form of the Einstein tensor for unperturbed FRW metric given in Appendix A, one can show that the stress energy tensor driving the Universe through unperturbed FRW phase can be written in a perfect fluid form [5, 9]. Einstein's equations require that there be perturbations in the stress energy tensor corresponding to the perturbations over the FRW metric. These perturbations can be written as [5]

$$\begin{aligned} \delta T_{ij} &= \bar{p}a^2 h_{ij}+a^2\Big(\delta_{ij}\delta p+\partial_i\partial_j\pi^s+2\partial_{(i}\pi^V_{j)}+\pi^T_{ij}\Big) \\ \delta T_{0i} &= \bar{p}a^2 h_{0i}-a(\bar{p}+\bar{\rho})(\partial_i\delta u+\delta u^V_i) \\ \delta T_{00} &= -\bar{\rho}a^2 h_{00}+a^2\delta\rho\,, \end{aligned} \tag{1.2}$$

where $\partial^i\delta u^V_i=\partial^i\pi^V_i=\partial^i\pi^T_{ij}=\partial^j\pi^T_{ij}=0$, $\pi^T_{ij}\delta^{ij}=0$ and $\pi^T_{ij}=\pi^T_{ji}$. The quantities $\bar{p}$ and $\bar{\rho}$ are the unperturbed pressure and energy density that drive the background spacetime through a purely FRW phase and all other quantities in the above expressions are perturbations.

Using the above expressions for the perturbed stress energy tensor and the perturbed metric in Einstein's equations and the conservation equations of the stress energy tensor and maintaining terms up to first order in perturbations, one can find the equations which govern the evolution of perturbations [5, 6, 10]. One important issue in cosmological perturbation theory is that of gauge fixing. One finds that one can modify the scalar and vector perturbations by changing the coordinate system for the background FRW metric and hence some of the scalar and vector perturbations ought to be unphysical. The tensor perturbations, however, remain unaffected by coordinate transformations and are all physical except for the constraints imposed by the transverse and traceless properties [5, 8]. We can get rid of the unphysical scalar and vector modes by choosing to work in a particular gauge. There exist many choices for gauges in the literature, for example, the Newtonian gauge, Synchronous gauge, etc [5, 8, 20, 21]. Particularly, for Newtonian gauge, one has $B = E = 0$ and the only scalar perturbations that remain are $\Phi \equiv \phi$ and $\Psi \equiv \psi$.

Let us take a scalar field, called the inflaton field [2], which drives the inflationary phase of the early universe with the following action

$$S = \int d^4x\sqrt{-g}\Big(-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\Big)\,. \tag{1.3}$$

The equation of motion for the above scalar field is given by

$$\varphi'' + 2\frac{a'}{a}\varphi' - \nabla^2\varphi + a^2 V_{,\varphi} = 0\,, \tag{1.4}$$

where $'$ on φ and a denotes a derivative with respect to η and $V_{,\varphi}$ represents the derivative of V with respect to φ. The stress energy tensor for the above scalar field is given by

$$T_{\alpha\beta} = \partial_\alpha\varphi\partial_\beta\varphi - g_{\alpha\beta}\Big(\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\Big)\,. \tag{1.5}$$

For this scalar field to drive the homogeneous and isotropic inflationary phase, it has to be a function only of the time coordinate i.e., $\varphi(x) = \bar{\varphi}(\eta)$. Thus, the inflaton field satisfies the following equation of motion i.e.,

$$\bar{\varphi}'' + 2\frac{a'}{a}\bar{\varphi}' + a^2\frac{\partial V}{\partial\bar{\varphi}} = 0\,. \tag{1.6}$$

The energy density and pressure corresponding to the inflaton field are given by

$$\bar{\rho} = \left(\frac{\bar{\varphi}'^2}{2a^2} + V(\bar{\varphi})\right), \tag{1.7}$$

$$\bar{p} = \left(\frac{\bar{\varphi}'^2}{2a^2} - V(\bar{\varphi})\right). \tag{1.8}$$

Using the above form of the energy density and the Friedmann equation (2.9) for flat FRW spacetimes i.e., $k = 0$, we obtain that

$$\frac{a'^2}{a^4} = \frac{8\pi G}{3}\left(\frac{\bar{\varphi}'^2}{2a^2} + V(\bar{\varphi})\right). \tag{1.9}$$

The equation of state parameter corresponding to the inflaton field is given by

$$w = \frac{p}{\rho} = \frac{\frac{\bar{\varphi}'^2}{2a^2} - V(\bar{\varphi})}{\frac{\bar{\varphi}'^2}{2a^2} + V(\bar{\varphi})}. \tag{1.10}$$

In order to have an exponentially accelerating expansion of the Universe i.e., the de Sitter phase, the equation of state parameter, w, should be -1 (see Chapter 2). We see that, for $\bar{\varphi}'^2 << |V(\bar{\varphi})|$, we can have $p \approx -\rho$ and $w \approx -1$. In fact, one can define a set of parameters, ε_1 and ε_2, called slow-roll parameters,

$$\varepsilon_1 = \frac{1}{16\pi G}\left(\frac{V_{,\varphi}}{V}\right)^2 \quad \text{and} \quad \varepsilon_2 = \frac{1}{8\pi G}\frac{V_{,\varphi\varphi}}{V}, \tag{1.11}$$

such that $\varepsilon_1, |\varepsilon_2| << 1$ is called the slow-roll condition and it ensures that the spacetime expansion is approximately exponentially accelerating [5, 6, 8, 22]. There exists a large number of inflaton field models which differ from each other via the form of their potentials that are used to generate the inflationary phase (See [17] and references therein).

1.1.1 Mukhanov-Sasaki equation

Let us consider perturbation, $\delta\varphi(x)$, to the inflaton field that drives the metric perturbations in the FRW spacetimes i.e.,

$$\varphi(x) = \bar{\varphi}(\eta) + \delta\varphi(x). \tag{1.12}$$

One can define a gauge invariant quantity called the 'comoving curvature perturbation' which is given by

$$R = -\Psi - \frac{a'\delta\varphi}{a\bar{\varphi}'}, \tag{1.13}$$

and in fact, it can be shown to satisfy the following equation of motion

$$R'' + 2\frac{z'}{z}R' - \nabla^2 R = 0\,, \tag{1.14}$$

where $z = \frac{a^2\bar{\varphi}'}{a'}$. This equation is the famous Mukhanov-Sasaki equation. Another quantity $v = zR$ is also easily seen to satisfy the following equation of motion

$$v'' - \nabla^2 v - \frac{z''}{z}v = 0\,, \tag{1.15}$$

which is the same equation as that of a massless scalar field in an FRW spacetime with scale factor z (see equation (2.24)). For a purely de Sitter phase, we consider the solutions of the above equation in the next chapter and choose a particular vacuum state called the Bunch-Davies vacuum [5, 11] . On superhorizon scales i.e., $-k\eta << 1$, the power spectrum of the quantity, $\psi \equiv \frac{v}{a}$, for the Bunch-Davies vacuum is found to be [6, 8]

$$\langle \psi_{\vec{k}} \psi_{\vec{k}'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') \frac{H^2}{2k^3}\,. \tag{1.16}$$

From the above expression, we see that the quantity ψ has a scale-invariant power spectrum. In fact, one also finds that, on super-horizon scales, the power spectrum of the comoving curvature perturbations, $\Delta_R^2(k)$, is [6]

$$\Delta_R^2(k) = \frac{1}{2m_{pl}^2 \varepsilon_1} \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{n_R - 1}, \tag{1.17}$$

where $n_R - 1 = 2\varepsilon_2 - 6\varepsilon_1$ and n_R is called the spectral index of the comoving curvature perturbation. This power spectrum can be related to the temperature anisotropies in the CMB data [8]. Thus, we obtain an almost scale-invariant power spectrum which agrees with the statistics of the CMB data [23]. The Planck collaboration estimates $n_R = 0.965 \pm 0.004$ [23].

1.1.2 Gravitational waves

The tensor perturbations to the background FRW dynamics of the Universe can also be quantized in a similar manner. The dynamics of the tensor perturbations, h_{ij}, is decided by the following action

$$S = \frac{1}{8} \int d\eta\, d^3\vec{x}\, a^2 \left((h_{ij}')^2 - (\nabla h_{ij})^2 \right), \tag{1.18}$$

where $h_{ij}\delta^{ij} = 0$ and $\partial^i h_{ij} = \partial^j h_{ij} = 0$. The above action for each component of the perturbations is just the same as that of a massless scalar field in an FRW background and hence they satisfy the same equation of motion. However, the transverse traceless conditions on the tensor perturbations leave only two independent components and hence the gravitational waves are dynamically equivalent to two massless scalar fields in FRW backgrounds.

In addition to the temperature anisotropy map of the CMB, the polarization map of the CMB is also expected to provide important information which would help constrain different inflationary models. The electric field associated with CMB photons is polarized in the plane orthogonal to their direction of motion [5]. The polarization of the electric field in these orthogonal planes can be decomposed in E-modes and B-modes [24, 25]. The E-modes and B-modes have different parity transformations in the plane of electric fields [8]. It can be shown [5, 24, 25] that the B-modes do not get any contribution from scalar fluctuations and get contribution only from tensor fluctuations. Hence, the detection of B-mode polarization in CMB data provides a probe of primordial tensor perturbations. The auto-correlation of B-mode polarization can be related to the power spectrum of the tensor fluctuations [8] which we now calculate for our inflaton field driven inflationary model.

The quantized gravitational waves can be expanded as

$$\hat{h}_{ij}(\vec{x},\eta) = \sum_{\lambda=+,\times} \int d^3\vec{q}\; e_{ij}(\hat{q},\lambda)\left(e^{i\vec{q}.\vec{x}} h_q(\eta)\hat{b}_{\vec{q},\lambda} + e^{-i\vec{q}.\vec{x}} h_q^*(\eta)\hat{b}^\dagger_{\vec{q},\lambda}\right), \tag{1.19}$$

where $\lambda = +,\times$ refer to two polarization states. The $e_{ij}(\vec{q},\lambda)'s$ satisfy [5]

$$\sum_{\lambda=+,\times} e_{ij}(\hat{q},\lambda)e_{kl}(\hat{q},\lambda) = \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl} + \delta_{ij}\hat{q}_k\hat{q}_l + \delta_{kl}\hat{q}_i\hat{q}_j - \delta_{ik}\hat{q}_j\hat{q}_l - \delta_{il}\hat{q}_j\hat{q}_k - \delta_{jk}\hat{q}_i\hat{q}_l - \delta_{jl}\hat{q}_i\hat{q}_k + \hat{q}_i\hat{q}_j\hat{q}_k\hat{q}_l\,. \tag{1.20}$$

Also, $\hat{b}_{\vec{q},\lambda}$ and $\hat{b}^\dagger_{\vec{q},\lambda}$ are the annihilation and creation operators for a state with wave-vector, $\vec{q}$, and polarization, λ. The $h_q(\eta)$ is the time dependent component of the mode functions. Since the dynamics of gravitational waves is just that of a massless scalar field in FRW spacetimes, we can choose the Bunch-Davies vacuum for them as well. On super-horizon scales, the power spectrum of the tensor perturbations, $\Delta_T^2(k)$, is given by [6]

$$\Delta_T^2(k) = \frac{8}{m_{pl}^2}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{n_T}. \tag{1.21}$$

where $n_T = -2\varepsilon_1$ is called the spectral index of tensor perturbations. Thus, the power spectrum for tensor perturbations is almost scale-invariant as well.

One can consider the ratio of the tensor and scalar power spectra, on horizon crossing, called the tensor-scalar ratio and denoted by r. It is given by

$$r = \Delta_T^2(k = aH)/\Delta_R^2(k = aH) = 16\varepsilon_1\,. \tag{1.22}$$

One of the major goals of the ongoing and planned missions [26] to probe the more detailed statistics of the CMB is to determine the values of spectral index n_R and the tensor-scalar ratio r. Since the power spectra for scalar and tensor perturbations are defined in terms of the correlations of these perturbations, they carry important information about the quantum correlations of these perturbations. Apart from these observables, there are other avenues where quantum correlations are decisive in setting up of observable effects and we discuss some of them next. All these different effects that we discuss below depend upon the quantum correlations of metric and matter fluctuations and/or other different fields (like Maxwell field) during the inflationary universe. So observing such effects tells us about the quantum nature of the early Universe too, just as the temperature anisotropies in the CMB map, etc. Additional information about the workings of the early Universe is expected to be obtained by observing the below-discussed effects.

1.2 Some important avenues capturing quantum features in early universe physics

- **Trans-Planckian issues for inflationary cosmology**
  The trans-Planckian issues in inflationary cosmology [27, 28] refers to the projection of modes of super-Planck scales at the start of the inflation to the sub-Planckian and sub-Horizon scales which are available for observations today. Models of inflation that are generally considered to explain the observed CMB data last for a 'sufficiently' long period. During this period of near exponential expansion of the Universe, the physical distances expand exponentially so much so that the distances which are smaller than the Planck distance at the start of the inflation get expanded to distances which are larger than the Planck distance and in fact, by the end of the inflation, they become even larger than the Hubble radius (see Fig. (1.1)). These modes remain super-horizon for most of the Universe's evolution after the end of the inflationary phase and have only recently become sub-horizon to become available to be observed. This picture presents to us some serious conceptual issues as we explain the cosmological observations by applying the standard quantum field theory techniques during the inflationary phase of the Universe which are, however, expected to fail at the distances (energies) which are smaller (higher) than the Planck distance

(energy). As the below-Planck scale lengths are expanded to cosmologically observable scales during inflation, it is expected that they may carry signatures of their past below-Planck length life with them in their new above-Planck length abode. Hence, other than raising doubts about the applicability of known laws (which have been verified only in the above-Planck distance regimes) to these crossing modes, this scenario provides us an opportunity to probe trans-Planckian physics through cosmological observations at the scales which are not achievable with particle physics accelerators. These issues have been addressed with a number of proposals for dealing with initially trans-Planckian modes which have now become sub-Planckian. These include using modified dispersion relations for these crossing modes to account for a possibly different physics at trans-Planckian scales.

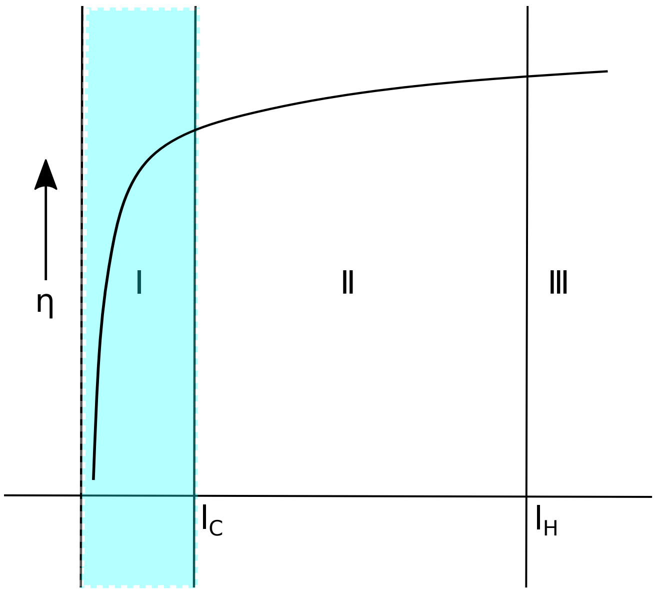


Figure 1.1: In this figure, l_C represents a cutoff length (say, Planck length) and l_H represents the Hubble radius at the end of inflation. Therefore, region I (the shaded area) is below-Planck length regime, region II is above-Planck length and sub-Hubble regime and region III is both above-Planck length and super-Hubble regime. The curve in the figure represents the projection of below-Planck wavelengths to above-Planck lengths and super-Hubble regimes during the inflationary phase. Here y-axis represents time. This figure is drawn along the lines of a similar figure in [27].

For example, [27, 29] discuss two types of modifications of dispersion relations where for one type, the power spectrum is not affected by the new physics at trans-Planckian scales whereas the other type of modification leads to changes in the prediction of scale-invariant power spectrum for primordial fluctuations. Particularly, [27] divides the length scale into three regions with two parameters l_C (represents the Planck-scale) and l_H (represents the Hubble Radius). From Eq. (1.15), for any particular comoving wavenumber, one obtains

$$v''_{\vec{k}} + k^2 v_{\vec{k}} - \frac{z''}{z} v_{\vec{k}} = 0\,. \tag{1.23}$$

Then, [27] assumes that the effect of different physics in super-Planckian regimes is to modify the dispersion relation i.e., $k = a(\eta)F(k/a(\eta))$. Using the Unruh dispersion relation [30] i.e.,

$$F(k) = k_C\, tanh^{\frac{1}{p}}\left[\left(\frac{k}{k_C}\right)^p\right], \tag{1.24}$$

where $k_C = \frac{2\pi}{l_C}$ and p is an arbitrary parameter and considering the minimum energy state with this dispersion relation, [27] shows that the power spectrum, calculated for modes that are outside the horizon at the end of inflation, remains the same as that with standard dispersion relation i.e., $F(k) = k$. Other studies [31–33] have also investigated the imprints of Planck scale physics in terms of modified dispersion relations for trans-Planckian modes on the power spectrum of primordial fluctuations. Sometimes, as in [34–37], the high-energy trans-Planckian physics is encoded through modified position and momentum commutation relations and their implications are investigated for cosmological observations. Other proposals [38–41] to take into account possible trans-Planckian physics are also considered like considering non-standard vacua for initially trans-Planckian modes.

- **Choice of initial state**

  In order to obtain the almost scale-invariant power spectrum for primordial fluctuations, generally all the modes are chosen to start in the Bunch-Davies vacuum in nearly de Sitter inflationary phase of the Universe. But one can start with more general states than the Bunch-Davies vacuum state and still find predictions which are in good agreement with the observed data. For example, [42] argues that coherent states with some constraints are as good as Bunch-Davies state with regard to explaining the observed data. In particular, [42] considers a coherent state, $|C\rangle$, such that $\hat{a}_{\vec{k}}\,|C\rangle = C(\vec{k})\,|C\rangle$ and $\langle C|C\rangle = 1$ where $\hat{a}_{\vec{k}}$ are the annihilation operators corresponding to the Bunch-Davies vacuum. Now imposing the constraint that $C(\vec{k}) = C^*(-\vec{k})$, one can show that, in superhorizon limit, the power spectrum of the comoving curvature perturbation is given by

$$\Delta^2_R(k) = \frac{1}{2m^2_{pl}\varepsilon_1}\left(\frac{H}{2\pi}\right)^2, \tag{1.25}$$

  which is the same as the power spectrum for the Bunch-Davies state, Eq. (1.17), with $n_R \approx 1$. Similarly, [43] considers an initial state with a thermal distribution and finds bounds on the corresponding temperature to fit the predictions with the observations. These more general states would also have their own characteristic features which are expected to be available for probing in future observations. For example, [16] considers the effects of modifications in the initial state on the CMB anisotropy, the distortion in the CMB black body radiation and in the Large scale structure (LSS) of the Universe. Similarly, [14, 44–47] explore non-Gaussianity in CMB data because of non-vacuum choice for the initial state of the primordial fluctuations. These studies are expected to provide useful insight into the pre-inflationary era as what happens during that phase of the Universe's history would decide the initial state at the start of the inflation. This line of investigation is also closely related to the trans-Planckian issues as the Planck scale provides a natural cutoff with respect to which one can treat the sub-Planckian and trans-Planckian modes differently and place them in different vacuum states as pointed out above. For example, [15] considers non-vacuum states by assuming an inherent scale in the initial state with respect to which some modes are taken to be vacuous whereas some are taken to be non-vacuous with non-zero particle content and their power spectra are compared with the observations. Different types of initial states for inflationary era and their effects on power spectrum have also been considered by assuming a radiation-dominated pre-inflationary era [48–50].

- **Inflationary magnetogenesis**

  There have been many proposals [51–53] to explain the magnetic fields present in the Universe by considering a primordial inflationary origin for them. Since the standard electromagnetic action is conformally invariant, it can be shown that the energy density of magnetic fields decreases due to the inflationary expansion of the early Universe [54, 55]. However, by breaking the conformal invariance of the standard Maxwell's action with modified actions, one can hope to have observationally significant magnetic fields. For example, one can consider the following action for electromagnetic fields [54–56] i.e.,

$$S = -\frac{1}{16\pi}\int d^4x\sqrt{-g}\, J^2(\varphi)F_{\mu\nu}F^{\mu\nu}\,, \tag{1.26}$$

where $J(\varphi)$ breaks the conformal invariance of the standard Maxwell's action and φ is the inflaton field which drives the universe through inflationary phase. The inclusion of a term like $J(\varphi)$ modifies the equation of motion for the vector potential and the mode functions for the vector potential are no longer the plane wave solutions of the flat spacetime. Quantizing the vector potential, one can find the energy density of both magnetic and electric fields by taking the vacuum expectation value of the stress energy operator corresponding to the above action. In fact, specializing to the de Sitter spacetime with scale factor $a(\eta) = -(1)/(H\eta)$ and taking $J(\eta) \propto (\eta)^{-2}$, one finds that the power spectrum for the magnetic field is scale invariant [55] i.e.,

$$P_{magnetic}(k) = \frac{9H^4}{4\pi^2}, \tag{1.27}$$

at horizon crossing. Thus, with the chosen action, one can find a scale invariant power spectrum of magnetic field generated during the inflationary phase. As already said, there exist many proposals for modifying Maxwell's actions with conformal breaking terms or parity violating terms etc [51, 52, 57]. The purpose of these considerations is to obtain a mechanism for primordial generation of observed magnetic fields in the Universe while also respecting the constraints coming from other considerations like CMB power spectra, etc.

In addition to the above considerations, there are other quantum effects that are believed to be important during the early inflationary (near de Sitter) phase of the Universe and are expected to be detected in more precise future observations. For example, the linear and uncoupled equations for scalar and tensor perturbations that we discussed above are obtained by expanding the action up to second order in perturbations but one can also study the contribution to the action coming from expansion of the action to third or higher orders in perturbations. The resultant new terms [7, 58] would provide self-interactions of the scalar and tensor perturbations and also couple the scalar and tensor perturbations. These interaction terms give rise to non-Gaussianities [7, 58, 59] in CMB data and we should be able to test them with the availability of more detailed data.

Most of the studies which consider quantum effects in cosmological contexts are generally considered for the early Universe physics. It can be shown [60–62] that, starting with the Bunch-Davies state as the initial state for quantized metric/matter perturbations, the quantum state evolves to become highly squeezed for all those modes which cross the Hubble radius by the end of inflation. The highly-squeezed modes remain frozen as such for much of the evolution of the Universe until they re-enter the region inside the Hubble radius to become available to be observed. For highly squeezed states, the quantum averages are equivalent to the averages of classical ensemble of perturbations with stochastic Gaussian amplitudes [61–63]. Also, for expanding Universe, the comoving distances get enhanced to large physical separations and the quantum correlations are expected to decay for large physical separations [64]. With all these observations, one expects that the quantum treatment of perturbations during later phases (after inflation) of the Universe (for example, radiation and matter dominated epochs) should not be much different from a classical treatment. However, in a recent work [65], it has been shown that the quantum correlations of scalar fields during these later phases of the Universe (e.g., for matter dominated era and/or dark energy era) may still remain significant. Thus, it should be interesting to probe whether quantum effects show revival during these phases of the Universe. Keeping this motivation in mind and the fact that these later phases of the Universe are modelled by FRW spacetimes, this thesis studies quantum correlations of fields in de Sitter and FRW spacetimes through noise kernel and Unruh deWitt detectors as they depend upon the quantum correlations of background fields.

1.3 Outline of the thesis

We divide this thesis in 7 chapters including this one.

- In chapter 2, we discuss very briefly some basic notions about the objects of study in this thesis i.e., the noise kernel and UdW detectors. We also review basic facts about FRW spacetimes and discuss the dynamics of both scalar and spinor fields in de Sitter spacetime.
- In chapter 3, we calculate the behaviour of the noise kernel for certain massive scalar fields in de Sitter spacetime. We find that as the mass of the scalar field is decreased, the noise kernel shows a transition from vanishing to divergent behaviour. We also study the behaviour of the noise kernel for massless scalar fields in power-law type FRW spacetimes. The most interesting behaviour is obtained for massless fields in nearly matter-dominated spacetimes for which the noise kernel is found to diverge as the spacetimes approach closer and closer to the matter dominated limit
- In chapter 4, we calculate the behaviour of the noise kernel for massive spinor fields in de Sitter spacetime. We find that the noise kernel always decays as the spacetime expands and the decay occurs irrespective of the mass of the spinor field. Then we carry out a similar study for massless spinor fields in all types of FRW spacetimes. It is found that the noise kernel behaves in a manner that is opposite to the behaviour of the scale factor i.e., the noise kernel decays for expanding spacetimes and grows for contracting spacetimes.
- In chapter 5, we obtain the expressions of the response rates for conventional and derivatively coupled Unruh deWitt detectors in different FRW spacetimes, particularly, de Sitter and matter dominated spacetimes. We find that in the response rate, for derivatively coupled detectors, the dominating infrared term vanishes in the de Sitter Universe but, for massless scalar fields in nearly matter dominated cases, the infrared divergent term contributes to the transition probability and

leads to very rapid transitions within the states of the detector.

- In chapter 6, we study how the metric perturbations over FRW spacetimes couple with atoms and calculate the rate for the atom to make transitions between its different shells caused by metric perturbations. We obtain that the coupling of perturbations with atoms has an Unruh deWitt like form and using the results of chapter 5, we conclude that the atoms undergo very rapid transitions during the nearly matter dominated eras of the Universe.

- In chapter 7, we conclude this thesis by providing a detailed chapter-wise summary of all the results obtained in this thesis and discuss what new insights have been understood about quantum fields evolving over FRW spacetimes. We also provide some future prospects of our study.

Chapter 2

Operational Tools

Since the primary goal of this thesis is to study correlations of quantum fields living on de Sitter and FRW spacetimes, let us collect here, in this chapter, some very basic properties of these spacetimes and of quantum fields living on them. In particular, how these spacetimes are defined qualitatively and how their line elements look in most commonly used coordinate systems is discussed. We focus on de Sitter spacetime and the dynamics of different quantum fields living on this spacetime. One can then use these studies of quantum fields in de Sitter spacetime to analyze similar studies of quantum fields living on other FRW spacetimes using certain mappings that exist between quantum fields in de Sitter and FRW spacetimes (which are discussed in more details in the following chapters). In this chapter, primarily scalar and spinor fields in de Sitter spacetime are considered. We write the action for these fields, their equations of motion and solutions to these equations. Since fixing arbitrary constants, which appear in the solutions to these equations, corresponds to choosing particular particle and anti-particle modes and hence corresponding particular vacuum state, one particular choice for mode functions is discussed below which defines what is called the Bunch-Davies vacuum. The expressions of Bunch-Davies Wightmann functions for both scalar and spinor fields are also provided. Since we study the correlations of quantum fields in de Sitter and FRW spacetimes through their noise kernel and by coupling them with Unruh deWitt detectors, we also provide some preliminary notions about them.

2.1 FRW spacetimes

The cosmological principle states that the spatial slices of the universe at any given instant of time must be both homogeneous and isotropic i.e., these slices exhibit translational as well as rotational symmetries and are, hence, maximally symmetric. We assume that, at least upto zeroth order, the universe obeys the cosmological principle and hence propose that there exists a coordinate system in which the spacetime metric of the universe is given by :

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\right), \tag{2.1}$$

where t, the time coordinate, is called cosmic time and $a(t)$ is referred to as cosmic scale factor. Here $d\Omega_2^2$ is the line element for a 2-sphere and k can take values $0, +1$ and -1 corresponding to flat, spherical and hyperbolic spaces. One can show [9, 66, 67] that there are only three types of maximally symmetric spaces, to which are associated the mentioned three values of k, in any given dimension and the form of the metric for these maximally symmetric spatial slices can be taken to be as given in the brackets above. Any metric of the above type, with all types of functional forms for $a(t)$, is called Friedmann Robertson Walker (in short, FRW) metric and the coordinates in which the spacetime metric assumes this mathematical form are called comoving coordinates. It is also important to note that the range of the coordinate r is different for different values of k. For $k = 0$ i.e., flat space, r can take values from 0 to ∞, for $k = 1$ i.e., sphere, r can take values from 0 to 1, and for $k = -1$ i.e., hyperboloid, r can take values from $-\infty$ to ∞. The isometry groups for these three cases are also different e.g., for $k = 0$, the isometry group is $SO(3) * R^3$, for $k = 1$, the isometry group is $SO(4)$ while, for $k = -1$, the isometry group is $SO(1,3)$. We can write the FRW metric in the following very frequently used form i.e.,

$$ds^2 = -dt^2 + a^2(t)\left(d\xi^2 + f^2(\xi) d\Omega_2^2\right), \tag{2.2}$$

where

$$f(\xi) = \begin{cases} \xi & \text{for } k = 0 \\ \sin\xi & \text{for } k = 1 \\ \sinh\xi & \text{for } k = -1 \end{cases} . \tag{2.3}$$

We can now make transformation from cosmic time coordinate, t, to what is called conformal time, η. The motivation for introducing conformal time becomes readily apparent when we write the FRW metric in the following manner:

$$ds^2 = a^2(t)\Big(-\frac{dt^2}{a^2(t)} + \frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\Big)\,. \tag{2.4}$$

The above form naturally suggests to define some new time coordinate, η, s.t. $d\eta = \frac{dt}{a(t)}$ and the FRW metric takes the following conformally flat avatar

$$ds^2 = a^2(\eta)\Big(-d\eta^2 + \frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\Big) = a^2(\eta)\Big(-d\eta^2 + d\xi^2 + f^2(\xi) d\Omega_2^2\Big)\,. \tag{2.5}$$

In general, conformally flat metrics have the following form

$$ds^2 = a^2(x)(-d\eta^2 + g_{ij}dx^i dx^j)\,. \tag{2.6}$$

where g_{ij} is the metric for spatial slices in more general coordinates and the scale factor, $a(x)$, can depend on both time and spatial coordinates. For more details on FRW spacetimes, the form of the metric in some other popular coordinates, their causal structure (in terms of Penrose diagrams), their geodesics and how they fit with Hubble expansion etc., refer to [9, 66].

In order to study FRW spacetimes further, we need to calculate Christoffel connections, Riemann tensor, Einstein tensor etc. for these spacetimes. Using Einstein equations and the form of the Einstein tensor for FRW spacetimes, the expression of the corresponding energy momentum content of the universe can be obtained. The expressions for these quantities are provided in Appendix A in both conformal and cosmic time coordinates. In order to be definitive with further discussion, we choose to work with cosmic time coordinates in the remainder of this section and will give the corresponding conformal time coordinate results at the end of this section. From the expressions of the components of the Einstein tensor, given in Appendix A, the form of the energy momentum tensor of the 'fluid' driving the universe through this FRW 'ride' can be written as

$$T_{\mu\nu} = (\rho(t) + p(t))u_\mu u_\nu + p(t) g_{\mu\nu}\,, \tag{2.7}$$

where $u_\mu = (-1,0,0,0)$ and $\rho(t)$ and $p(t)$ have natural interpretation of energy density and pressure, respectively.

From the Einstein's equations, one obtains the following two equations

$$-3\frac{\ddot{a}}{a} = 4\pi G(\rho+3p)-\Lambda\,, \tag{2.8}$$

$$\frac{\dot{a}^2}{a^2}+\frac{k}{a^2} = \frac{8\pi G}{3}\rho+\frac{\Lambda}{3}\,, \tag{2.9}$$

where the presence of Λ in the equations just allows for the possibility of the existence of a cosmological constant. These equations are called the Friedmann equations. We get one more equation from the conservation of the energy momentum tensor i.e., $\nabla_\mu T^{\mu\nu}=0$. Conservation laws for $\nu=i$ i.e., spatial indices don't contribute anything whereas for $\nu=0$, we obtain that

$$\dot{\rho} = -3H(\rho+p)\,, \tag{2.10}$$

where H, called Hubble parameter, is just a shorthand notation for $\dot{a}/a$. It can be easily shown that certain manipulations of the 2nd and 3rd equations from the above three equations, give rise to the 1st one in the collection. In that sense only two of these three equations are independent.

In order to deduce anything from the above derived Friedmann equations, we need to provide some information about the 'nature' of the fluid i.e., the matter content of the universe. This information is encoded in what is called the equation of state of the fluid i.e., the functional dependence of the pressure on the energy density, $p=f(\rho)$. We use a particular class of equations of state where pressure and energy density are linearly related i.e.,

$$p = w\rho\,, \tag{2.11}$$

where w is called the equation of state parameter. For these types of fluid, we see that, using (2.10),

$$\rho = Ca^{-3(1+w)}\,, \tag{2.12}$$

where C is some constant. Substituting this dependence of energy density on scaling factor $a(t)$, in equation (2.9), we find that

$$\dot{a}^2 = \frac{8\pi G}{3}\sum_b C_b a^{-(1+3w_b)}+\frac{\Lambda}{3}a^2-k\,, \tag{2.13}$$

where the summation is over different types of fluids with linearly related equations of state. In fact, we can drop the 2nd and the 3rd terms on the RHS of the above equation and include them in the summation with $w_b=-1$ and $w=-1/3$, in the summation, giving rise to the corresponding terms, respectively.

After the most remarkable discovery of the present day expansion of the universe, deduced from the red-shifts of the light rays coming from distant galaxies, by Hubble [68], many astronomical surveys and cosmological observations have suggested that some of the important eras through which the universe has evolved are when the RHS in the above equation is dominated by a term with

- $w = \frac{1}{3}$**:** This case is called the radiation dominated case. Since, in this case, $\dot{a}^2 \propto a^{-2}$, we find that $a(t) \propto t^{\frac{1}{2}}$. The universe is believed to have been dominated by radiation type of fluid from the end of the inflation to the redshift of $z \approx 3400$ [69].

- $w = 0$**:** This case is called the dust (or matter) dominated case. Since, in this case, $\dot{a}^2 \propto a^{-1}$, we find that $a(t) \propto t^{\frac{2}{3}}$. The universe is believed to have been dominated by matter type of fluid between $z \approx 3300$ to $z \approx 0.3$ [69].

- $w = -1$**:** This case is called the cosmological constant dominated case. Since, in this case, $\dot{a}^2 \propto a^2$, we find that $a(t) \propto e^{Ht}$. The present day universe is believed to be dominated by the dark energy. The transition from matter dominated phase to dark-energy dominated phase modelled by cosmological constant is believed to have taken place at around $z \approx 0.3$.

There exist many models (see [70] and references therein) to explain the present day dark-energy driven accelerating expansion of the Universe. In fig. (2.1), we plot the evolution of the average equation of state parameter as a function of redshift (on log scale) with the contribution coming from all radiation dominated, matter dominated and dark-energy driven fluids. We have considered models for dark-energy with constant equation of state parameter (blue and red curves in the figure) with $w_\Lambda = -1.03$ and $w_\Lambda = -0.5$ as well as a dynamical dark-energy equation of state parameter model (magenta curve) with $w_\Lambda = -1 - 0.5\ \mathrm{Log}(1+z)$ [70, 71]. Even though the dust (or radiation) type of fluid can still be the dominant matter for some intervals of z outside the left (or right) shaded area in the figure, the shaded area represents only that interval of z for which the average equation of state parameter is approximately 0 (or $1/3$). This window, too, for which the average equation of state parameter is approximately 0 vary for different models of dark-energy.

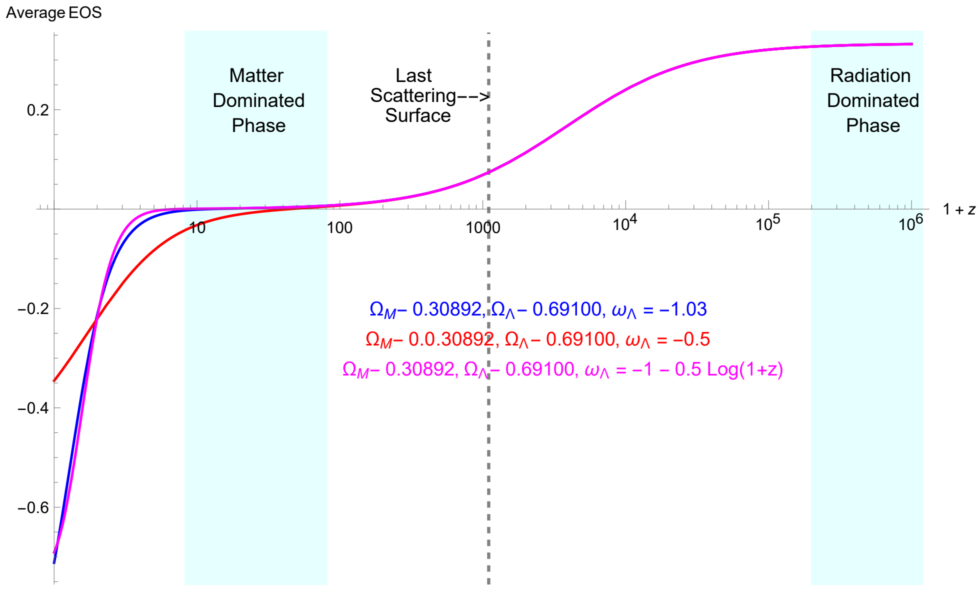


Figure 2.1: In this figure, we have plotted the average equation of state parameter with the evolution of universe in terms of redshift parameter, z (on log scale). The left and right shaded areas represent the values of z corresponding to which the average equation of state parameter is approximately 0 and $1/3$, respectively. Three different curves correspond to different models for dark energy component of the Universe.

Other than the above deduced dependencies of $a(t)$ on t for the mentioned important eras of the universe, we find that, for an arbitrary w, the scaling factor, $a(t) \propto t^{\frac{2}{3(1+w)}}$. In fact, one can also find the dependence of the scaling factor on the conformal time coordinate, η, by observing that $d\eta = (dt)/(a(t)) \propto (dt)/(t^{\frac{2}{3(1+w)}})$ gives $\eta \propto t^{\frac{1+3w}{3(1+w)}}$. Substituting the just derived relation between the cosmic time coordinate and the conformal time coordinate in the expression for the scaling factor, it is seen that $a(\eta) \propto \eta^{\frac{2}{1+3w}}$. From this relation, one sees that, for radiation dominated era $a(\eta) \propto \eta$, for dust (or matter) dominated era $a(\eta) \propto \eta^2$ and for cosmological constant dominated era $a(\eta) \propto \eta^{-1}$.
In order to simplify notations for further studies in this thesis, we express $a \propto t^p \propto \eta^{-q}$ which implies that

$$p = \frac{2}{3(1+w)} \qquad and \qquad q = -\frac{2}{1+3w}. \tag{2.14}$$

Removing w in the above expressions, we obtain $q = (p)/(p-1)$.
It is in these FRW spacetimes that we aim to study the fluctuations of quantum fields. Like mentioned at the beginning of this chapter, we shall study quantum fields living on FRW spacetimes in terms of quantum fields living on de Sitter spacetimes through certain mappings that we will come to in the following chapters. Before discussing de Sitter spacetime in greater details in the next section of this chapter, let us just point out that the $w = -1$ case i.e., the cosmological constant dominated case with $a \propto e^{Ht} \propto \eta^{-1}$ is what corresponds to the de Sitter case (or at least, a half of it). This is an

extremely important case to study as it is believed that the present day dark energy dominated expansion of the universe can be well approximated by this de Sitter phase. Also, the proposal of sufficiently long inflationary phase of the universe is also modelled by a near de Sitter spacetime.

2.2 de Sitter spacetime

In this section, we introduce some basic aspects of the geometry of de Sitter spacetime. An n dimensional de Sitter spacetime is described as the following embedding in $(n+1)$ dimensional Minkowskian spacetime i.e., $R^{(1,n)}$,

$$-(X^0)^2+\sum_{i=1}^{n}(X^i)^2=H^{-2}\,. \tag{2.15}$$

From the fact that the Minkowskian metric is symmetric under the group $SO(1,n)$ and that the collection of points defining de Sitter spacetime, through the above equation, is also closed(mapped to the same collection) under the action of $SO(1,n)$, we can conclude that the metric induced on the de Sitter spacetime, from the Minkowskian metric, is also symmetric under the group $SO(1,n)$. Since the group $SO(1,n)$ is generated by $((n)(n+1))/2$ number of vector fields and the fact that it is also the maximum number of Killing vectors possible for any spacetime of dimension n, we conclude that the de Sitter spacetime has $((n)(n+1))/2$ Killing vectors and it is a maximally symmetric spacetime. One important fact about maximally symmetric spacetimes is that the Ricci scalar is a constant for these spacetimes, therefore, the Ricci scalar for de Sitter spacetime is also a constant and is given by $2(n-1)(n-2)H^2$. One can also show that the de Sitter spacetime is a solution of the vacuum Einstein equations with a positive cosmological constant, given by $((n-1)(n-2)/2)H^2$. Though this last remark about the de Sitter spacetime can be proven as a theorem, we will see that it becomes quite apparent when we express the de Sitter metric in the planar coordinates which resembles the above discussed (in the last section) cosmological constant dominated FRW metric. If we move the $-X_0^2$ to the RHS in the above definition for the de Sitter spacetime, we observe that the topology of the de Sitter spacetime is $R\times S^{n-1}$. Now let us discuss the planar coordinates which can be used to cover the de Sitter spacetime. For more details, refer to [9, 66, 72, 73].

2.2.1 Planar coordinates with cosmic time and conformal time

In this subsection, we introduce the planar coordinates for the de Sitter spacetime. This is the most commonly used coordinate system in cosmological applications. This coordinate system is defined by relating the Minkowski Cartesian coordinates and the planar coordinates in the following way

$$
\begin{aligned}
X^n - X^0 &= \pm\frac{e^{Ht}}{H}, X^i = x^i e^{Ht}, i = 1, \ldots\ldots, n-1, \\
X^n + X^0 &= \pm\Big(\frac{e^{-Ht}}{H} - x_i x^i H e^{Ht}\Big).
\end{aligned}
\tag{2.16}
$$

Now calculating that $dX^n - dX^0 = \pm dt e^{Ht}, dX^i = dx^i e^{Ht} + x^i H e^{Ht} dt$ and $dX^n + dX^0 = \pm\Big(-e^{-Ht}dt - x_i x^i H^2 e^{Ht} dt - 2x_i dx_i e^{Ht} H\Big)$, we find that the metric induced on de Sitter spacetime is given by

$$
ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2 . \tag{2.17}
$$

One sees that the form of the metric for the spatial sheets, written in the planar coordinates, has the general form of an FRW metric with $k = 0$ i.e., the metric on the spatial slices is that of flat space R^{n-1}. This is the primary reason of referring to this coordinate system as the planar coordinates.

Now let's make transformation to the conformal time coordinate, η, by defining $d\eta = dt/a(t)$ which casts the above metric in the following conformally flat form $ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2)$, where $d\eta = dt/e^{Ht}$ implies that $\eta = -1/(He^{Ht})$ and $a(\eta) = -1/(H\eta)$. Since t lies between $(-\infty, \infty)$, we have that, from $\eta = -1/(He^{Ht})$, η lies between $(-\infty, 0)$. The region covered by any single planar coordinates chart with its corresponding metric is what one usually understands as de Sitter spacetime in cosmological contexts. In fact, this is the spactime that is used to approximately model the present day dark energy driven universe as well as the early universe inflationary phase.

Before we end this section, let's consider the Minkowski distance function between any two points of the de Sitter spacetime which is given by

$$
d(x,y) = \eta_{ab}(X^a(x) - X^a(y))(X^b(x) - X^b(y)) = 2H^{-2}(1 - Z(x,y)), \tag{2.18}
$$

where x and y are the coordinates of the considered points of the de Sitter spacetime in some coordinate system laid out on it and $X^a(x)$ and $X^a(y)$ are the coordinates of the same points in the ambient Minkowski space Cartesian coordinate system. Here $Z(x,y)$ is easily seen to be equal to $H^2\eta_{ab}X^a(x)X^b(y)$. One can argue that this function is a biscalar under the action of the de Sitter isometry group. Under an isometry, σ, of the de Sitter spacetime, the points x and y go to the points $\sigma(x)$ and $\sigma(y)$ and the corresponding Minkowski space Cartesian coordinates go to $X^a(\sigma(x)) = \Lambda^a{}_b(\sigma)X^b(x)$ and $X^a(\sigma(y)) = \Lambda^a{}_b(\sigma)X^b(y)$ where $\Lambda(\sigma)$ is the element of $SO(1,n)$ corresponding to the de Sitter isometry, σ. This implies that $Z(\sigma(x),\sigma(y)) = Z(x,y)$ i.e., it is a biscalar and hence $d(x,y)$ is also a biscalar. In fact, any biscalar of the de Sitter spacetime can be written as a function of Z. For more details, refer [72, 74]. We end this section by noticing that the function $Z(x,y)$, in planar coordinates, is given by:

$$Z(x,y) = 1 - \frac{-(\eta-\eta')^2 + (\vec{x}-\vec{y})^2}{2\eta\eta'}. \tag{2.19}$$

2.3 Quantum fields in de Sitter spacetime

Study of quantum field theories on curved spacetimes is an old, rich and still a very thriving area of research [12, 75, 76]. Analyses of this kind have led to deeper insights into the understanding of quantum field theories and have given rise to phenomena such as Hawking radiation, Unruh effect etc[77–79]. A major portion of the analyses of QFT on curved spacetimes is, in one way or another, related to the possibility of choosing different vacua and the lack of 'natural' choices for vacuum in these setups. These aspects of QFT are not exclusive to the presence of gravitational fields but to all kinds of external fields, for example presence of external electric field gives rise to non-perturbative Schwinger effect [80, 81] etc. In this section, we consider quantum fields in de Sitter spacetime which, other than being an important arena for theoretical investigations [11, 13, 75, 82–85], is immensely important for inflationary cosmology [1, 2, 5, 7, 8, 10]. We are mainly considering scalar and spinor fields in de Sitter spacetime and in these considerations, one is presented with the task of making a choice of vacuum before start building a quantum theory. In this thesis, we work with what is called the Bunch-Davies vacuum for both scalar and spinor fields and the motivation for making the Bunch-Davies choice is the fact that, in the asymptotic past, the equations of motion in both these cases resemble the Minkowski spacetime equations of motion. The Bunch-Davies choice for vacuum corresponds to choosing the arbitrary constants in such a way that, in the asymptotic past, the mode functions behave like Minkowski space positive and negative frequency mode functions. Here are also provided outlines for deriving the form of the Wightmann functions in both these cases.

2.3.1 Scalar fields

Let us consider a scalar field in an FRW spacetime, in conformal time coordinates with spatially flat slices. The dynamics of a scalar field in curved spacetime is governed by the following action:

$$\begin{aligned} S &= -\frac{1}{2}\int d^4x\sqrt{-g}\left(g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi+V(\phi)\right) \\ &= -\frac{1}{2}\int d^4x a^4\left(a^{-2}\eta^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi+V(\phi)\right) \\ &= -\frac{1}{2}\int d^4x a^2\left(-(\phi^{'})^2+(\nabla\phi)^2+a^2V(\phi)\right) \qquad (2.20) \\ &= -\frac{1}{2}\int d^4x a^2\left(-(\phi^{'})^2+(\nabla\phi)^2+a^2V(\phi)\right) \qquad (2.21) \end{aligned}$$

where ϕ' represents derivative of ϕ with respect to η. Writing $\phi = a^{-1}v$, the above action reduces to

$$S=-\frac{1}{2}\int d^4x\left(-(v^{'})^2-\frac{a'^2}{a^2}v^2+\frac{2a'v'v}{a}+(\nabla v)^2+a^4V(v/a)\right). \qquad (2.22)$$

In the above equation, v' and a' represent derivatives of v and a with respect to η. With the above action, we find that the conjugate momentum is $\pi = v' - (a'/a)v$ and the Hamiltonian is given by

$$H=\frac{1}{2}\int d^4x\left(\pi^2+(\nabla v)^2+\frac{a'}{a}(\pi v+v\pi)+a^4V(v/a)\right). \qquad (2.23)$$

The equation of motion (e.o.m), for the above action, is given by

$$v''-\nabla^2v-\frac{a''}{a}v+\frac{1}{2}a^3V'(v/a)=0\,, \qquad (2.24)$$

where v'' and a'' represent double derivatives of v and a with respect to η whereas $V'(v/a)$ represents a derivative of V with respect to ϕ and after taking the derivative with respect to ϕ, we replace ϕ with v/a. Considering a massive scalar field with no interaction i.e., taking $V(\phi) = m^2\phi^2$, and going to the spatial Fourier space i.e., taking Fourier transform with respect to spatial coordinates, the e.o.m becomes

$$v_{\vec{k}}''+k^2v_{\vec{k}}-\frac{a''}{a}v_{\vec{k}}+m^2a^2v_{\vec{k}}=0\,. \qquad (2.25)$$

The above Fourier space equation of motion is valid for all types of FRW spacetimes expressed in conformal time coordinates with flat spatial slices. However, we are interested in studying the dynamics of quantum scalar fields only in de Sitter spacetime and will study quantum fields in other FRW spacetimes only later on and, that too, in terms of quantum fields living on de Sitter spacetime through a mapping between these spacetimes. Therefore, let us consider the de Sitter case i.e., $a(\eta) = -1/(H\eta)$, for which the equation of motion is given by

$$v''_{\vec{k}} + \Big(k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2}\Big)v_{\vec{k}} = 0 \tag{2.26}$$

where $\nu^2 = (9/4) - (m^2/H^2)$. This is Bessel equation and the most general solution to this equation is given by

$$v_{\vec{k}} = c_{\vec{k}}\sqrt{-\eta}H_\nu^{(1)}(-k\eta) + d_{\vec{k}}\sqrt{-\eta}H_\nu^{(2)}(-k\eta)\,. \tag{2.27}$$

Now, in order to build quantum field theory for the above setup, we express the field operator as follows

$$v(\eta,\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3}\Big[a_{\vec{k}}v_{\vec{k}}(\eta)e^{i\vec{k}.\vec{x}} + a^\dagger_{\vec{k}}v^*_{\vec{k}}(\eta)e^{-i\vec{k}.\vec{x}}\Big]\,. \tag{2.28}$$

where $a_{\vec{k}}$ and $a^\dagger_{\vec{k}}$ are creation and annihilation operators. Here the above form of the field operator is determined by requiring that it be Hermitian which is the quantum analog of the requirement that the considered scalar field is real. Imposing the equal time canonical commutation relations between field operator and its conjugate momentum operator and also requiring the bosonic statistics, it is found that $v'_{\vec{k}}s$ need to satisfy the following condition

$$i(v^*_{\vec{k}}v'_{\vec{k}} - h.c.) = 1\,. \tag{2.29}$$

The above condition holds true for all time if it holds true at any one instant of time. Noticing that

$$v'_{\vec{k}} = -k\sqrt{-\eta}\big(c_{\vec{k}}H^{(1)}_{\nu-1}(-k\eta) + d_{\vec{k}}H^{(2)}_{\nu-1}(-k\eta)\big) + \frac{1}{\eta}\Big(-\nu + \frac{1}{2}\Big)v_{\vec{k}}\,, \tag{2.30}$$

the above condition implies that

$$ik\eta\Big(|c_{\vec{k}}|^2\big(H^{(1)}_{\nu-1}(-k\eta)H^{(1)*}_{\nu}(-k\eta) - H^{(1)*}_{\nu-1}(-k\eta)H^{(1)}_{\nu}(-k\eta)\big) \\ + |d_{\vec{k}}|^2\big(H^{(2)}_{\nu-1}(-k\eta)H^{(2)*}_{\nu}(-k\eta) - H^{(2)*}_{\nu-1}(-k\eta)H^{(2)}_{\nu}(-k\eta)\big)\Big) = 1\,. \tag{2.31}$$

Since the restriction given by the above constraint is satisfied at all times if it is satisfied at any one instant of time, let us evaluate it at $\eta \to -\infty$ limit and find, using the large argument expansion of the Hankel functions given in Appendix B, that

$$\Big(|c_{\vec{k}}|^2 - |d_{\vec{k}}|^2\Big) = \frac{\sqrt{\pi}}{2}\,. \tag{2.32}$$

Thus, even after having considered all the aspects that are needed to be considered for building a consistent quantum theory, we still have some arbitrariness left to define a unique vacuum. This particular aspect is not specific to curved spacetimes only but appear in flat spacetime case as well. In order to fix a vacuum state, we see that, in the $\eta \to -\infty$ limit, the equation of motion (2.26) reduces to

$$v''_{\vec{k}} + k^2 v_{\vec{k}} = 0\,, \tag{2.33}$$

which resembles equation of motion in flat spacetime. Therefore, requiring that the positive frequency modes behave like flat spacetime positive frequency modes i.e.,$\approx (e^{-ik\eta})/(\sqrt{2k})$, in the $k\eta \to -\infty$ limit and using the large argument expansion of the Hankel functions (see Appendix B), one concludes that

$$v_k(\eta) = \frac{\sqrt{\pi}}{2} e^{i\pi(\frac{\nu}{2}+\frac{1}{4})} \sqrt{-n} H_\nu^{(1)}(-k\eta)\,. \tag{2.34}$$

Notice that it is the magnitude of the wavevector, k, rather than the wavevector, $\vec{k}$, itself in the subscript of the mode functions because the RHS is dependent only on the magnitude of the wavevector, k. Another point is that these mode functions are defined only upto arbitrary phase factors. These mode functions are generally referred to as the Bunch-Davies mode functions. Before we calculate the Wightmann function corresponding to the Bunch-Davies vacuum for a massive scalar field, let us see the form of the Bunch-Davies mode functions for a massless scalar field. For a massless scalar field, the most general solution to the equation of motion is given by

$$v_{\vec{k}} = c_{\vec{k}} e^{-ik\eta} \left(1 - \frac{i}{k\eta}\right) + d_{\vec{k}} e^{ik\eta} \left(1 + \frac{i}{k\eta}\right). \tag{2.35}$$

Now performing the same analysis as done for massive scalar fields, the Bunch-Davies positive frequency modes are found to be given by

$$v_k = \frac{1}{\sqrt{2k}} e^{-ik\eta} \left(1 - \frac{i}{k\eta}\right). \tag{2.36}$$

These Bunch-Davies mode functions for the massless case can be obtained smoothly from the Bunch-Davies mode functions for massive field case in the limit $m \to 0$. Therefore, to obtain massless case expressions, one can simply take the $m \to 0$ limit of the corresponding quantities in the massive case. Let us now go to the calculation of the form of the Wightmann function for massive case. The Wightmann function is defined as $G(x,y) = \langle 0|\, \phi(x)\phi(y)\, |0\rangle$. Therefore, using the form of the field operator given above with mode functions taken to be the Bunch-Davies modes, we find that

$$G(x,y) = \frac{\pi H^2 (\eta\eta')^{\frac{3}{2}}}{4} \int \frac{d^3\vec{k}}{(2\pi)^3} H_\nu^{(1)}(-k\eta_x) H_\nu^{(2)}(-k\eta_y) e^{i\vec{k}.(\vec{x}-\vec{y})}, \quad (2.37)$$

$$= \frac{H^2 (\eta\eta')^{\frac{3}{2}}}{8\pi r} \int_0^\infty kdk H_\nu^{(1)}(-k\eta_x) H_\nu^{(2)}(-k\eta_y) sin(kr), \quad (2.38)$$

where $r = |\vec{x} - \vec{y}|$. Obtaining a closed form expression for the above integral is an involved exercise which is carried out at many places [11, 65]. The proof involves expressing the Hankel functions in some integral representations, given in standard places like [86–89], so that one now has a triple integral and then after some change of variables and further manipulations one can write the expression again as a single integral which is then recognizable as an integral representation of the Legendre function. Some further manipulations give us the commonly encountered following form of the Wightmann function $G(x,y)$ [11]

$$G(x,y) = \frac{H^2}{16\pi^2} \Gamma(a)\Gamma(b)_2F_1\left(a,b,2,\frac{1+Z(x,y)}{2}\right), \quad (2.39)$$

where $a,b = (3/2) \pm \nu$ and $Z(x,y)\Big(= 1 - (-(\eta - \eta')^2 + (\vec{x} - \vec{y})^2)/(2\eta\eta')\Big)$ is the invariant distance function between the points x and y [13]. We will make heavy use of this Wightmann function in the following chapters as we will mostly be concerned with the Bunch-Davies vacuum. Let us consider an alternative method to arrive at the above Wightmann function [13]. The field operator satisfies the following equation

$$(\Box - m^2)\phi(x) = 0. \quad (2.40)$$

Since in the Wightman function, the spacetime dependence comes only from the field operators, one concludes that the Wightmann function also satisfies the above equation i.e.,

$$(\Box - m^2)G(x,x') = 0. \quad (2.41)$$

Considering a vacuum state that is invariant under the full de Sitter isometry group, we can conclude that the Wightmann function for such a state is a biscalar (for more information, refer [13]) and, by the arguments presented above, one obtains that $G(x,y) = G(Z(x,y))$. Therefore, using the chain rule, the above equation reduces to

$$(Z^2 - 1)\frac{\mathrm{d}^2 G}{\mathrm{d}Z^2} + 4Z\frac{\mathrm{d}G}{\mathrm{d}Z} + \frac{m^2}{H^2}G(Z) = 0. \quad (2.42)$$

Substituting $Z = 2Y - 1$, we obtain

$$Y(1-Y)\frac{\mathrm{d}^2 G}{\mathrm{d}Y^2} + (2 - (a+b+1)Y)\frac{\mathrm{d}G}{\mathrm{d}Y} - abG(Y) = 0, \quad (2.43)$$

where $a = 3/2 + \sqrt{9/4 - m^2/H^2}$ and $b = 3/2 - \sqrt{9/4 - m^2/H^2}$ or vice-versa. This has the form of the hypergeometric equation and the most general solution to this equation is given by $G(Z) = a_2F_1(a,b,2,\frac{1+Z}{2}) + b_2F_1(a,b,2,\frac{1-Z}{2})$. We notice that, for $b = 0$ and $a = (H^2/16\pi^2)\Gamma(a)\Gamma(b)$, the Wightmann function has the form of the above obtained Bunch-Davies Wightmann function. Different choices of a and b give the Wightmann functions for different de Sitter invariant vacua [13, 82, 84, 90–92].

2.3.2 Spinor fields

Let us now consider a spinor field in FRW spacetimes. Spinors in curved spacetime are generally studied using tetrad formalism. The action for a minimally coupled spinor field is given by

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\left[i\bar{\psi}\gamma^\mu\nabla_\mu\psi - m\bar{\psi}\psi\right] \\ &= \int d^4x\frac{\sqrt{-g}}{2}\left[i\bar{\psi}\gamma^\mu\nabla_\mu\psi - i(\nabla_\mu\bar{\psi})\gamma^\mu\psi - 2m\bar{\psi}\psi\right] + \int d^4x\frac{\sqrt{-g}}{2}\nabla_\mu\left(i\bar{\psi}\gamma^\mu\psi\right). \end{aligned} \tag{2.44}$$

where $\gamma^\mu = e^\mu_a\Gamma^a$ and $\{\Gamma^a,\Gamma^b\} = -2\eta^{ab}$. Here e^μ_a are a tetrad basis and satisfy $e^\mu_a e^\nu_b\eta^{ab} = g^{\mu\nu}$. The covariant derivative, $\nabla_\mu = \partial_\mu - (1/8)\omega^{ab}_\mu[\Gamma_a,\Gamma_b]$ where the spin connections, $\omega^{ab}_\mu = e^a_\lambda e^{\tau b}\Gamma^\lambda_{\tau\mu} - e^{\tau b}\partial_\mu e^a_\tau$. The equation of motion corresponding to the above action is given by

$$(i\gamma^\lambda\nabla_\lambda - m)\psi = 0. \tag{2.45}$$

Let us consider conformally flat spacetimes i.e., $ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2)$. For these spacetimes, one finds that tetrad basis can be taken to be $e^\mu_a = (1/a)\delta^\mu_a$ which implies that $e^a_\mu = a\delta^a_\mu$, $e^{a\mu} = (1/a)\eta^{a\mu}$ and $e_{a\mu} = a\eta_{a\mu}$. With these expressions for tetrad basis and using the fact that $\Gamma^\lambda_{\mu\nu} = (a'/a)(\delta^\lambda_\nu\delta^0_\mu + \delta^\lambda_\mu\delta^0_\nu - \eta_{\mu\nu}\eta^{\lambda 0})$ (see Appendix A), we find that the spin connections, $\omega_{\mu ab} = (a'/a)(\eta_{a\mu}\delta^0_b - \eta_{b\mu}\delta^0_a)$. With this expression for spin connections, one obtains that

$$\begin{aligned} i\gamma^\lambda\nabla_\lambda &= \frac{i}{a}\Gamma^\mu\left(\partial_\mu - \frac{1}{8}\omega_{\mu ab}[\Gamma^a,\Gamma^b]\right) \\ &= \frac{i}{a}\left(\Gamma^\mu\partial_\mu + \frac{a'}{4a}\eta_{\mu a}\Gamma^\mu[\Gamma^0,\Gamma^a]\right) \\ &= \frac{i}{a}\left(\Gamma^\mu\partial_\mu + \frac{3a'}{2a}\Gamma^0\right). \end{aligned} \tag{2.46}$$

With the above expression, the equation of motion is seen to be given by

$$\left(i\Gamma^{\mu}\partial_{\mu}+\frac{3ia'}{2a}\Gamma^{0}-am\right)\psi=0\,. \tag{2.47}$$

In Fourier space, it becomes

$$\left(i\Gamma^{0}\partial_{0}-\vec{k}.\vec{\Gamma}+\frac{3ia'}{2a}\Gamma^{0}-am\right)\psi_{\vec{k}}(\eta)=0\,, \tag{2.48}$$

and by the field transforamtion , $\chi_{\vec{k}}(\eta)=a^{\frac{3}{2}}(\eta)\psi_{\vec{k}}(\eta)$, it acquires the following form

$$\left(i\Gamma^{0}\partial_{0}-\vec{k}.\vec{\Gamma}-am\right)\chi_{\vec{k}}(\eta)=0\,. \tag{2.49}$$

Taking the Weyl representation of the Gamma matrices i.e.,

$$\Gamma^{0}=\begin{bmatrix}0&1\\1&0\end{bmatrix}\otimes\begin{bmatrix}1&0\\0&1\end{bmatrix}\text{ and }\Gamma^{i}=\begin{bmatrix}0&1\\-1&0\end{bmatrix}\otimes\sigma_{i}\,, \tag{2.50}$$

where $\otimes$ is the tensor product symbol and $\sigma_i' s$ stand for the $2*2$ Pauli matrices, and decomposing $\chi_{\vec{k}}(\eta)$ as given below

$$\chi^{h}(\vec{k},\eta)\equiv\chi_{\vec{k}}(\eta)=\begin{bmatrix}\chi_{L,h}(\vec{k},\eta)\\\chi_{R,h}(\vec{k},\eta)\end{bmatrix}\otimes\xi_{h}\,, \tag{2.51}$$

where $(\hat{k}.\vec{\sigma})\xi_h=h\xi_h$, we obtain two coupled linear differential equations which are

$$\begin{aligned}i\partial_0\chi_{R,h}(\vec{k},\eta)-kh\chi_{R,h}(\vec{k},\eta)-am\chi_{L,h}(\vec{k},\eta)&=0\,,\\ i\partial_0\chi_{L,h}(\vec{k},\eta)+kh\chi_{L,h}(\vec{k},\eta)-am\chi_{R,h}(\vec{k},\eta)&=0\,.\end{aligned} \tag{2.52}$$

Considering the linear combinations of $\chi_{L,h}(\vec{k},\eta)$ and $\chi_{R,h}(\vec{k},\eta)$ i.e.,

$$u_{\pm h}(k,\eta)=\frac{\chi_{L,h}(\vec{k},\eta)\pm\chi_{R,h}(\vec{k},\eta)}{\sqrt{2}}\,,$$

and taking the case of the de Sitter spacetime i.e., $a(\eta)=-(1/(H\eta))$, one obtains the following equations

$$u''_{\pm h}+\left(k^{2}+\frac{\frac{1}{4}-(\frac{1}{2}\mp\frac{im}{H})^{2}}{\eta^{2}}\right)u_{\pm h}=0. \tag{2.53}$$

These are Bessel's equation and the most general solutions for $u_{\pm h}$ are

$$u_{\pm h}(k,\eta) = \alpha^h_{\pm k}\sqrt{-k\eta}H^{(1)}_{\nu_\pm}(-k\eta) + \beta^h_{\pm k}\sqrt{-k\eta}H^{(2)}_{\nu_\pm}(-k\eta)\,, \tag{2.54}$$

where $\nu_\pm = (1/2)\mp((im)/(H))$ and $\alpha^h_{\pm k}$'s and $\beta^h_{\pm k}$'s are arbitrary constants.

We, again, have the standard conundrum of making a choice for the vacuum state. For this case, let us take the fermionic counterpart of the scalar field Bunch-Davies vacuum by observing that the equation of motion for spinors become that of flat spacetime equation of motion in the asymptotic past. Hence, we take $\alpha^h_{\pm k}$'s and $\beta^h_{\pm k}$'s to be such that, in the asymptotic past, the spinor mode functions have the form of flat spacetime positive and negative frequency modes. From (2.53), it is seen that in the asymptotic past i.e., $k\eta \to -\infty$ limit, $u_{\pm h}$ satisfies $u''_{\pm h} + k^2 u_{\pm h} = 0$ equation. Thus, in the asymptotic past, one has a criterion to define the positive and negative frequency modes in the same way that one defines them in flat spacetime. In the $k\eta \to -\infty$ limit, the argument of the Hankel functions goes to large values and therefore, using the expansion of the Hankel functions for large arguments (see Appendix B) i.e.,

$$H^{(1)}_\nu(z) = \sqrt{\frac{2}{\pi z}}e^{i(z-\frac{\pi}{2}\nu-\frac{\pi}{4})}\left[1+O\left(\frac{1}{z}\right)\right], \tag{2.55}$$

$$H^{(2)}_\nu(z) = \sqrt{\frac{2}{\pi z}}e^{-i(z-\frac{\pi}{2}\nu-\frac{\pi}{4})}\left[1+O\left(\frac{1}{z}\right)\right], \tag{2.56}$$

$$for\ |z|\to\infty,\ Re(\nu) > -\frac{1}{2}\ and\ |arg(z)| < \pi,$$

it is observed that taking $\alpha^h_{\pm k}$'s to be non-zero and $\beta^h_{\pm k}$'s to be zero, the mode functions $u_{\pm h}$ behave like $e^{-ik\eta}$ in the considered asymptotic past limit. For this choice of arbitrary constants i.e., for $u_{\pm h}(k,\eta) = \alpha^h_{\pm k}\sqrt{-k\eta}H^{(1)}_{\nu_\pm}(-k\eta)$, one can show, using properties of Hankel functions and the equations of motion, that $\alpha^h_{-k} = ih\alpha^h_{+k}e^{i\pi\nu_-}$. Imposing that $\lim_{k\eta\to-\infty} u_{+h}(k,\eta) = \left((e^{-ik\eta})/(\sqrt{2})\right)$, we end up requiring $\alpha^h_{+k} = \sqrt{\pi/4}e^{i\frac{\pi}{2}(\nu_+ + 1/2)}$ and the mode functions are given by

$$u_{+h}(k,\eta) = \sqrt{\frac{-\pi k\eta}{4}}e^{i\frac{\pi}{2}(\nu_+ + 1/2)}H^{(1)}_{\nu_+}(-k\eta) \equiv f(k,\eta)\,, \tag{2.57}$$

$$u_{-h}(k,\eta) = -h\sqrt{\frac{-\pi k\eta}{4}}e^{i\frac{\pi}{2}(\nu_- + 1/2)}H^{(1)}_{\nu_-}(-k\eta) \equiv -hg^*(k,\eta)\,. \tag{2.58}$$

Thus, one obtains that

$$\chi_{L,h}(k,\eta) = \frac{f-hg^*}{\sqrt{2}}\,, \tag{2.59}$$

$$\chi_{R,h}(k,\eta) = \frac{f+hg^*}{\sqrt{2}}\,. \tag{2.60}$$

To obtain the negative frequency modes, a similar analysis can be carried out as has been done above for positive frequency modes. For negative frequency modes, let us denote the Fourier coefficient of $e^{-i\vec{k}.\vec{x}}$ by $v_{\vec{k}}(\eta)$ and the differential equations, written again in the helicity and chirality basis, have $-k$ as opposed to k. Therefore, if we swap the placement of left and right handed fermions in the four column spinor i.e., defining

$$v^h(\vec{k},\eta) \equiv v_{\vec{k}}(\eta) = \begin{bmatrix} v_{R,h}(\vec{k},\eta) \\ v_{L,h}(\vec{k},\eta) \end{bmatrix} \otimes \xi_h\,, \tag{2.61}$$

the set of differential equations for $v_{R,h}$ and $v_{L,h}$ is same as was for $\chi_{R,h}$ and $\chi_{L,h}$ i.e., (2.52). Now, requiring the linear combinations of $v_{R,h}$ and $v_{L,h}$ to behave like $(e^{ik\eta})/(\sqrt{2})$ in the asymptotic past, we obtain that

$$v_{L,h}(k,\eta) = \frac{hf^* + g}{\sqrt{2}}\,, \tag{2.62}$$

$$v_{R,h}(k,\eta) = \frac{-hf^* + g}{\sqrt{2}}\,. \tag{2.63}$$

With these 'Bunch-Davies modes', the field operator and its conjugate are expressed as below

$$\hat{\psi}(\eta,\vec{x}) = a^{-\frac{3}{2}}(\eta)\int \frac{d^3\vec{k}}{(2\pi)^3}\sum_h \left[\hat{a}_{\vec{k},h}\chi^h(\vec{k},\eta)e^{i\vec{k}.\vec{x}} + \hat{b}^{\dagger}_{\vec{k},h}v^h(\vec{k},\eta)e^{-i\vec{k}.\vec{x}}\right], \tag{2.64}$$

and

$$\hat{\bar{\psi}}(\eta,\vec{x}) = a^{-\frac{3}{2}}(\eta)\int \frac{d^3\vec{k}}{(2\pi)^3}\sum_h \left[\hat{a}^{\dagger}_{\vec{k},h}\bar{\chi}^h(\vec{k},\eta)e^{-i\vec{k}.\vec{x}} + \hat{b}_{\vec{k},h}\bar{v}^h(\vec{k},\eta)e^{i\vec{k}.\vec{x}}\right]. \tag{2.65}$$

Here $\hat{a}$'s and $\hat{b}$'s represent the annihilation operators whereas $\hat{a}^{\dagger}$'s and $\hat{b}^{\dagger}$'s represent the creation operators corresponding to the Bunch-Davies modes. The Bunch-Davies vacuum, $|0\rangle$, is defined to be the state which is annihilated by all the annihilation operators i.e., $\hat{a}_{\vec{k},h}|0\rangle = 0$ and $\hat{b}_{\vec{k},h}|0\rangle = 0$.
The Wightman function for the Bunch-Davies vacuum is given by

$$S_{ij}(x,x') \equiv \langle\psi_i(x)\bar{\psi}_j(x')\rangle = a^{-\frac{3}{2}}(\eta)a^{-\frac{3}{2}}(\eta')\int \frac{d^3\vec{k}}{(2\pi)^3}\sum_h \chi_i^h(\vec{k},\eta)\bar{\chi}_j^h(\vec{k},\eta')e^{i\vec{k}.(\vec{x}-\vec{x}')}\,, \tag{2.66}$$

and

$$\begin{aligned} R_{ji}(x',x) \equiv \langle\bar{\psi}_j(x')\psi_i(x)\rangle &= a^{-\frac{3}{2}}(\eta)a^{-\frac{3}{2}}(\eta')\int \frac{d^3\vec{k}}{(2\pi)^3}\sum_h v_i^h(-\vec{k},\eta)\bar{v}_j^h(-\vec{k},\eta')e^{i\vec{k}.(\vec{x}-\vec{x}')} \\ &= -S_{ij}(x,x')\,. \end{aligned} \tag{2.67}$$

Using the expressions of the above obtained Bunch-Davies modes and performing some calculations, one finds that

$$\begin{aligned} S_{ij}(x,x') &= \langle \psi_i(x)\bar{\psi}_j(x')\rangle \\ &= a(\eta_x)\left[i\gamma^{\lambda}\overrightarrow{\nabla}^{x}_{\lambda}+m\right]\frac{H^2}{\sqrt{a(\eta_x)a(\eta_{x'})}}\left[S_+(x,x')\frac{1+\Gamma^0}{2}+S_-(x,x')\frac{1-\Gamma^0}{2}\right], \end{aligned} \tag{2.68}$$

where

$$S_{\pm}(x,x') = \frac{\Gamma\left(2\pm i\frac{m}{H}\right)\Gamma\left(1\mp i\frac{m}{H}\right)}{(4\pi)^2}\,{}_2F_1\left(2\pm i\frac{m}{H},1\mp i\frac{m}{H},2,Z(x,x')\right), \tag{2.69}$$

and $Z(x,x') = 1+\Big(((\eta-\eta')^2-(\Delta\vec{x})^2)/(4\eta\eta')\Big)$. Making use of (2.46), the above expression can be cast in the following form

$$\begin{aligned} S_{ij}(x,x') &= \left[i\Gamma^{\lambda}\partial^x_{\lambda}+\frac{3ia'(\eta_x)}{2a(\eta_x)}\Gamma^0+a(\eta_x)m\right]\frac{H^2}{\sqrt{a(\eta_x)a(\eta_{x'})}}\left[S_+(x,x')\frac{1+\Gamma^0}{2}+S_-(x,x')\frac{1-\Gamma^0}{2}\right] \\ &= \frac{H^2}{\sqrt{a(\eta_x)a(\eta_{x'})}}\left[i\Gamma^{\lambda}\partial^x_{\lambda}+i\frac{a'(\eta_x)}{a(\eta_x)}\Gamma^0+a(\eta_x)m\right]\left[S_+(x,x')\frac{1+\Gamma^0}{2}+S_-(x,x')\frac{1-\Gamma^0}{2}\right]. \end{aligned} \tag{2.70}$$

For more details, refer to [93, 94]. We make very frequent use of the expressions of the Wightman functions (both for scalar and spinor fields) in the following chapters.

2.4 Stress energy tensor

In classical general relativity (GR), the gravity is assumed to be encoded in the geometry of the spacetime in terms of the spacetime metric, $g_{\mu\nu}$. The evolution of spacetime background is given by the Einstein's equations i.e.,

$$G_{\mu\nu}+\Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}\,, \tag{2.71}$$

where $G_{\mu\nu}$ is the Einstein tensor, Λ is the cosmological constant and $T_{\mu\nu}$ is the stress energy tensor which is defined as the variation of the matter action with respect to the metric variation i.e.,

$$T_{\mu\nu}(x) = -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\mu\nu}(x)}\,. \tag{2.72}$$

Here S_M is the action for the dynamics of the matter content of the universe. The stress energy tensor accounts for all the energy momentum content that is present in the spacetime. The Einstein's equations are a set of classical equations in the sense that nothing quantum mechanical has been assumed about either the matter or the spacetime (metric). But we believe that everything is fundamentally quantum mechanical in nature and to account for this quantum mechanical behaviour of, at least, the matter, we can treat the matter present in the spacetime to be quantum mechanical and keep the spacetime backround as classical and this approach is called semiclassical approach. For example, in this approach, we can take matter in the spacetime as a quantum field evolving over a fixed classical spacetime background. These considerations lead us to an important arena of quantum field theory in curved spacetime. This coming together of quantum mechanical ideas and general relativity concepts has given rise to a plethora of theoretically and conceptually important phenomena like cosmological particle creation, Hawking radiation, etc. [77–79, 95]. Most of these phenomena have their origin in the ambiguity in defining a vacuum for all times i.e., the lack of the existence of a state which satisfies the notion of a vacuum for both asymptotic past and asymptotic future in a dynamical spacetime. In this thesis, we focus on QFT in FRW spacetimes. As discussed in the previous chapter, cosmology provides an area where these notions based on the interplay of the quantum theory and general relativity play an important role. QFT on FRW backgrounds has been investigated very extensively in the literature [12, 65, 75, 96–100].

The correlations of quantum fields over the inflationary de Sitter and Friedmann–Robertson–Walker (FRW) spacetimes (which model the different stages of the evolution of the Universe) are the most important objects in these quantum fields in curved spacetime analyses. The study of correlations of quantum fields for the considered FRW spacetimes is the main topic of this thesis. We study the behaviour of these correlations, particularly in terms of the noise kernel (which we define below) and also see that the noise kernel captures the quantum effects of the quantum fields on spacetime. Another line of investigation to study the behaviour of these correlations that is considered in this thesis is the Unruh deWitt coupling of discrete localized quantum systems with the quantum fields in these cosmological spacetimes.

All that we have said till now has focused only on the dynamics of quantum fields in fixed curved spacetime background. But it is natural to ask how the dynamics of quantum fields affect the background spacetime over which it is evolving. One resolution to this question leads to what is called the semiclassical gravity paradigm. In this approach, one considers the quantum field over a class of spacetimes and calculate the expectation value of the stress energy operator (which one obtains by replacing the classical fields by field operators in the classical expression for the stress-energy tensor (2.72)), denoted by $\hat{T}_{ab}$, corresponding to any state in which the quantum field is placed. One then replaces the classical stress energy tensor in the Einstein's equations, (2.71), by the expectation value of the stress energy operator and obtains the modified Einstein's equations of semiclassical gravity i.e.,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \langle \hat{T}^R_{ab} \rangle \,, \tag{2.73}$$

where $\langle \hat{T}^R_{ab} \rangle$ represents the properly regularized and renormalized expectation value of stress energy operator with respect to some state. In semiclassical gravity, the goal is to solve the dynamical equation of the quantum field and the just introduced modified Einstein's equations in a consistent manner. However, this approach takes into account only the expectation value of the stress energy operator and its effect on the background spacetime. But the higher n-point correlations of stress energy operators at n-spacetime points also carry quantum signatures of the field. In case these n-point correlations of stress energy operators are relatively large, we expect the semiclassical gravity approach to break down and realize the need to incorporate the quantum effects of the fields captured by higher n-point correlations into our analysis. Only by taking into account quantum effects corresponding to higher correlations, we can expect our theory to be able to make robust claims and predictions related to the behaviour of the fields and spacetimes in these settings. As a first step to go beyond the semiclassical gravity, we look at the fluctuations in the stress energy operator in terms of the two point correlations of stress energy operators with respect to any state in which the field is placed. In fact, one can define such correlations of stress energy operators through the noise kernel of quantum fields which we discuss in the next section. In this thesis, we do not concern ourselves with the effects of three or higher-point correlations of the stress energy operators. We call the effects based only on the expectation value of the stress energy operator as first order quantum effects. As we discuss below that, in the stochastic gravity paradigm, the noise kernel induces metric perturbations to the semiclassical gravity results to linear order, we call the incorporation of the effects of the two-point correlations of stress energy operators, encoded in the noise kernel, as second order effects. Only a fully quantum theory is expected to take into account the effects of all n-point correlations but this is not what this thesis tries to achieve.

2.5 The noise kernel

As discussed in [101], the scope and the validity of the semiclassical gravity results cannot be definitely known in the absence of a full quantum gravity theory. However, one criterion to be able to make some remarks about the validity of the results of semiclassical gravity approach is to consider their stability in the light of quantum metric fluctuations. The stochastic gravity approach [101, 102] is such a theory which tries to do just that. More definitely, consider that $g_{ab}(x)$ is a solution of the modified Einstein's equation (2.73) and $h_{ab}(x)$ are perturbations over it so that the full metric is given by $g_{ab}(x)+h_{ab}(x)$. In stochastic gravity, the dynamics of this full metric is given by the following equation, called the Einstein-Langevin equation [101] i.e.,

$$G_{ab}(g+h,x)+\Lambda(g_{ab}(x)+h_{ab}(x)) = 8\pi G\big(\langle \hat{T}^R_{ab}(g+h,x)\rangle + \xi_{ab}(g,x)\big)\,, \tag{2.74}$$

where $G_{ab}(g+h,x)$ and $\langle \hat{T}^R_{ab}(g+h,x)\rangle$ have the same meaning as in Eq. (2.72) but with spacetime metric $g_{ab}(x)+h_{ab}(x)$ (shown by an explicit dependence of these quantities on $g+h$). The quantity $\xi_{ab}(x)$[1] is a stochastic Gaussian tensor field of rank 2 such that it satisfies the following relation

$$\langle \xi_{ab}(x)\rangle_s = 0 \quad \text{and} \quad \langle \xi_{ab}(x)\xi_{cd}(x')\rangle_s = N_{abcd}(x,x')\,, \tag{2.75}$$

where $\langle \dots \rangle_s$ means statistical average. The quantity $N_{abcd}(x,x')$, which is related to the two point statistical average of the stochastic tensor field, is called the noise kernel and is one of the main objects in this thesis to study the quantum correlations of fields in FRW spacetimes. The noise kernel is defined as follows

$$N_{abcd}(x,x') \equiv \frac{1}{2}\langle \{\hat{t}_{ab}(x),\hat{t}_{cd}(x')\}\rangle\,, \tag{2.76}$$

where $\hat{t}_{ab}(x) = \hat{T}_{ab}(x) - \langle \hat{T}_{ab}(x)\rangle$ and the curly brackets represent the anti-commutator i.e., $\{A,B\} = AB+BA$. All the expectations are taken with respect to a state in which the field is placed.

In stochastic gravity paradigm, the Einstein-Langevin equation is considered to linear order in perturbations $h_{ab}(x)$ and hence, its dynamics is given by [101]

$$h_{ab}(x) = h^0_{ab}(x) + 8\pi G\int d^4x'\sqrt{-g(x')}G_{abcd}(x,x')\xi^{cd}(x')\,, \tag{2.77}$$

where $h^0_{ab}(x)$ is the solution of the homogeneous equation obtained by considering the Einstein's Langevin equation upto linear order in h. The quantity $G_{abcd}(x,x')$ is the retarded propagator of the mentioned linear equation in h [101] and the integral in the above expression represents the inhomogeneous solution in the presence of the stochastic source term. From the above expression, it can also be shown that the two-point statistical average of $h_{ab}(x)$ is related to the noise kernel of the quantum field. Thus, we see that the stochastic field ξ_{ab} provides the backreaction effects of the two-point quantum correlations of the stress energy operator on the dynamics of the spacetime. Since we consider linear perturbations

[1] We have suppressed the explicit dependence of the stochastic field on the background metric g.

over the expectation value based semiclassical gravity results induced by the noise kernel, we call the quantum effects encapsulated in the noise kernel as second order. In this thesis, we mainly study the behaviour of the noise kernel and we will not solve the Einstein-Langevin equation for the derived noise kernel for the quantum fields in the considered de Sitter and FRW spacetimes. For more details about the stochastic gravity and the Einstein-Langevin equation, one should refer to the monumental work done on it in [101]. We also define the following two point correlations of stress energy operators

$$\begin{aligned} \langle \hat{t}_{abcd}(x,x') \rangle &\equiv \langle \hat{t}_{ab}(x) \hat{t}_{cd}(x') \rangle \\ &= \langle 0|\hat{T}_{ab}(x)\hat{T}_{cd}(x')|0\rangle - \langle 0|\hat{T}_{ab}(x)|0\rangle \langle 0|\hat{T}_{cd}(x')|0\rangle \,. \end{aligned} \tag{2.78}$$

It is in terms of this above defined correlation of stress energy operators that we study the behaviour of the correlations of quantum fields in de Sitter and other FRW spacetimes. The noise kernel is a bi-tensor quantity i.e., it is a rank 2 tensor at spacetime point x and simultaneously it is a rank 2 tensor at spacetime point x'. We will show that one can contract the noise kernel with some timelike vector field and build an invariant quantity from the noise kernel. We come to this invariant correlator in chapters 3 and 4. A number of important observations can be made about the behaviour of a quantum field in a specific spacetime by studying its noise kernel in that spacetime. For example, the fluctuation in stress energy operator at some space-time point x is equal to $\langle (\hat{t}_{ab}(x))^2 \rangle$, as is true for any quantum operator, and it can be obtained from the noise kernel if we take $a = c$ and $b = d$ and evaluate the noise kernel in the coincidence limit $x' \to x$ i.e., $\lim_{x' \to x} \langle \hat{t}_{abab}(x,x') \rangle$. But the coincidence limit, $x' \to x$, leads to UV divergences in the quantities involving product of quantum fields and hence one needs to handle them properly by employing proper regularization procedures as is usually required to be done for handling these UV divergences in quantum field theory [12, 102, 103]. In this thesis, however, we do not concern ourselves with the coincidence limit of the noise kernel and are mainly focused on studying the behaviour of the noise kernel for well separated spacetime points. Let us now discuss another approach by which also, one can study the correlations of quantum fields in cosmologically interesting spacetimes. This approach is that of Unruh deWitt (UdW) detectors.

2.6 Unruh deWitt (UdW) detector

An Unruh deWitt detector is a point like object following a classical trajectory in spacetime with an internal discrete quantum structure and it interacts with a quantum field evolving in the spacetime [75, 104, 105]. The conventional UdW coupling with a quantum field has the following Hamiltonian in the interaction picture

$$\hat{H}_I = c\chi(\tau)\hat{\mu}(\tau)\hat{\phi}(x(\tau)) \tag{2.79}$$

where c is the coupling constant, $\chi(\tau)$ is a switching (real valued) function which determines for how long the detector interacts with the field, $\hat{\mu}(t) = e^{i\hat{H}_D\tau}\hat{\mu}(0)e^{-i\hat{H}_D\tau}$ with H_D being the Hamiltonian for the internal structure of the detector and $\hat{\mu}(0)$ determines the transitions between the internal states of the detector. Here $\hat{\phi}(x(\tau))$ is a quantum scalar field at spacetime point $x(\tau)$ which is the point along the detector's trajectory at which the detector is at proper time τ. One can consider the transition probability for the detector to make a transition from some state $|0\rangle_D$ with energy 0 to another state $|\Omega\rangle_D$ with energy Ω whereas the quantum field starts in its vacuum state $|0\rangle$ and ends up in some arbitrary state $|\psi\rangle$. Using the time dependent perturbation theory upto first order in coupling constant, c, it can be shown that the transition amplitude for the above transition is given by

$$\begin{aligned} A_{|0\rangle_D\otimes|0\rangle\to|\Omega\rangle_D\otimes|\psi\rangle} &= -ic\int_{-\infty}^{\infty} d\tau\chi(\tau)\,\langle\Omega|_D\,\hat{\mu}(\tau)\,|0\rangle_D\,\langle\psi|\,\hat{\phi}(x(\tau))\,|0\rangle \\ &= -ic\,\langle\Omega|_D\,\hat{\mu}(0)\,|0\rangle_D\int_{-\infty}^{\infty} d\tau\chi(\tau)e^{i\Omega\tau}\,\langle\psi|\,\hat{\phi}(x(\tau))\,|0\rangle\,. \end{aligned} \tag{2.80}$$

Therefore, the transition probability for the detector to go from $|0\rangle_D$ to $|\Omega\rangle_D$ irrespective of the final state of the field is given by

$$\begin{aligned} \frac{P_{|0\rangle_D\to|\Omega\rangle_D}}{|c|^2|\,\langle\Omega|_D\,\hat{\mu}(0)\,|0\rangle_D\,|^2} &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\tau_1 d\tau_2\chi(\tau_1)\chi(\tau_2)e^{-i\Omega(\tau_1-\tau_2)} \\ &\qquad \sum_{|\psi\rangle}\langle 0|\,\hat{\phi}(x(\tau_1))\,|\psi\rangle\,\langle\psi|\,\hat{\phi}(x(\tau_2))\,|0\rangle\,. \end{aligned} \tag{2.81}$$

Using the completeness of the final states i.e., $\sum_\psi |\psi\rangle\langle\psi| = 1$, the above expression becomes

$$\frac{P_{|0\rangle_D\to|\Omega\rangle_D}}{|c|^2|\,\langle\Omega|_D\,\hat{\mu}(0)\,|0\rangle_D\,|^2} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\tau_1 d\tau_2\chi(\tau_1)\chi(\tau_2)e^{-i\Omega(\tau_1-\tau_2)}G(x(\tau_1),x(\tau_2))\,, \tag{2.82}$$

where $G(x(\tau_1),x(\tau_2)) = \langle 0|\,\hat{\phi}(x(\tau_1))\hat{\phi}(x(\tau_2))\,|0\rangle$ is the Wightman function of the quantum field. Therefore, the probability for the above considered process is related to the integrals of the Wightman function of the quantum field and as such the probability would encode in itself the behaviour of the correlations of the quantum fields. Thus, in addition to the noise kernel of the quantum fields, finding the expression of the above probability for quantum fields in curved spacetimes will also provide important insights into the behaviour of the quantum fields. The correlations of quantum fields at different spacetime points can also be studied in a similar setting in terms of the generation of entanglement between spatially separated localized quantum systems [106–109]. One can also consider other modified UdW couplings like the derivative UdW coupling [110–113] which we, in fact, analyze later in the thesis in the context of quantum fields in FRW spacetimes only. Indeed, there exist many different types of UdW detectors depending upon whether the detector quantum space is like that of a qubit [105, 114] or a quantum harmonic oscillator [115, 116], etc. Other features like the point size or finite spatial size [117–120] of the detector also gives rise to different UdW detectors. One important factor on which the response of a UdW detector depends is the choice of timelike trajectory along which the UdW detector is moving. In this work, the UdW detectors are taken to be moving along what are called the comoving trajectories which will be explained later in the thesis. In this thesis, we also explore the UdW type interaction of metric perturbations over the otherwise isotropic and homogeneous FRW spacetimes with localized quantum systems.

Chapter 3

Stress energy correlator of scalar fields in FRW spacetimes

In this chapter, we look at the behaviour of the stress energy correlator or noise kernel of scalar fields in FRW spacetimes. Particularly, we focus on massive scalar fields in de Sitter spacetime and massless scalar fields in power-law type FRW spacetimes. We will carry out the analysis for scalar fields in de Sitter spacetime while placing the field in the Bunch-Davies vacuum discussed in the first chapter. We discuss an equivalence that exists between massless scalar fields in power-law type FRW spacetimes with massive scalar fields in de Sitter spacetime. Using this equivalence, we place the massless scalar fields of power-law type FRW spacetimes in the Bunch-Davies state of the corresponding massive scalar field of the de Sitter spacetime. In case of de Sitter spacetime, we also provide the results for non minimally coupled scalar fields. Finally, we discuss the implications of the obtained results.

3.1 Introduction

As motivated in the previous chapter, one way to study the correlations of quantum fields is through their noise kernel. The noise kernel, in fact, provides how these correlations in quantum fields can cause fluctuations in the otherwise stable metric solutions of the semiclassical gravity. In that sense, if the noise kernel or stress energy correlations are large relative to the expectation of the stress energy operator, we may find ourselves in a situation where the self-consistent solutions of the semiclassical gravity may no longer have any relevance and we may need to revisit the conclusions based on the first order semiclassical gravity. Therefore, studying the noise kernel, which captures the second order quantum effects of the fields, is an important exercise. Particularly, we want to carry out this analysis for quantum fields in the early Universe as well as for other epochs of the cosmological expansion. Observations [121] suggest that the Universe went through a near de Sitter configuration during the early stages of its evolution. We also understand that the vacuum fluctuations in the quantum fields of the metric and matter perturbations (over the otherwise maximally symmetric configuration of the de Sitter Universe) sowed the seed of the large scale structure (LSS) of the Universe during the early phase of the Universe [12, 67, 99, 122]. Therefore, studying the behaviour of the noise kernel for quantum fields in the early Universe is worth spending time on because if the second order quantum effects are significant, it may have important implications for how we understand the cosmic microwave background (CMB) radiations and the origin of structures in the Universe. Quantization of scalar fields over de Sitter spacetime and studying its implications has been a focus of investigation for a long time [82] and a lot of effort has gone into it [13, 84, 90, 122–136]. One of the most important object for studying free quantum fields over any spacetime is its Wightman function. It contains in itself the information about how the quantum field at different spacetime points are correlated with each other. In fact, the vacuum expectation value of the stress energy operator that appears in the modified Einstein's equations of semiclassical gravity can also be written as some derivative operators acting on the Wightman function of the field [12]. But the Wightman functions of quantum fields need to be handled carefully because of the many divergences that they carry in cosmological settings [12, 137]. For example, the Wightman function is an ill-defined object in the coincidence limit i.e., at the same space-time point, because, in this limit, the Wightman function diverges in a quadratic as well as in a logarithmic manner which is also called the Hadamard form [12]. These short-distance divergences are related to the high-energy limit of the theory and can be removed by proper regularization and renormalization techniques in most physical situations [12, 75, 138]. In case of scalar fields in de Sitter spacetime, the situation gets even worse as the Wightman function shows a divergence even for different spacetime points when one considers the

case of a minimally coupled massless scalar field [13, 84, 137]. This divergence for minimally coupled massless scalar fields is often called the infra-red problem of the de Sitter spacetime and this infrared divergence of the Wightman function can also be seen in its power spectrum [12, 65]. Some methods have been worked out ([12, 139]) to make sense of these divergences and obtain physically sensible results out of them. Using an equivalence between scalar fields in de Sitter and FRW spacetimes given in [65], the Wightman functions of scalar fields in de Sitter and FRW spacetimes can be related and it can be shown that the infrared divergence of the de Sitter case is inherited by scalar fields in some FRW spacetimes. Now if we wish to analyze whether these infrared divergences in de Sitter and some FRW spacetimes have the potential to make these spacetimes unstable or not, we can study the behaviour of the noise kernel for the corresponding situations and see whether the second order quantum effects encoded in the noise kernel are strong enough or not especially in the light of the infra-red problem.
Using the point split form of the stress energy tensor, one can write the noise kernel as a sum of products of derivatives of Wightman functions [140] and because of this fact, we expect the noise kernel, in general, to be also infected by the divergences of the Wightman functions. A lot of efforts has gone into studying the backreation of the quantum fields [91, 96–98, 100, 138, 141]. In addition to de Sitter spacetime [74, 142], the noise kernel has been investigated for other maximally symmetric spacetimes [143, 144] as well. In this work, rather than explicitly calculating the backreaction in terms of the metric fluctuations induced by the noise kernel, we mainly concentrate our attention to calculate the behaviour of the noise kernel which carries the information about the stochastic part of the Einstein Langevin equations of the stochastic gravity approach [101] and only qualitatively study the implications of its behaviour.
To calculate the noise kernel, we follow in the footsteps of [140] but generalize our studies to the de Sitter and FRW spacetimes. First, we take up the case of massive scalar fields in de Sitter spacetime and calculate the noise kernel for it. We compare our results with a similar study carried out in [74], point out the points of departure in our results compared with this study and explain the reasons. We are interested in studying whether these second order quantum effects grow or decay as the physical distances between any two spatially separated comoving points increase with the increasing scale factor in de Sitter expansion. We also want to analyze similar study for FRW spacetimes. These observations will provide insights into the stability or instability of the considered spacetimes.

We divide this chapter into 6 sections including this one. In section 3.2, we give a brief introduction of the noise kernel for scalar fields in curved spacetime. In section 3.3, we obtain the late time behaviour of the noise kernel for a minimally coupled massive scalar field in de Sitter spacetime and discuss its behaviour for various masses. We also perform a similar analysis for non-minimally coupled scalar field case. In section 3.4, we carry out the analysis for massless scalar fields in power-law type FRW spacetimes and discuss the stability of these spacetimes in light of the obtained results. In section 3.5, we define an invariant object called the energy energy correlator and calculate its behaviour for all the spacetimes considered in the sections before it. We discuss the main results obtained in this work and further future prospects in Section 3.6.

3.2 The noise kernel

Let us consider the action of a minimally coupled massive scalar field in a general spacetime metric i.e.,

$$S[g_{\alpha\beta},\phi] = -\frac{1}{2}\int d\eta d^3\vec{x}\sqrt{-g}\left(g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi + m^2\phi^2\right). \tag{3.1}$$

Now using the formula for stress energy tensor (2.72) i.e.,

$$T_{\alpha\beta}(x) = -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\alpha\beta}(x)}, \tag{3.2}$$

we see that the stress energy tensor for the above action of a minimally coupled scalar field in general spacetime is given by

$$T_{\alpha\beta}(x) = \nabla_\alpha\phi\nabla_\beta\phi - \frac{1}{2}g_{\alpha\beta}(g^{\gamma\delta}\nabla_\gamma\phi\nabla_\delta\phi + m^2\phi^2). \tag{3.3}$$

If we specialize to the case of Minkowski spacetime i.e., $g_{\alpha\beta} = \eta_{\alpha\beta}$, we can write the stress energy tensor in the following point-split form

$$T_{ab}(x) = \lim_{y\to x} P_{ab}(x,y)\phi(x)\phi(y), \tag{3.4}$$

where

$$P_{ab}(x,y) = \left(\delta^c_{(a}\delta^d_{b)} - \frac{1}{2}\eta_{ab}\eta^{cd}\right)\nabla^x_c\nabla^y_d - \frac{1}{2}\eta_{ab}m^2. \tag{3.5}$$

We can obtain the quantum stress energy operator from the above stress energy tensor expression by replacing the classical field with the field operator i.e.,

$$\hat{T}_{ab}(x) = \lim_{y\to x} P_{ab}(x,y)\hat{\phi}(x)\hat{\phi}(y). \tag{3.6}$$

Now using the above formula and the definition of the two point correlations of stress energy operators i.e., (2.78), in a vacuum state, we see that the stress energy two point correlator is given by

$$\langle \hat{t}_{abcd}(x,x') \rangle = \lim_{\substack{y\to x \\ y'\to x'}} P_{ab}(x,y) P_{cd}(x',y') \langle 0|\hat{\phi}(x)\hat{\phi}(y)\hat{\phi}(x')\hat{\phi}(y')|0\rangle$$
$$- \lim_{\substack{y\to x \\ y'\to x'}} P_{ab}(x,y) P_{cd}(x',y') \langle 0|\hat{\phi}(x)\hat{\phi}(y)|0\rangle \langle 0|\hat{\phi}(x')\hat{\phi}(y')|0\rangle \,. \quad (3.7)$$

Using the Wick's theorem, the above expression can also be written as [140]

$$\langle \hat{t}_{abcd}(x,x') \rangle = 2 \lim_{\substack{y\to x \\ y'\to x'}} P_{ab}(x,y) P_{cd}(x',y') G(x,x') G(y,y') \,, \quad (3.8)$$

where

$$G(x,x') = \langle 0|\, \hat{\phi}(x)\hat{\phi}(x')\, |0\rangle \,, \quad (3.9)$$

i.e., the Wightman function of the scalar field in the considered vacuum.

3.3 Behaviour of the noise kernel for de Sitter spacetime

Let us write down the expression of the stress energy correlator[1] i.e., (2.78), for a conformally flat FRW space-time i.e., for $g_{\alpha\beta} = a(\eta)^2 \eta_{\alpha\beta}$. For a minimally coupled scalar field in a conformally flat space-time, the stress energy operator can again be written in the point split form (3.6) with

$$P_{ab}(x,y) = \left(\delta^c_{(a}\delta^d_{b)} - \frac{1}{2}\eta_{ab}\eta^{cd}\right)\nabla^x_c \nabla^y_d - \frac{1}{2}\left(\frac{a(\eta)+a(\eta')}{2}\right)^2 \eta_{ab} m^2 \,. \quad (3.10)$$

We find that the stress energy correlator for a conformally flat FRW spacetime is also given by (3.8) with the above $P_{ab}(x,y)$. For de Sitter space-time i.e., $a(\eta) = -1/(H\eta)$, the explicit expression of the stress

[1] The noise kernel can be obtained from the stress energy correlator by simply comparing (2.76) with (2.78).

energy correlator is given by

$$\langle \hat{t}_{abcd}(x,x')\rangle_{dS} = \Bigg(\nabla_b\nabla'_c G(x,x')\nabla_a\nabla'_d G(x,x') + \nabla_b\nabla'_d G(x,x')\nabla_a\nabla'_c G(x,x')$$
$$- \eta_{cd}\eta^{\rho\sigma}\nabla_a\nabla'_\rho G(x,x')\nabla_b\nabla'_\sigma G(x,x') - \frac{1}{H^2\eta'^2}m^2\eta_{cd}\nabla_a G(x,x')\nabla_b G(x,x')$$
$$- \eta_{ab}\eta^{\gamma\delta}\nabla_\gamma\nabla'_c G(x,x')\nabla_\delta\nabla'_d G(x,x') + \frac{1}{2}\eta_{ab}\eta^{\gamma\delta}\eta_{cd}\eta^{\rho\sigma}\nabla_\gamma\nabla'_\rho G(x,x')\nabla_\delta\nabla'_\sigma G(x,x')$$
$$+ \frac{1}{2H^2\eta'^2}m^2\eta_{ab}\eta^{\gamma\delta}\eta_{cd}\nabla_\gamma G(x,x')\nabla_\delta G(x,x') - \frac{1}{H^2\eta^2}m^2\eta_{ab}\nabla'_c G(x,x')\nabla'_d G(x,x')$$
$$+ \frac{1}{2H^2\eta^2}m^2\eta_{ab}\eta_{cd}\eta^{\rho\sigma}\nabla'_\rho G(x,x')\nabla'_\sigma G(x,x') + \frac{1}{2H^4\eta^2\eta'^2}m^4\eta_{ab}\eta_{cd}G(x,x')G(x,x')\Bigg). \quad (3.11)$$

Now we want to find out what happens to the correlations of the stress energy operators as the Universe expands in this de Sitter case. For the present work, we consider spacetime points with equal time coordinates i.e., we work on equal time sheets. We take the points on equal time sheets to be co-moving with finite spatial distances between them and want to analyse whether $\langle \hat{t}_{abcd}(x,x')\rangle_{dS}$ grows or not with the growth of the physical distances between the fixed co-moving points as the de Sitter spacetime expands i.e., in the $a(\eta) \to \infty$ limit.

Minimal coupling

To make any inferences about the behaviour of the spacetime in the light of the magnitude of the noise kernel or stress energy correlations, we must not have knowledge of the exact expressions of all components of the noise kernel. We find that with only knowledge of the behaviour of the $\langle \hat{t}_{0000}\rangle$ component, we are able to make a qualitative assessment about the stochastic corrections to the first order quantum effects of the semiclassical gravity. In fact, the degree of the divergences for other components is either sub dominant or equal to that of $\langle \hat{t}_{0000}\rangle$ (Refer Appendix D). We can also relate the $\langle \hat{t}_{0000}\rangle$ with the coordinate independent invariant or energy energy correlator which is introduced below. Thus, we focus on studying the behaviour of the $(a=0,\ b=0,\ c=0,\ d=0)$ component of the noise kernel. For de Sitter spacetime, the $\eta \to 0$ limit is what corresponds to the late time that is the limit in which the scale factor grows to large values. Thus, our goal is to analyse the behaviour of the $(a=0,\ b=0,\ c=0,\ d=0)$ component of the noise kernel on constant time sheets (i.e., $\eta = \eta'$) with finite spatial distances (i.e., $\Delta\vec{x} \neq 0$) in the $\eta \to 0$ limit.

Using the equation (3.11) and formulas from Appendix C, we see that

$$\langle \hat{t}_{00}(\eta,\vec{x})\hat{t}_{00}(\eta,\vec{x}')\rangle_{dS} = \Bigg((G'')^2\left[\frac{(\Delta\vec{x})^6}{4\eta^{10}}+\frac{(\Delta\vec{x})^8}{32\eta^{12}}+\frac{(\Delta\vec{x})^4}{2\eta^8}\right]+G^2\left[\frac{m^4}{2H^4\eta^4}\right]$$
$$+(G')^2\left[\frac{3(\Delta\vec{x})^2}{2\eta^6}+\frac{(\Delta\vec{x})^4}{8\eta^8}+\frac{2}{\eta^4}+\frac{m^2}{H^2}\left(\frac{(\Delta\vec{x})^4}{4\eta^8}+\frac{(\Delta\vec{x})^2}{\eta^6}\right)\right]+(G''G')\left[-\frac{5(\Delta\vec{x})^4}{4\eta^8}-\frac{(\Delta\vec{x})^2}{\eta^6}-\frac{(\Delta\vec{x})^6}{8\eta^{10}}\right]\Bigg). \tag{3.12}$$

In the above equation, a single prime ($'$) on G represents a single derivative with respect to Z and similarly, two primes represent a double derivative with respect to Z. Now we need to make a choice for the vacuum state of the scalar field and we choose the vacuum state to be the Bunch-Davies vacuum introduced in chapter 2. The Wightman function of the scalar field in the Bunch-Davies vacuum is given by equation (2.39) and therefore, we have

$$G(Z) = \frac{H^2}{16\pi^2}\Gamma\left(\frac{3}{2}+\nu\right)\Gamma\left(\frac{3}{2}-\nu\right){}_2F_1\left(\frac{3}{2}+\nu,\frac{3}{2}-\nu,2,\frac{1+Z}{2}\right), \tag{3.13}$$
$$G'(Z) = \frac{H^2}{64\pi^2}\Gamma\left(\frac{5}{2}+\nu\right)\Gamma\left(\frac{5}{2}-\nu\right){}_2F_1\left(\frac{5}{2}+\nu,\frac{5}{2}-\nu,3,\frac{1+Z}{2}\right), \tag{3.14}$$
$$G''(Z) = \frac{H^2}{384\pi^2}\Gamma\left(\frac{7}{2}+\nu\right)\Gamma\left(\frac{7}{2}-\nu\right){}_2F_1\left(\frac{7}{2}+\nu,\frac{7}{2}-\nu,4,\frac{1+Z}{2}\right), \tag{3.15}$$

where $\nu=\sqrt{(9/4)-(m^2/H^2)}$. Using the late time limit (which corresponds $Z\to-\infty$ limit) for the ${}_2F_1$ function [87] i.e.,

$${}_2F_1(a,b,c,z) = \frac{\Gamma(b-a)\Gamma(c)(-z)^{-a}}{\Gamma(b)\Gamma(c-a)}\left(\sum_{k=0}^{\infty}\frac{(a)_k(a-c+1)_k z^{-k}}{k!(a-b+1)_k}\right)$$
$$+\frac{\Gamma(a-b)\Gamma(c)(-z)^{-b}}{\Gamma(a)\Gamma(c-b)}\left(\sum_{k=0}^{\infty}\frac{(b)_k(b-c+1)_k z^{-k}}{k!(b-a+1)_k}\right), \tag{3.16}$$

it can be shown [145] that

$$\langle \hat{t}_{00}(\eta,\vec{x})\hat{t}_{00}(\eta,\vec{x}')\rangle_{dS}\Big|_{\text{late time}} = \frac{H^4\Gamma^2(\nu)\Gamma^2(\frac{5}{2}-\nu)}{\pi^5}\Bigg[\frac{9\eta^{2-4\nu}}{32(\Delta\vec{x})^{6-4\nu}}$$
$$+\frac{21(3-2\nu)\eta^{4-4\nu}}{16(\Delta\vec{x})^{8-4\nu}}+\frac{(656\nu^3-3244\nu^2+5168\nu-2655)\eta^{6-4\nu}}{64(\nu-1)(\Delta\vec{x})^{10-4\nu}}+O(\eta^2)\Bigg]. \tag{3.17}$$

From the above expression, we observe that the correlations of the stress energy operators on equal time sheets undergo a transition at $\nu = 1/2$. The exponent of the most dominant term, $\eta^{2-4\nu}$, in the above expression is positive for $\nu < 1/2$, e.g., for $\nu = 0$,

$$\langle t_{00}(\eta,\vec{x})t_{00}(\eta,\vec{x}')\rangle_{dS}\big|_{\text{late time}} = \lim_{\eta\to 0}\left[O(\eta^2)\right], \tag{3.18}$$

and hence the stress energy correlations vanish for $\nu < 1/2$. Whereas, for $\nu = 1/2$, the correlations approach an $\eta-$ independent leading term (of course, in the late time limit i.e., $\eta \to 0$ limit) i.e.,

$$\langle t_{00}(\eta,\vec{x})t_{00}(\eta,\vec{x}')\rangle_{dS}\big|_{\text{late time}} = \lim_{\eta\to 0}\left[\frac{9H^4}{32\pi^4(\Delta\vec{x})^4} + O(\eta)\right]. \tag{3.19}$$

This should not come as a surprise because, for $\nu = 1/2$, the resultant scalar field theory in de Sitter spacetime (with $m^2/H^2 = 2$) is a conformally invariant theory and as such should be ignorant of the presence of $a(\eta)$. On the other hand, the leading order behaviour of the correlations diverges for $\nu > 1/2$. For example, for $\nu = 3/2$ (which corresponds to a massless theory in de Sitter spacetime), the correlations are given as follows [145]

$$\begin{aligned}
&\langle t_{00}(\eta,\vec{x})t_{00}(\eta,\vec{x}')\rangle_{dS}\big|_{\text{late time}} = \\
&\quad \lim_{\eta\to 0}\lim_{\varepsilon\to 0}\left[\frac{9H^4}{128\pi^4\eta^4}\Big[1-4\varepsilon\Big] + \frac{21H^4\varepsilon}{32\pi^4(\Delta\vec{x})^2\eta^2} + \frac{H^4}{16\pi^4(\Delta\vec{x})^4}\left[\frac{3}{2}+14\varepsilon\right] + O(\eta)\right] \\
&\quad = \lim_{\eta\to 0}\left[\frac{9H^4}{128\pi^4\eta^4} + \frac{H^4}{16\pi^4(\Delta\vec{x})^4}\left[\frac{3}{2}\right] + O(\eta)\right] \to \infty.
\end{aligned} \tag{3.20}$$

In the above expression, we start with $v = 3/2 - \varepsilon$ where $\varepsilon << 1$ i.e., we start with nearly massless scalar fields and we obtain the massless field case by taking $\varepsilon \to 0$. Thus, we observe that, for spacetime points on constant time sheets with finite spatial distances between them and in the late time, $\eta \to 0$, limit, the $(a = 0,\ b = 0,\ c = 0,\ d = 0)$ component of the noise kernel (for a minimally coupled scalar field in de Sitter spacetime placed in the Bunch-Davies vacuum) shows a transition from a decaying to a divergent behaviour as ν is changed between $[0, 3/2]$ with transition taking place at $\nu = 1/2$. These observations should be seen especially in the light of the well known fact that the de Sitter spacetime is unstable against the particle creation of light mass particles [85, 90, 146–148].

Comparison with large co-moving distance case

It is important that we compare our results with the ones obtained in [74]. In this work, the noise kernel is expressed in terms of functions P,Q,R,S,T (which are sums of products of Wightman function and its first and 2nd order derivatives with respect to the geodesic distance)

$$\langle t_{abcd}(x,x'))\rangle_{dS} = P(\mu)n_a n_b n_c n_d + Q(\mu)(n_a n_b g_{c'd'} + n_{c'} n_{d'} g_{ab})$$

$$+R(\mu)(n_s n_{c'} g_{bd'} + n_b n_{d'} g_{ac'} + n_a n_{d'} g_{bc'} + n_b n_{c'} g_{ad'}) + S(\mu)(g_{ac'} g_{bd'} + g_{bc'} g_{ad'}) + T(\mu) g_{ab} g_{c'd'}\,, \tag{3.21}$$

where μ is the distance of the geodesic which connects the points x and x'. The quantities $n_a, n_{c'}, g_{ab}, g_{ad'}$, $g_{c'd'}$, etc. appearing in the above expression are described with respect to the geodesic which connects the points x and x'. Then, the unit vectors n_a and $n_{a'}$ are tangent vectors to this geodesic at the points x and x', respectively. The object $g_{ac'}$ is defined by its action which is to move a vector from x' to x along the above geodesic in parallel transport manner.
In [74], it is shown that

$$P,Q,T \sim Z^{-2h_-} \text{ and } R \sim Z^{-2h_- -1} \text{ and } S \sim Z^{-2h_- -2}. \tag{3.22}$$

for $Z << -1$. Using this, [74] argues that *the fluctuations decay faster with the distance as mass increases*. However, such a conclusion is reached by ignoring the dependence of the coefficients of $P(\mu), Q(\mu)$ etc., in the above equation, on η $\big($and hence on Z as $Z = 1 + ((\eta-\eta')^2 - (\Delta\vec{x})^2)/(2\eta\eta')\big)$ and which is justifiable only for the case in which the $Z \to -\infty$ by keeping the η and η' fixed and taking $(\Delta\vec{x})^2 \to \infty$ limit. However, for the case in which we do not hold η and η' fixed, then we will also have to worry about the behaviour of the coefficients of the $P(\mu), Q(\mu)$ etc. to reach to any conclusion about the overall behaviour of the stress energy correlations in the $Z << -1$ regime. Particularly, $Z << -1$ case can also be achieved by keeping the spatial distance finite $\Delta\vec{x}$ $(\neq 0)$ and fixed and working on constant $\eta-$ sheets and taking $a(\eta) \to \infty$. The just mentioned case is the same one that we have considered in our analysis and this work has also obtained the same divergences in this case as we have obtained, for appropriate mass ranges.

Below we provide the calculations for arriving at the behaviour of the noise kernel for the case of non-minimally coupled scalar fields. We find that though the technical details are a bit different but the overall qualitative conclusions can be carried over from the previous case by the replacement $m^2 \to m^2 + 12\xi H^2$.

Non-minimal coupling

The action for a non-minimally coupled massive scalar field is given by[2]

$$S^{nm}[g_{\alpha\beta},\phi] = -\frac{1}{2}\int d\eta d^3\vec{x}\sqrt{-g}\left(g^{\alpha\beta}\,\nabla_\alpha\phi\,\nabla_\beta\phi + m^2\phi^2 + \xi R\phi^2\right). \tag{3.23}$$

The corresponding equation of motion for the field, ϕ, is

$$\left[\Box - (12\xi H^2 + m^2)\right]\phi(x) = 0\,, \tag{3.24}$$

from which one can show that the Wightman function is now given by

$$G(Z(x,x')) = \frac{H^2}{16\pi^2}\Gamma(a)\Gamma(b)_2F_1\left(a,b,2,\frac{1+Z}{2}\right), \tag{3.25}$$

where $a = 3/2 + \sqrt{9/4 - (12\xi H^2 + m^2)/(H^2)}$ and $b = 3/2 - \sqrt{9/4 - (12\xi H^2 + m^2)/(H^2)}$.

Using the definition of the stress-energy tensor i.e., equation (2.72), we obtain

$$T^{nm}_{\alpha\beta}(x) \,=\, \nabla_\alpha\phi\nabla_\beta\phi \,-\, \frac{1}{2}g_{\alpha\beta}(g^{\gamma\delta}\nabla_\gamma\phi\nabla_\delta\phi \,+\, m^2\phi^2) + \xi\left(G_{\alpha\beta}\phi^2 \,+\, g_{\alpha\beta}g^{\gamma\delta}\nabla_\gamma\nabla_\delta\phi^2 \,-\, \nabla_\alpha\nabla_\beta\phi^2\ \right), \tag{3.26}$$

where $G_{\alpha\beta}$ is the Einstein tensor. Making use of the fact that, for de Sitter spacetime i.e, $g_{\alpha\beta} = \eta_{\alpha\beta}/H^2\eta^2$, $G_{\alpha\beta} = -3H^2 g_{\alpha\beta}$, we can write the stress energy tensor in the following point-split form

$$T^{nm}_{\alpha\beta}(x) = \lim_{y\to x} P^{nm}_{ab}(x,y)\phi(x)\phi(y) = \lim_{y\to x}\left(P_{ab}(x,y) + M_{ab}(x,y)\right)\phi(x)\phi(y)\,, \tag{3.27}$$

where

$$P_{ab}(x,y) = \left[\left((1-2\xi)\delta^r_{(a}\delta^s_{b)} - (\frac{1}{2} - 2\xi)\eta_{ab}\eta^{rs}\right)\nabla^x_r\nabla^y_s - \frac{2(3H^2\xi + \frac{m^2}{2})}{(H\eta)^2 + (H\eta')^2}\eta_{ab}\right], \tag{3.28}$$

and

$$M_{ab}(x,y) = \left[2\xi\,\eta_{ab}\eta^{rs} - 2\xi\,\delta^r_{(a}\delta^s_{b)}\right]\frac{\nabla^x_r\nabla^x_s + \nabla^y_r\nabla^y_s}{2}\,. \tag{3.29}$$

[2]Here superscript nm refers to non-minimal coupling.

We see that, for $\xi = 0$, we obtain the minimally coupled case back which should, in fact, be the case. Particularly, $M_{ab}(x,y)$ vanishes whereas the $P_{ab}(x,y)$ goes to the corresponding expression of the minimally coupled case.

We obtain the stress energy operator by replacing the classical fields by the field operator in the above expression and find that the stress energy correlator is given by

$$\begin{aligned}
\langle \hat{t}^{nm}_{ab}(x)\hat{t}^{nm}_{cd}(x')\rangle &= 2\lim_{\substack{y\to x\\ y'\to x'}} P^{nm}_{ab}(x,y)P^{nm}_{cd}(x',y')G(x,x')G(y,y') \\
&= 2\lim_{\substack{y\to x\\ y'\to x'}} \Big(P_{ab}(x,y)P_{cd}(x',y') + P_{ab}(x,y)M_{cd}(x',y') \\
&\quad + M_{ab}(x,y)P_{cd}(x',y') + M_{ab}(x,y)M_{cd}(x',y')\Big)G(x,x')G(y,y') . \qquad (3.30)
\end{aligned}$$

We provide the exact expressions of the $P_{ab}P_{cd}$, $P_{ab}M_{cd}$, $M_{ab}P_{cd}$ and $M_{ab}M_{cd}$ terms in the Appendix D. As in the case of minimal coupling, we find (by using power counting argument) that the most dominant power of η (in the limit $\eta \to 0$) is still $2-4\nu$ $\big($where $\nu = \sqrt{9/4-(12\xi H^2+m^2)/(H^2)}\big)$ i.e.,

$$\begin{aligned}
\langle t^{nm}_{ab}(x)t^{nm}_{cd}(x')\rangle\big|_{\text{late time}} &= \lim_{\eta\to 0}\Bigg[\frac{\eta^{2-4\nu}H^4}{512\pi^5(\Delta\vec{x})^{6-4\nu}}\Big[32(12\xi-1)\Gamma\big(\frac{5}{2}-\nu\big)\Gamma\big(\frac{7}{2}-\nu\big) \\
&+\Big(16\frac{m^4}{H^4}+8\frac{m^2}{H^2}(24\xi+(3-2\nu)^2)-48\xi(3-2\nu)^2+(3-2\nu)^2(29-20\nu+4\nu^2) \\
&+32\xi^2(27-12\nu+4\nu^2))\Big)\Gamma^2\big(\frac{3}{2}-q\big)\Big]\Gamma[\nu]^2+O(\eta^{4-4\nu})\Bigg] . \qquad (3.31)
\end{aligned}$$

Thus, we find that the behaviour of the noise kernel for non-minimally coupled case is the same as the behaviour of the noise kernel for minimally coupled case with the difference that m^2/H^2 is now replaced by $m^2/H^2+12\xi$. Therefore, we, again, observe the transition of the noise kernel from decaying to divergent behaviour as $m^2/H^2+12\xi$ is varied in the range $[0,3/2]$. From the expression of ν, we see that, for conformal coupling i.e., for $\xi = 1/6$, the value of $\nu = 1/2$ for massless scalar field and therefore, for this case, we do not have divergent behaviour for the noise kernel (in the $\eta \to 0$ limit). As the mass of the field is taken to non-zero values, the value of ν becomes less than 1/2 which implies that the stress energy correlations for these cases vanish for late time limit. It is clear that there exists a range of mass values for which we can have divergent behaviour of the stress energy correlations in those cases for which $\xi < 1/6$. These divergences are, of course, different from the UV divergences of the Wightman function as we are using the (a) Regularized Stress Energy Tensor (RSET), and (b) this divergence appears only for finite co-moving distance in the large scale factor limit. The above analysis shows that the growth or the decay of the correlations between stress energy operators (as the physical distances between the considered fixed co-moving points become large) depends on the value of the coupling ξ and mass m of the field. We have discussed the de Sitter spacetime but the Universe is believed to have undergone other FRW epochs as well. To study the issue of possible quantum backreaction in these FRW phases, we turn to the case of the behaviour of the noise kernel for FRW spacetimes.

3.4 Behaviour of the noise kernel for FRW spacetimes

In this section, we calculate the behaviour of the noise kernel of massless scalar fields in FRW spacetimes. For this, as we have already mentioned, we make use of an equivalence between massless scalar fields in power-law type FRW spacetimes with massive scalar fields in de Sitter spacetime[3]. According to this equivalence, for a power-law type FRW spacetime with scaling factor, $a(\eta) = (H\eta)^{-q}$, the mass of the corresponding scalar field in de Sitter space-time is given by $m^2 = H^2(1-q)(2+q)$. Similarly, the relationship between the Wightman functions in the two settings is given by

$$G^{P.L.}(x,x') = (H\eta)^{q-1}(H\eta')^{q-1}G(x,x'). \tag{3.32}$$

With the use of the above relation, we find that

$$\nabla'_\mu G^{P.L.} = (H)^{2q-2}[(q-1)(\eta)^{q-1}(\eta')^{q-2}G\delta_{\mu 0} + (\eta)^{q-1}(\eta')^{q-1}\nabla'_\mu G], \tag{3.33}$$

[3]This equivalence can be established by noticing that the action of a massless scalar field in a power-law type FRW spacetime i.e., $g_{\alpha\beta} = a^2\eta_{\alpha\beta}$ with $a(\eta) = (H\eta)^{-q}$, is

$$S = -\frac{1}{2}\int d^4x a^4 \left(a^{-2}\eta^{\alpha\beta}\partial_\alpha\phi\,\partial_\beta\phi\right).$$

and effecting a field redefinition $\phi(x) = (H\eta)^{-1+q}\,\psi(x)$, the action takes the following form

$$S = -\frac{1}{2}\int d^4x b^4 \left(b^{-2}\eta^{\alpha\beta}\partial_\alpha\psi\,\partial_\beta\psi - m_{eff}^2\psi^2\right),$$

where $b(\eta) = (H\eta)^{-1}$ and $m_{eff}^2 = H^2(1-q)(2+q)$. Thus, we have been able to show that a massless scalar field in an FRW spacetime with scaling factor, $a(\eta) = (H\eta)^{-q}$, can be mapped to a massive scalar field in a de Sitter spacetime with the use of the above considered field redefinition. This field redefinition i.e., $\phi(x) = (H\eta)^{-1+q}\,\psi(x)$, also provide the relation between the Wightman functions for the corresponding settings i.e., $G^{P.L.}(x,x') = (H\eta)^{q-1}(H\eta')^{q-1}G(x,x')$. For more details on this, one can refer the Appendix A.2 of [65] where a similar equivalence for the non-minimal case is also considered.

and

$$\begin{aligned}\nabla_\nu\nabla'_\mu G^{P.L.} &= H^{2q-2}[(q-1)^2(\eta)^{q-2}(\eta')^{q-2}G\delta_{\mu 0}\delta_{\nu 0}+(q-1)(\eta)^{q-1}(\eta')^{q-2}\delta_{\mu 0}\nabla_\nu G\\ &\quad +(q-1)(\eta)^{q-2}(\eta')^{q-1}\delta_{\nu 0}\nabla'_\mu G+(\eta)^{q-1}(\eta')^{q-1}\nabla_\nu\nabla'_\mu G]\\ &= (H\eta H\eta')^{q-1}\Big(\frac{(q-1)^2}{\eta\eta'}G\delta_{\mu 0}\delta_{\nu 0}+\frac{(q-1)}{\eta'}\delta_{\mu 0}\nabla_\nu G+\frac{(q-1)}{\eta}\delta_{\nu 0}\nabla'_\mu G+\nabla_\nu\nabla'_\mu G\Big).\end{aligned} \tag{3.34}$$

Since we are interested in calculating the stress energy correlations between spacetime points which lie on constant time sheets and have fixed spatial distances, we take $\eta=\eta'$. Different components of the above expression on constant time sheets are given as follows

$$\begin{aligned}\nabla_0\nabla'_0 G^{P.L.} &= (H\eta)^{2q-2}\Big[\frac{(q-1)^2}{\eta^2}G+\frac{(q-1)}{\eta}\nabla_0 G+\frac{(q-1)}{\eta}\nabla'_0 G+\nabla_0\nabla'_0 G\Big],\\ \nabla_0\nabla'_j G^{P.L.} &= (H\eta)^{2q-2}\Big[\frac{q-1}{\eta}\nabla'_j G+\nabla_0\nabla'_j G\Big],\\ \nabla_i\nabla'_0 G^{P.L.} &= (H\eta)^{2q-2}\Big[\frac{q-1}{\eta}\nabla_i G+\nabla_i\nabla'_0 G\Big],\\ \nabla_i\nabla'_j G^{P.L.} &= (H\eta)^{2q-2}\Big[\nabla_i\nabla'_j G\Big].\end{aligned} \tag{3.35}$$

Thus, we have the expressions for the covariant derivatives of the Wightman function for a massless scalar field in a power-law type FRW spacetime in terms of the covariant derivatives of the Wightman function for the corresponding field in de Sitter spacetime. Using these expressions, the expressions given in Appendix C, and the expression of the $(a=0,\ b=0,\ c=0,\ d=0)$ component of the stress energy correlator for a massless scalar field in considered FRW spacetime, we obtain that the stress energy correlations (for $\eta=\eta'$) is

$$\begin{aligned}\langle\hat{t}_{00}(\eta,\vec{x})\hat{t}_{00}(\eta,\vec{x}')\rangle_{P.L.} &= (H\eta)^{4(q-1)}\Bigg[\frac{G^2}{2\eta^4}(q-1)^4+GG'\Big[\frac{(2q^3-7q^2+8q-3)(\Delta x)^2}{2\eta^6}-\frac{(q-1)^2}{\eta^4}\Big]\\ &\quad +GG''\frac{(q-1)^2(\Delta\vec{x})^4}{4\eta^8}+G'G''\Big[\frac{(q-\frac{3}{2})(\Delta\vec{x})^6}{4\eta^{10}}+\frac{(q-\frac{9}{4})(\Delta\vec{x})^4}{\eta^8}-\frac{(\Delta\vec{x})^2}{\eta^6}\Big]\\ &\quad +(G')^2\Big[\frac{2}{\eta^4}+\frac{(q^2-5q+\frac{11}{2})(\Delta\vec{x})^2}{\eta^6}+\frac{(2q^2-6q+\frac{9}{2})(\Delta\vec{x})^4}{4\eta^8}\Big]\\ &\quad +(G'')^2\Big[\frac{(\Delta\vec{x})^6}{4\eta^{10}}+\frac{(\Delta\vec{x})^8}{32\eta^{12}}+\frac{(\Delta\vec{x})^4}{2\eta^8}\Big]\Bigg].\end{aligned} \tag{3.36}$$

Using the above expression, we can analyze the behaviour of the correlations of stress energy operators (for points on constant time sheets) for different values of q i.e., for different types of power-law FRW spacetimes. We notice that it is only for $q \in [-2,1]$ that the square of the mass of the corresponding field in de Sitter spacetime is positive and the corresponding values of ν lie in the range $[-3/2,3/2]$ which is also the range that we have considered for the de Sitter case. The values of $|\nu| > 3/2$ correspond to the cases in which q lie outside $[-2,1]$. Since our goal is to study the correlations of stress energy operators in the limit of physical distances (between co-moving points with same time coordinate) going to very large values i.e., in the late time universe, we conclude that, for $q \in (0,1]$, it is the $\eta \to 0$ limit which corresponds to late time limit and for $q \in [-2,0)$, the $\eta \to \infty$ limit represents the late time limit. Below we discuss the behaviour of the stress energy correlator for different FRW spacetimes.

- $q = 1$: This case is that of a massless scalar field in de Sitter space-time which have already discussed in the previous section for $\nu = 3/2$. So, we already know that, in this case, the correlations diverge as η^{-4} or a^4 as $\eta \to 0$ limit.

- $q \in (0,1)$: Using (3.36) along with (3.13) (expression of the Wightman function in the Bunch-Davies vacuum), we see that [145]

$$\begin{aligned}\langle \hat{t}_{00}(\eta,\vec{x})\hat{t}_{00}(\eta,\vec{x'})\rangle_{P.L.}\big|_{\text{late time}} = \\ \lim_{\eta\to 0}\frac{(H\eta)^{4q-4}}{(\Delta\vec{x})^4}\Bigg[\frac{H^4\eta^{4-4q}(\Delta\vec{x})^{4q-4}}{8\pi^5}\Big((11-12q+4q^2)(\Gamma(2-q))^2(\Gamma(0.5+q))^2\Big) \\ + \frac{4^{4q}\eta^{4q+4}H^4}{32\pi^5(\Delta\vec{x})^{4+4q}}\Big((1+2q)^4(\Gamma(2+q))^2(\Gamma(-0.5-q))^2\Big) + O(\eta^{6-4q})\Bigg].\end{aligned} \tag{3.37}$$

The leading term, in the $\eta \to 0$ limit, is $\big((H^{4q}(\Delta\vec{x})^{4q-8})/(8\pi^5)\big)\Big((11-12q+4q^2)(\Gamma(2-q))^2(\Gamma(0.5+q))^2\Big)$ which is $\eta-$ independent and implies that even though the physical distances between co-moving points on constant time sheets increase with time, the correlations between the stress energy operators for these points saturates to a constant. Thus, for these cases, we may have the scenario where the stochastic term in the Einstein Langevin equation may become important provided that the value it settles to is of the same magnitude or more as compared to the expectation values used in the semiclassical analysis.

For these cases, in the late time limit, we find that the Wightman function (and hence the stress energy correlator) does not depend on time (or the scale factor). For spacetime points with same

η coordinate i.e., on constant time-sheets, the Wightman function is given by

$$G^{P.L.}(\eta,\vec{x},\eta',\vec{x}') = \frac{H^2(H\eta)^{2q-2}}{16\pi^2}\ {}_2F_1(2+q,1-q,2,1-\frac{(\Delta\vec{x})^2}{4\eta^2})\,. \tag{3.38}$$

Considering the late time i.e., $\eta \to 0$ limit, we obtain

$$\begin{aligned} G^{P.L.}(\eta,\vec{x},\eta',\vec{x}') = &\frac{H^2(H\eta)^{2q-2}}{16\pi^2}\Gamma(2+q)\Gamma(1-q) \\ &\Big[\frac{\Gamma(-1-2q)\big(\big(\frac{(\Delta\vec{x})^2}{4\eta^2}\big)\big)^{-2-q}}{\Gamma(1-q)\Gamma(-q)}\sum_{k=0}^{\infty}\frac{(2+q)_k(1+q)_k\big(-\frac{(\Delta\vec{x})^2}{4\eta^2}\big)^{-k}}{k!(2+2q)_k} \\ &+\frac{\Gamma(1+2q)\big(\frac{(\Delta\vec{x})^2}{4\eta^2}\big)^{-1+q}}{\Gamma(2+q)\Gamma(1+q)}\sum_{k=0}^{\infty}\frac{(1-q)_k(-q)_k\big(-\frac{(\Delta\vec{x})^2}{4\eta^2}\big)^{-k}}{k!(-2q)_k}\Big]. \end{aligned} \tag{3.39}$$

Using the form of the scale factor i.e. $a(\eta) = (H\eta)^{-q}$, we can write $H\eta = a^{-1/q}$ and substituting this in the above expression, we find the above expansion in terms of the physical distance on constant time sheets, i.e. $a^2(\Delta\vec{x})^2$, and scale factor $a(\eta)$ i.e.,

$$\begin{aligned} G^{P.L.}(\eta,\vec{x},\eta',\vec{x}') = &\frac{H^2}{16\pi^2}\Gamma(2+q)\Gamma(1-q) \\ &\Big[\frac{\Gamma(-1-2q)\big(\frac{H^2}{4}\big)^{-2-q}a^{2q-2/q}}{\Gamma(1-q)\Gamma(-q)(a^2(\Delta\vec{x})^2)^{2+q}}\sum_{k=0}^{\infty}\frac{(2+q)_k(1+q)_k\big(-\frac{H^2}{4}\big)^{-k}(a^2(\Delta\vec{x})^2)^{-k}(a^{-2+2/q})^{-k}}{k!(2+2q)_k} \\ &+\frac{\Gamma(1+2q)\big(\frac{H^2}{4}\big)^{-1+q}a^{2-2q}}{\Gamma(2+q)\Gamma(1+q)(a^2(\Delta\vec{x})^2)^{1-q}}\sum_{k=0}^{\infty}\frac{(1-q)_k(-q)_k\big(-\frac{H^2}{4}\big)^{-k}(a^2(\Delta\vec{x})^2)^{-k}(a^{-2+2/q})^{-k}}{k!(-2q)_k}\Big]. \end{aligned} \tag{3.40}$$

From the above expression, it is easy to see that, for the range of q under consideration and in the $\eta \to 0$ limit, the most dominant term is the leading term of the second series in the square bracket, which has no a dependence and hence the above expression is a independent for late times. Though, the above expression was $\eta-$ dependent for the prior times and it is only for late times that the η dependence gradually wears off and we obtain a constant term as the leading order term in the $\eta \to 0$ limit. Thus, at late times, even though the physical distances between any two co-moving points with small coordinate distances are large, the correlations between them survive.

- $q = 0$: This case (i.e., $a(\eta) = 1$) corresponds to the flat Minkowski spacetime which has already been studied previously [140, 149, 150]. In the Minkowski spacetime, the Wightman function for a massless scalar field is $G(x,x') = (1)/(4\pi^2(-(\eta-\eta')^2 + (\Delta\vec{x})^2))$. The expression for the correlations between stress energy operators for spacetime points with same time coordinate i.e., on constant time-sheets, and with finite spatial distance, is as follows

$$\langle \hat{t}_{00}(\eta,\vec{x})\hat{t}_{00}(\eta,\vec{x'})\rangle_{P.L.} = \frac{3}{2\pi^4(\Delta\vec{x})^8}\,. \tag{3.41}$$

In case of the Minkowski spacetime, the coordinate distances and the physical distances are the same and for co-moving points, the correlator has no dynamics i.e., it remains the same for all the times. Also, the correlations decay with increasing co-moving distances.

- $q \in (-2,0)$: For these negative values of q, the exponent of the scale factor i.e., $a(\eta) = (H\eta)^{-q}$ is positive and hence it is the $\eta \to \infty$ limit which corresponds to the scale factor going to large values i.e., the late time limit. The stress energy correlator for these cases is given by [145]

$$\langle \hat{t}_{00}(\eta,\vec{x})\hat{t}_{00}(\eta,\vec{x'})\rangle_{P.L.}\big|_{\text{late time}} = \lim_{\eta\to\infty}(H\eta)^{4q-4}\Bigg[\frac{3H^4\eta^4}{2\pi^4(\Delta\vec{x})^8} + \frac{\eta^2 H^4(3q+4q^2)}{8\pi^4(\Delta\vec{x})^6}$$

$$+\frac{H^4 q}{64\pi^4(\Delta\vec{x})^4}\Big((-4-7q+6q^2+11q^3)$$

$$+2(1+q)(-1+q)^2\big[2\gamma + log(\frac{(\Delta\vec{x})^2}{4\eta^2}) + \psi^{(0)}(1-q) + \psi^{(0)}(2+q))\big]\Big) + O(\eta^{-2})\Bigg]. \tag{3.42}$$

In the above expression, γ is the Euler gamma symbol whereas $\psi^{(0)}(z)$ is the PolyGamma function. For $\eta \to \infty$, the most dominant term is $\mathscr{O}(\eta^{4q})$ which, for fixed Δx, implies that no quantum correlations survive between stress energy operators in the late time limit.

- $q = -2$: We treat this case by considering massless scalar fields in nearly matter dominated spacetimes i.e., $q = -2+\varepsilon$ (with $\varepsilon \to 0$) which then get mapped to nearly massless scalar fields in de Sitter spacetime as can be seen from the formula, $m^2 = H^2(1-q)(2+q) \approx 3H^2\varepsilon \to 0$. For this case, the expression of the correlations between stress energy operators has the following expansion [145]

$$
\begin{aligned}
\langle \hat{t}_{00}(\eta,\vec{x})\hat{t}_{00}(\eta,\vec{x'})\rangle_{P.L.}\big|_{\text{late time}} = \lim_{\eta\to\infty}\lim_{\varepsilon\to 0} H^{-12}\Bigg[\frac{3H^4}{2\pi^4\eta^8(\Delta\vec{x})^8}+ \\
\frac{4}{(\Delta\vec{x})^6\eta^{10}}\Big(\frac{5H^4}{16\pi^4}+O(\varepsilon)\Big)+\frac{1}{\eta^{12}(\Delta\vec{x})^4}\Big(\frac{\mathbf{9H^4}}{\mathbf{16\pi^4\varepsilon}}+\frac{9(6H^4+H^4 log(\frac{(\Delta\vec{x})^2}{4\eta^2}))}{16\pi^4}+O(\varepsilon)\Big) \\
+\frac{1}{4(\Delta\vec{x})^2\eta^{14}}\Big(-\frac{\mathbf{27H^4}}{\mathbf{8\pi^4\varepsilon}}-\frac{27(7H^4+2H^4 log(\frac{(\Delta\vec{x})^2}{4\eta^2}))}{16\pi^4}+O(\varepsilon)\Big) \\
+\frac{1}{16\eta^{16}}\Big(\frac{\mathbf{81H^4}}{\mathbf{8\pi^4\varepsilon^2}}+\frac{27H^4(10+3log(\frac{(\Delta\vec{x})^2}{4\eta^2})))}{4\pi^4\varepsilon}+O(\varepsilon^0)\Big)+O(\eta^{-18})\Bigg].
\end{aligned}
\tag{3.43}
$$

For large but finite η, we see that the correlations between stress energy operators become infinitely large due to presence of $1/\varepsilon$ terms in the limit $\varepsilon\to 0$ and hence one has to necessarily perform the stochastic gravity analysis to take into account the second order quantum effects to make any reliable conclusion for this setting. One can easily see that this divergence is, in fact, present at all times. The origin of this divergence is the infrared problem of the massless scalar field theory in de Sitter spacetime. To see this, notice that the Wightman function for massless scalar field in $q=-2$ space-time is related to the Wightman function of the corresponding scalar field in de Sitter case by the relation

$$
G^{q=-2}_{m=0}(x,x') = (H^2\eta\eta')^{-3}G^{dS}_{m=0}(x,x'), \tag{3.44}
$$

and therefore the term giving rise to infrared divergence in the de Sitter spacetime (which is spacetime independent) develops a time dependence in the $q=-2$ spacetime case. Because of this fact, this infrared term does not go away under the action of the derivative operators of (3.8). One finds that, for spacetimes with $q<-2$ and $q>1$ corresponding to $|\nu|>3/2$, similar spacetime dependent divergence shows up (refer to Appendix E) and the semiclassical analysis for these cases is also vulnerable to breakdown in light of these large quantum fluctuations.

3.5 The invariant or energy-energy correlator

Up until this point, we have looked at the behaviour of $\langle \hat{t}_{0000} \rangle$ component of the noise kernel for certain massive scalar fields in de Sitter spacetime and massless scalar fields in a class of power-law type FRW spacetimes. But this quantity is a particular component of a bitensor quantity whose functional dependence changes from one coordinate system to another which is also clear from the fact that it is, in some of the cases considered above, dependent only on the co-ordinate separation (Δx) which is not a coordinate invariant object. It is, however, not very difficult to construct invariant scalar objects out of this bitensor quantity. We define one such invariant scalar out of the stress energy correlators. For this purpose, we consider a vector field whose components (in conformal coordinates) are $t^{\alpha}(\eta,\vec{x}) = (1/a(\eta),0,0,0)$ and hence $g_{\alpha\beta}t^{\alpha}t^{\beta} = -1$ for all spacetime points. It is easy to see that the considered vector field is normalized tangent vector field to co-moving observers whose spatial coordinates remain fixed. By contracting both the indices of stress energy operator with this vector field i.e., $\hat{T}_{\alpha\beta}(x)t^{\alpha}(x)t^{\beta}(x)$ $(= \hat{T}_{00}(\eta,\vec{x})/a^2(\eta)$ in conformal coordinates), we obtain a coordinate invariant object for all spacetime points. In fact, it can be shown that, in the considered spacetimes, the vacuum expectation value of the stress energy tensor for scalar and spinor fields has the perfect fluid form with the above chosen vector field i.e., $\langle \hat{T}_{\mu\nu} \rangle = (\rho + p)t_{\mu}t_{\nu} + pg_{\mu\nu}$ and therefore, the quantity $\langle \hat{T}_{\alpha\beta} \rangle t^{\alpha}t^{\beta}$ is equal to the energy density i.e., $-\langle \hat{T}^{0}_{0} \rangle = \rho$. Thus, we can now define the following 'invariant' correlator

$$\begin{aligned}
\langle \Big(\hat{T}_{\mu\nu}(x)t^{\mu}(x)t^{\nu}(x)\Big)\Big(\hat{T}_{\alpha\beta}(y)t^{\alpha}(y)t^{\beta}(y)\Big)\rangle - \langle \Big(\hat{T}_{\mu\nu}(x)t^{\mu}(x)t^{\nu}(x)\Big)\rangle \langle \Big(\hat{T}_{\alpha\beta}(y)t^{\alpha}(y)t^{\beta}(y)\Big)\rangle \\
= \Big(\langle \hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)\rangle - \langle \hat{T}_{\mu\nu}(x)\rangle\langle \hat{T}_{\alpha\beta}(y)\rangle\Big)t^{\mu}(x)t^{\nu}(x)t^{\alpha}(y)t^{\beta}(y) \\
= \langle \hat{t}_{\mu\nu\alpha\beta}(x,y)\rangle t^{\mu}(x)t^{\nu}(x)t^{\alpha}(y)t^{\beta}(y)\,. \qquad (3.45)
\end{aligned}$$

We will refer this coordinate independent object as the invariant correlator or energy-energy correlator for the rest of this thesis. In conformal coordinates, it acquires the following form

$$\langle \hat{t}_{\mu\nu\alpha\beta}(x,y)\rangle t^{\mu}(x)t^{\nu}(x)t^{\alpha}(y)t^{\beta}(y) = \frac{\langle \hat{t}_{0000}(x,y)\rangle}{a^2(\eta_x)a^2(\eta_y)}\,. \qquad (3.46)$$

At this point, it is important that we emphasize that the quantity which is coordinate invariant is

$$\langle \hat{t}_{\mu\nu\alpha\beta}(x,y)\rangle t^{\mu}(x)t^{\nu}(x)t^{\alpha}(y)t^{\beta}(y)\,, \qquad (3.47)$$

but not $(\langle \hat{t}_{0000}(x,y)\rangle)/(a^2(\eta_x)a^2(\eta_y))$. It is just that, in conformal coordinates, the invariant correlator is given by $(\langle \hat{t}_{0000}(x,y)\rangle)/(a^2(\eta_x)a^2(\eta_y))$.

We now present the results for the behaviour of the invariant correlator in all the cases considered above in this chapter. For this purpose, we use the results for $\langle \hat{t}_{0000}\rangle$ obtained in the previous section. Thus, we have [145]

- $q = 1$: This case corresponds to de Sitter spacetime. From equation (3.46), we find that, in the late time i.e, $\eta \to 0$ limit, the energy energy correlator is 0 for all the considered values of ν except for $\nu = 3/2$. For $\nu = 3/2$, the energy energy correlator attains a constant value which is $9H^8/128\pi^4$. The infrared problem of the de Sitter spacetime, present at the level of the Wightman function for $\nu = 3/2$, does not make its presence felt at the level of the stress energy correlator or the energy energy correlator. In order to treat this case, we need to regularize the Wightman function properly as a limiting case since the massless fields have no de Sitter invariant vacuum [13] but the action of the derivative operators in (3.12) remove the problematic part of regularized massless Wightman function. Similar issue is dealt within [151] and it is shown that the infrared piece does not contribute to the energy energy correlator.

- $q \in (0,1)$: Making use of the formula (3.46), we find that, for this case, the energy energy correlator vanishes in the late time $\eta \to 0$ limit.

- $q = 0$: For this case, we have $a(\eta) = 1$ and hence the energy energy correlator is same as the stress energy correlator $\langle \hat{t}_{0000}\rangle$.

- $q \in (-2,0)$: For this case also, results from the previous section and the formula (3.46) imply that the energy energy correlator vanishes in the late time $\eta \to \infty$ limit.

- $q = -2$: In this case, the energy energy correlator is divergent in the late time limit (holds, in fact, for all values of η) because of the infrared divergent factor present in the Wightman function at $\nu = 3/2$. The energy energy correlator similarly diverges for the cases $q < -2$ and $q > 1$.

From the above results, we see that the above defined invariant or energy energy correlator vanishes for most of the cases and hence, for these cases, we do not expect large corrections coming from this correlator. However, it diverges for a number of cases which we discuss about more in the following.

3.5.1 Some implications

From chapter 2, we know that the equation of state parameter, w, for an ideal fluid which drives a particular FRW spacetime is related to the exponent of that spacetime by the formula (2.14) i.e.,

$$q = -2/(1+3w)\,. \tag{3.48}$$

As discussed in chapter 2, for any FRW phase of the Universe i.e., for a given exponent, the ideal fluid with the corresponding equation of state parameter represents the dominant matter content of the Universe during that phase. For example, dust phase ($q = -2$) of the Universe is driven by a fluid with $w = 0$ whereas the fluid with $w = 1/3$ drives the Universe through radiation ($q = -1$) phase. From the above formula, we find that the FRW spacetimes with $q \in (0,1]$ have the corresponding equation of state parameter w lying in the range $\in (-\infty,-1]$ whereas fluids with $w \in [0,\infty)$ are related to spacetimes with $q \in [-2,0)$. Thus, using the results obtained in the previous sections, we conclude that the second order quantum effects (($a = 0, b = 0, c = 0, d = 0$) component of the stress energy correlator or the noise kernel) may be important for $w \in (-\infty,-1]$ in the late time limit whereas the second order quantum fluctuations are absent for $w \in (0,\infty)$ in the late time limit. For the case of pressureless dust i.e., $w = 0$, driving the Universe through the $q = -2$ phase, we find that the semiclassical analysis is vulnerable to a complete breakdown because of the divergent noise kernel. Thus, conclusions derived on the basis of semi-classical or probably even classical analysis in dust driven spacetime need to be looked with suspicion and are to be modified by taking into account the stochastic analysis. Thus, for this spacetime, only a completely (at least including second order quantum effects) quantum analysis is needed to arrive at reliable results. We expect similar arguments to hold true for $q < -2$ or $q > 1$ cases which correspond to fluids with $w \in [-1,0]$. Therefore, it is necessary to perform a higher order quantum analysis for these spacetimes. Particularly interesting is the case of accelerating spacetimes (the ones corresponding to $w < -1/3$) which as per the analysis performed in this chapter necessarily require a completely quantum treatment. We can summarize the results of this section by the following diagram:

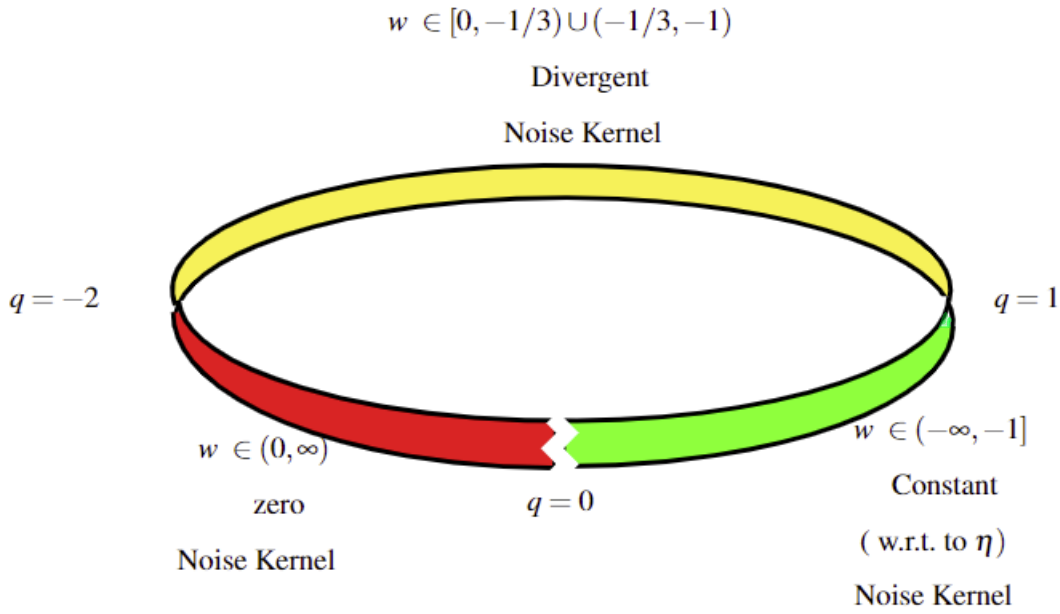


Figure 3.1: Relation between different types of fluid (and the corresponding Friedmann space-times) and the behaviour of noise kernel in these regions.

3.6 Summary

In this chapter, we focus on the second order quantum effects of scalar fields in FRW Universes and discuss whether these second order effects have the potential to significantly modify the first order semi-classical gravity analysis. For flat spacetime, we know that the quantum correlations between field (or stress energy operator) for fixed co-moving spacetime points with same time coordinates remain the same as the time evolves. In this case, the correlations decay for the points with large physical separations whereas they are significant only for points with smaller separations. For FRW spacetimes, the physical distances between the fixed comoving points with same time coordinates increase as the time evolves and points which have small physical separations initially grow further in physical separations with time. Because of this, there are situations, e.g. for conformal fields, where the correlations depend only on the coordinate distances but not on physical distances and thus, we see that the correlations remain the same between fixed comoving points even though the physical separation between them is increasing. What may even happen is that the quantum fluctuations between points with small coordinate distances can get amplified by the scale factors of the FRW spacetimes and develop a potential to completely breakdown the semiclassical analysis. We present that such behaviour is, in fact, manifested by certain FRW spacetimes in the late time limit. Below we present the main results obtained in this chapter :

- **Minimally Coupled Massive Scalar Fields in de Sitter Space-time :** First, we analyse the case of a minimally coupled massive scalar field in de Sitter spacetime. We study how the $(a = 0,\ b = 0,\ c = 0,\ d = 0)$ component of the noise kernel changes as the mass of the field is varied. In order to consider only those cases for which the mass is real, we restrict to $|\nu| \in [0, 3/2]$ (recall that $m^2/H^2 = 9/4 - \nu^2$). From our analysis, we conclude that the considered component of the stress energy correlator or the noise kernel, for finitely separated points with same time coordinates in the late time $\eta \to 0$ limit, shows a transition from zero to divergent behaviour as mass of the field is varied. More precisely, the considered noise kernel component is vanishing for $\nu \in [0, 1/2)$ and has a finite non-zero value at $\nu = 1/2$. However, the same component of the noise kernel diverges for $\nu > 1/2$ in the limit $\eta \to 0$. Similarly, the invariant or the energy energy correlator (defined above in the main text) vanishes in the $\eta \to 0$ limit for all the considered values of ν except for $\nu = 3/2$ for which it has the value $9H^8/128\pi^4$.

- **Non-Minimally Coupled Scalar Field in de Sitter Space-time:** We perform a similar analysis for studying the behaviour of the noise kernel for non-minimally coupled scalar field in de Sitter spacetime (again, for spacetime points with same time coordinates in the $\eta \to 0$ limit). This case has an extra term added to the minimally coupled Lagrangian which is $\xi R \phi^2$. In this case, one obtains a similar variation of the $(a = 0, b = 0, c = 0, d = 0)$ component of the noise kernel with ν as in the previous case of minimally coupled scalar field except that now $\nu = \sqrt{9/4 - (m^2 + 12\xi H^2)/H^2}$. Thus, we conclude that the considered noise kernel component vanishes for $(m^2/H^2 + 12\xi) > 2$ and is non-zero for $(m^2/H^2 + 12\xi) = 2$ but diverges for $(m^2/H^2 + 12\xi) < 2$. This observation implies that the noise kernel does not diverge for any mass value in the conformal case i.e., $\xi = 1/6$. Similar conclusions regarding the energy-energy correlator can be made e.g., it is 0 for $(m^2/H^2 + 12\xi) \in [0, 9/4)$ but is a non-zero constant for $m^2/H^2 + 12\xi = 0$.

- **Massless Scalar Fields in FRW Spacetimes:** We make use of an equivalence (shown in [65]) between massless scalar fields in FRW spacetimes with massive scalar fields in de Sitter spacetime to calculate the noise kernel for the former setting. We, again, consider cases with $|\nu| \in [0, 3/2]$ in de Sitter spacetime for which the corresponding FRW spacetimes have the exponent of the scaling factor $q \in [-2, 1]$. We notice that, for $q \in (0, 1]$, the late time limit (in which the scale factor becomes very large) corresponds to $\eta \to 0$ whereas for $q \in [-2,0)$, it corresponds to $\eta \to \infty$. We show that, in the late time limit, the considered noise kernel component goes to 0 for $q \in (-2,0)$ whereas it approaches a constant value for $q \in [0,1)$. As far as the energy energy correlator is concerned, we found that, for $q \in (-2,0) \cup (0,1)$, it is zero. For the remaining power-law type FRW spacetimes i.e., for $q \leq -2$ and $q \geq 1$ (with the correpsonding equation of state paramter lying in the range $-1 < w < 0$), the Wightman function contains a spacetime dependent divergent term in it. The conformal time dependent factors relating the de Sitter and the corresponding FRW Wightman functions provide extra time dependence to the divergent term of the de Sitter Wightman function and this term then dominates in the noise kernel and the invariant or energy energy correlator. Hence, these spacetimes are prone to significant quantum fluctuations at late times.

One can observe many interesting implications of these results. We find that the spacetimes, driven by fluids with equation of state parameter, $w > 0$, or $w < -1$ (corresponding to phantom Universes), do not get significant corrections from second order quantum effects and are likely stable against stochastic fluctuations provided that the expectation values are large. These implications, therefore, lend support to the structure of quantum fluctuations suggested in [65]. A number of other interesting conclusions can be drawn from this work. For example, the divergence in the de Sitter noise kernel, in the $\eta \to 0$ limit, for the cases with $\nu > 1/2$ has a time dependence (i.e., it is dynamical) as opposed to the spacetime independent divergence in the Wightman function which, too, is present only for the minimally coupled massless case. We also observe, in the case of phantom spacetimes i.e., for $w < -1$, the considered noise kernel component does not blow up but still has a non-zero value which is significant even at large physical distances corresponding to the points which have otherwise small coordinate separations. The most drastic effect of second order quantum fluctuations (and hence, leading to potentially large stochastic corrections to the semiclassical analysis and in fact, probably even disrupting the first order analysis altogether) is expected for the $q = -2$ case which has a divergence present in the noise kernel for all times. Similar behaviour is also expected for spacetimes driven by fluids with $w \in (-1,0]$. Thus, it is important that we take these second order quantum fluctuations into account to study the dynamics of massless scalar fields in these spacetimes and only with such an analysis, we will be able to make robust predictions about these spacetimes and the massless fields evolving in them.

Chapter 4

Stress energy correlator of spinor fields in de Sitter and FRW spacetimes

In this chapter, we evaluate the behaviour of the stress energy correlator of spinor fields in FRW spacetimes. We consider the cases of arbitrarily massive spinor fields in de Sitter spacetime and massless spinor fields in all types of FRW spacetimes. For spinor fields in de Sitter spacetime, we perform the calculations by placing the field in the fermionic Bunch-Davies vacuum which is discussed in chapter 2. For massless spinor fields, we make use of their conformal invariance in FRW spacetimes and place the massless fields in the Poincare vacuum of the corresponding massless spinor field in flat spacetime. Like in the scalar field study, we look at the behaviour of the stress energy correlator for spacetime points lying on constant time sheets. We also compare the obtained results for spinor fields with the ones derived for scalar fields in the previous chapter.

4.1 Introduction

We know that, other than the scalar fields, the fermionic fields are also present in the Universe and the second order quantum effects of these spinor fields, encoded in the noise kernel, may also affect the dynamics of the background spacetimes. Thus, it is important to analyze the behaviour of the stress energy correlator for spinor fields in FRW spacetimes because, on the basis of the behaviour of the stress energy correlator, we will be able to say something about the stability of the background spacetimes against second order quantum effects of spinor fields. Since spinor fields, just like any other field, can couple with the metric perturbations through their stress energy tensor, the stress energy correlator also provides corrections to the power spectrum of metric perturbations, particularly gravitational waves. There are some past works that have considered the modifications to gravitational waves by spinor fields. For example, in the works [152, 153], correction to the gravitational wave spectra by the spinor fields has been analyzed for the inflationary phase of the Universe. Similarly, in [154–156], the backreaction of spinor fields on gravitational waves have been looked at during the reheating era, assuming that the fermions are produced during this reheating phase of the Universe. Studying the stress energy correlator of spinor fields in FRW spacetimes can provide us with new insights into the dynamics of these FRW spacetimes. In this chapter, we study the behaviour of the stress energy correlator/noise kernel of spinor fields in general FRW spacetimes. For arbitrarily massive spinor fields in de Sitter spacetime, we perform this analysis for the case in which they are placed in the fermionic Bunch-Davies vacuum of chapter 2, which, as we saw, are defined analogously to how one defines scalar field Bunch-Davies vacuum. We also consider massless spinor fields in general FRW spacetimes. To perform the analysis for these cases, we use the conformal invariance of massless spinor fields in FRW spacetimes according to which one can relate a massless spinor field in any FRW spacetime with another massless spinor field in any other FRW spacetime through some time-dependent conformal factors. This equivalence also relates the Wightman functions in the two related settings. We also compare our results for spinor fields obtained in this chapter with the results for scalar fields obtained in chapter 3. This analysis of massless spinor fields in FRW spacetimes can be directly applied to cosmological contexts since different eras in the evolution history of the Universe can be approximated by FRW spacetimes. For example, the equation of state parameter for present day dark energy driven Universe is estimated to be equal to -1.03 ± 0.03 using the data from surveys [23] and hence, our analysis which includes both the quintessence regime ($-1 < w < -\frac{1}{3}$) and the phantom regime ($w < -1$) can be applied to the present day phase of the Universe [157]. This analysis, as applied to phantom Universes, has a physical appeal only if the phantom phase can prevent itself from its inherent big rip [157, 158] problem (in which the scale factor and the energy density diverges in a finite time interval) which is, in fact, possible if we have a dynamical equation of state parameter caused by certain potentials [159] for which the Universe indeed goes through a phantom phase but exits it before the big rip of the phantom phase arrives.

This chapter is divided into 6 sections including this one. In section 4.2, we make use of mapping between massless spinor fields in de Sitter and general FRW spacetimes to relate the expressions of the Wightman function in the two settings. In section 4.3, we express the noise kernel as a sum of some derivatives acting on product of Wightman functions using the point-split form of the stress energy operator. In section 4.4, we study the behaviour of the noise kernel for massive spinor fields in de Sitter spacetime placing them in the Bunch-Davies vacuum and analyse how it varies with the mass of the field. In section 4.5, we carry out a similar exercise for massless spinor fields in general FRW spacetimes. In section 4.6, we provide a summary of all our findings and discuss their possible implications.

4.2 Equivalence between massless spinor fields in FRW spacetimes and de Sitter spacetime

In the last chapter, we made use of an equivalence between massless scalar fields in power-law type FRW spacetimes with massive scalar fields in de Sitter spacetime. In this chapter, we employ the conformal invariance of massless spinor fields to relate the Wightman function in one FRW spacetime to the Wightman function in other FRW spacetime. Let us first consider a massive spinor field in an arbitrary FRW spacetime with $ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2)$ whose action is given by

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\left[i\bar{\psi}\gamma^\mu\nabla_\mu\psi - m\bar{\psi}\psi\right] \\ &= \int d^4x a^3\bar{\psi}\left[i\Gamma^\mu\partial_\mu + i\frac{3a'}{2a}\Gamma^0 - am\right]\psi . \end{aligned} \tag{4.1}$$

Under the field redefinition, $\psi = F(\eta)\Omega$, the above action becomes

$$S = \int d^4x a^3F^2\bar{\Omega}\left[i\Gamma^\mu\partial_\mu + i\frac{3a'}{2a}\Gamma^0 + i\frac{F'}{F}\Gamma^0 - am\right]\Omega , \tag{4.2}$$

and if we demand that $a^3F^2 = b^3$ and $\left[i\frac{3a'}{2a}\Gamma^0 + i\frac{F'}{F}\Gamma^0 - am\right] = \left[i\frac{3b'}{2b}\Gamma^0 - bm'\right]$, we have the action for a spinor field in an FRW spacetime with $ds^2 = b^2(\eta)(-d\eta^2 + d\vec{x}^2)$ i.e.,

$$S = \int d^4x b^3\bar{\Omega}\left[i\Gamma^\mu\partial_\mu + i\frac{3b'}{2b}\Gamma^0 - bm'\right]\Omega . \tag{4.3}$$

The condition that $a^3F^2 = b^3$ translates to

$$\frac{3a'}{2a} + \frac{F'}{F} = \frac{3b'}{2b} . \tag{4.4}$$

which, when used with the condition

$$\left[i\frac{3a'}{2a}\Gamma^0 + i\frac{F'}{F}\Gamma^0 - am\right] = \left[i\frac{3b'}{2b}\Gamma^0 - bm'\right] , \tag{4.5}$$

implies that $m' = \frac{a}{b}m$. We can make the following observations from this analysis

- Firstly, we see that a massless spinor field in an FRW spacetime can be related to another massless spinor field in any other arbitrary FRW spacetime, in particular we can map a massless spinor in any FRW spacetime to a massless spinor field in de Sitter or flat spacetime.
- Secondly, we can use the above mapping to study particle creation of spinor fields in FRW spacetimes in terms of similar questions in flat/de Sitter spacetimes. For example, if we take $b(\eta) = 1$ and $m = (1/a(\eta))$, then we have $m' =$ constant i.e., we have mapped a spinor field with time-dependent mass in an FRW spacetime to a spinor field in flat spacetime with constant mass. Similar types of equivalences can be used to study usual investigations of particle creation etc. in FRW spacetimes in terms of equivalent questions in relatively more easily tractable settings.
- Another very exciting possibility is to map a massless spinor (ψ) and a massless scalar field (ϕ) interacting through Yukawa coupling (with d being the coupling constant) in an FRW spacetime ($a(\eta) \propto \eta^{-q}$) to a massless spinor field and a massive scalar field interacting through Yukawa coupling in de Sitter spacetime. For this, we notice that the relevant action i.e.,

$$S = \int d^4x \left[a^3 \bar{\psi}\left(i\Gamma^\mu \partial_\mu + \frac{3ia'}{2a}\Gamma^0\right)\psi + \frac{a^2}{2}(-\eta^{\mu\nu}\partial_\mu \phi \partial \nu \phi) + a^4 d\phi\bar{\psi}\psi\right], \tag{4.6}$$

under the mapping $\psi = (-H\eta)^{-\frac{3}{2}(1-q)}\Omega$ (from above analysis) and $\phi = (-H\eta)^{-(1-q)}\zeta$ (Refer [65]), transforms to

$$\begin{aligned} S = \int d^4x \Big[(-H\eta)^{-3}\bar{\Omega}\Big(i\Gamma^\mu \partial_\mu - \frac{3i}{2\eta}\Gamma^0\Big)\Omega + \frac{(-H\eta)^{-2}}{2}\Big(-\eta^{\mu\nu}\partial_\mu \zeta \partial \nu \zeta \\ -\zeta^2(-H\eta)^{-2}(2+q)(1-q)H^2\Big) + (-H\eta)^{-4} d\zeta\bar{\Omega}\Omega\Big], \end{aligned} \tag{4.7}$$

which is just a massless spinor field and a massive, $m^2 = (2+q)(1-q)$, scalar field interacting through Yukawa coupling in de Sitter spacetime. A number of past works ([160–162]) have considered Yukawa coupling in FRW spacetimes and one can try to analyse how above considered transformations fit into these considerations.

All of these are interesting and possibly important directions to explore. However, in the present work, we are concerned with studying the behaviour of noise kernel of massless spinor fields in FRW spacetimes which, as we have seen, are related to massless spinor fields in de Sitter spacetime. Employing this equivalence we evaluate the behaviour of the noise kernel for massless spinor fields in general FRW spacetimes in terms of the de Sitter quantities i.e., we take $b(\eta) = -(1/(H\eta))$. The Wightman functions for the two settings are related as follows

$$\begin{aligned} S^{FRW}_{ij}(x,x') = \langle \psi_i(x)\bar{\psi}_j(x')\rangle &= (F(\eta)F(\eta'))\langle \Omega_i(x)\bar{\Omega}_j(x')\rangle \\ &= \left(\frac{b(\eta)}{a(\eta)}\right)^{\frac{3}{2}}\left(\frac{b(\eta')}{a(\eta')}\right)^{\frac{3}{2}} S^{dS}_{ij}(x,x') \\ &= \frac{1}{a^{\frac{3}{2}}(\eta_x)a^{\frac{3}{2}}(\eta_{x'})} S^{flat}_{ij}(x,x')\,. \end{aligned} \tag{4.8}$$

We also have

$$\begin{aligned} R^{FRW}_{ji}(x',x) = \langle \bar{\psi}_j(x')\psi_i(x)\rangle &= (F(\eta)F(\eta'))\langle \bar{\Omega}_j(x')\Omega_i(x)\rangle \\ &= \left(\frac{b(\eta)}{a(\eta)}\right)^{\frac{3}{2}}\left(\frac{b(\eta')}{a(\eta')}\right)^{\frac{3}{2}} R^{dS}_{ji}(x',x) \\ &= \frac{1}{a^{\frac{3}{2}}(\eta_x)a^{\frac{3}{2}}(\eta_{x'})} R^{flat}_{ji}(x',x)\,. \end{aligned} \tag{4.9}$$

We employ these relations later on when we evaluate the noise kernel for massless spinor fields in general FRW spacetimes.

4.3 Noise kernel for spinor fields

Using the formula (2.72) for the stres energy tensor i.e.,

$$T_{\mu\nu}(x) = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}(x)}\,, \tag{4.10}$$

we obtain that the stress energy tensor for minimally coupled spinor fields in curved spacetime (with the action (2.44)) is given by [12, 163]

$$\begin{aligned} T_{\mu\nu} &= -\frac{i}{2}g_{\mu\nu}[\bar{\psi}\gamma^\lambda \overrightarrow{\nabla}_\lambda\psi - \bar{\psi}\overleftarrow{\nabla}_\lambda\gamma^\lambda\psi] + \frac{i}{2}[\bar{\psi}\gamma_{(\mu}\overrightarrow{\nabla}_{\nu)}\psi - \bar{\psi}\overleftarrow{\nabla}_{(\nu}\gamma_{\mu)}\psi] + m\bar{\psi}\psi g_{\mu\nu} \\ &= -\frac{g_{\mu\nu}}{2}\bar{\psi}\Big[(i\gamma^\lambda\overrightarrow{\nabla}_\lambda - m) - (i\overleftarrow{\nabla}_\lambda\gamma^\lambda + m)\Big]\psi + \frac{i}{2}\bar{\psi}[\gamma_{(\mu}\overrightarrow{\nabla}_{\nu)} - \overleftarrow{\nabla}_{(\nu}\gamma_{\mu)}]\psi\,. \end{aligned} \tag{4.11}$$

Like in the case of scalar field, we write the spinor field stress energy tensor in a point-split form i.e.,

$$T_{\mu\nu}(x) = \lim_{x'\to x} P_{\mu\nu ij}(x,x')\bar{\psi}_i(x)\psi_j(x')\,, \tag{4.12}$$

where there is an implicit sum over i and j and

$$P_{\mu\nu ij}(x,x') = -\frac{g_{\mu\nu}}{2}\left[(i\gamma^\lambda \overrightarrow{\nabla}^{x'}_\lambda - m) - (i\overleftarrow{\nabla}^{x}_\lambda \gamma^\lambda + m)\right]_{ij} + \frac{i}{2}\left[\gamma_{(\mu}\overrightarrow{\nabla}^{x'}_{\nu)} - \overleftarrow{\nabla}^{x}_{(\nu}\gamma_{\mu)}\right]_{ij}. \tag{4.13}$$

We obtain the stress energy operator from the stress energy tensor by replacing the classical spinor fields by the corresponding quantum spinor fields. Now we use the above obtained point-split form of the stress energy operator in the definition of the stress energy correlator (2.78) i.e.,

$$\langle \hat{t}_{abcd}(x,y)\rangle = \langle \hat{T}_{ab}(x)\hat{T}_{cd}(y)\rangle - \langle \hat{T}_{ab}(x)\rangle\langle \hat{T}_{cd}(y)\rangle\,, \tag{4.14}$$

and obtain that it is given by

$$\langle \hat{t}_{abcd}(x,y)\rangle = \lim_{x'\to x}\lim_{y'\to y} P_{abij}(x,x')P_{cdkl}(y,y')\Big[\langle \hat{\bar{\psi}}_i(x)\hat{\psi}_j(x')\hat{\bar{\psi}}_k(y)\hat{\psi}_l(y')\rangle - \langle \hat{\bar{\psi}}_i(x)\hat{\psi}_j(x')\rangle\langle \hat{\bar{\psi}}_k(y)\hat{\psi}_l(y')\rangle\Big]. \tag{4.15}$$

Using Wick's theorem, the above expression becomes

$$\langle \hat{t}_{abcd}(x,y)\rangle = \lim_{x'\to x}\lim_{y'\to y} P_{abij}(x,x')P_{cdkl}(y,y')\Big[\langle \hat{\bar{\psi}}_i(x)\hat{\psi}_l(y')\rangle\langle \hat{\psi}_j(x')\hat{\bar{\psi}}_k(y)\rangle\Big]. \tag{4.16}$$

Identifying that the terms in the first square bracket in (4.13) are just the Dirac equation operator and its adjoint and the fact that $\hat{\psi}$ is a linear combination of the solutions of the Dirac equation and $\hat{\bar{\psi}}$ is a linear combination of the solutions of the adjoint of the Dirac equation, the terms in the first bracket in (4.13) gives zero when they act on $\hat{\psi}$ and $\hat{\bar{\psi}}$. Keeping this into account, we obtain that the noise kernel is given by

$$\begin{aligned}
\langle \hat{t}_{abcd}(x,y)\rangle &= \lim_{x'\to x}\lim_{y'\to y}\frac{1}{4}\left[\gamma_{(a}\overrightarrow{\nabla}^{x'}_{b)} - \overleftarrow{\nabla}^{x}_{(a}\gamma_{b)}\right]_{ij}\left[\gamma_{(c}\overrightarrow{\nabla}^{y'}_{d)} - \overleftarrow{\nabla}^{y}_{(c}\gamma_{d)}\right]_{kl} S_{jk}(x',y)S_{li}(y',x)\\
&= \lim_{x'\to x}\lim_{y'\to y}\frac{1}{4}\Big[Tr\big(\gamma_{(a}\overrightarrow{\nabla}^{x'}_{b)}S(x',y)\gamma_{(c}\overrightarrow{\nabla}^{y'}_{d)}S(y',x)\big)\\
&\quad -Tr\big(\gamma_{(a}\overrightarrow{\nabla}^{x'}_{b)}S(x',y)\overleftarrow{\nabla}^{y}_{(c}\gamma_{d)}S(y',x)\big) - Tr\big(S(x',y)\gamma_{(c}\overrightarrow{\nabla}^{y'}_{d)}S(y',x)\overleftarrow{\nabla}^{x}_{(a}\gamma_{b)}\big)\\
&\quad +Tr\big(S(x',y)\overleftarrow{\nabla}^{y}_{(c}\gamma_{d)}S(y',x)\overleftarrow{\nabla}^{x}_{(a}\gamma_{b)}\big)\Big].
\end{aligned} \tag{4.17}$$

The above expressions have been derived without assuming any particular spacetime metric and are valid for any general spacetime. Now we specialize to the case of arbitrarily massive spinor fields in de Sitter spacetime and provide the behaviour of the noise kernel for this case in the next section. We perform the noise kernel calculations for the fermionic Bunch Davies vacuum which we considered in Chapter 2. Like in the scalar field case, here also we calculate the behaviour of the $(a=b=c=d=0)$ component of the noise kernel for exactly the same reasons as for the scalar fields case.

4.4 Noise kernel for spinor fields in de Sitter spacetime

Let us now consider the case of masssive spinor fields in de Sitter spacetime. Using the above obtained expression of the noise kernel as a sum of derivatives acting on product of Wightman functions i.e., (4.17), and taking the vacuum to be the fermionic Bunch-Davies vacuum for which the Wightman function is given by (2.70) and also using that $\overrightarrow{\nabla}^x_0 = \partial^x_0$, we obtain that the $(a=b=c=d=0)$ component of the noise kernel is given by [164]

$$\begin{aligned}\langle \hat{t}_{0000}(x,y)\rangle &= \lim_{x'\to x}\lim_{y'\to y}\frac{a(\eta_x)a(\eta_y)}{4}\left[\partial_0^{x'}\partial_0^{y'}+\partial_0^{x}\partial_0^{y}-\partial_0^{x'}\partial_0^{y}-\partial_0^{x}\partial_0^{y'}\right]Tr\big(\Gamma_0 S(x',y)\Gamma_0 S(y',x)\big)\\ &= \lim_{x'\to x}\lim_{y'\to y}\frac{a_x a_y}{4}\left[\partial_0^{x'}\partial_0^{y'}+\partial_0^{x}\partial_0^{y}-\partial_0^{x'}\partial_0^{y}-\partial_0^{x}\partial_0^{y'}\right]\frac{H^4}{\sqrt{a_x a_{x'} a_y a_{y'}}}\\ &\quad Tr\left(\Gamma_0\left[i\Gamma^\lambda\partial_\lambda^{x'}+i\frac{a'_{x'}}{a_{x'}}\Gamma^0+a_{x'}m\right]\Big[\sum_{\varepsilon=\pm}S_\varepsilon(x',y)\frac{1+\varepsilon\Gamma^0}{2}\Big]\right.\\ &\quad\left.\Gamma_0\left[i\Gamma^\sigma\partial_\sigma^{y'}+i\frac{a'_{y'}}{a_{y'}}\Gamma^0+a_{y'}m\right]\Big[\sum_{\varepsilon=\pm}S_\varepsilon(y',x)\frac{1+\varepsilon\Gamma^0}{2}\Big]\right),\end{aligned} \tag{4.18}$$

where a_x stands for $-(1/(H\eta_x))$ (we take this convention for the rest of this section). By applying the operators in the big square brackets on the factor $((H^4)/(\sqrt{a_x a_{x'} a_y a_{y'}}))$ and using the product rule, we find that this factor cancels the factor $a_x a_y$ in the above expression in the above limits. Finally, we are left with the following expression

$$\begin{aligned}\langle \hat{t}_{0000}(x,y)\rangle &= \lim_{x'\to x}\lim_{y'\to y}\frac{H^4}{4}\left[\partial_0^{x'}\partial_0^{y'}+\partial_0^{x}\partial_0^{y}-\partial_0^{x'}\partial_0^{y}-\partial_0^{x}\partial_0^{y'}\right]\\ &\quad Tr\left(\Gamma_0\left[i\Gamma^\lambda\partial_\lambda^{x'}+i\frac{a'_{x'}}{a_{x'}}\Gamma^0+a_{x'}m\right]\Big[\sum_{\varepsilon=\pm}S_\varepsilon(x',y)\frac{1+\varepsilon\Gamma^0}{2}\Big]\right.\\ &\quad\left.\Gamma_0\left[i\Gamma^\sigma\partial_\sigma^{y'}+i\frac{a'_{y'}}{a_{y'}}\Gamma^0+a_{y'}m\right]\Big[\sum_{\varepsilon=\pm}S_\varepsilon(y',x)\frac{1+\varepsilon\Gamma^0}{2}\Big]\right).\end{aligned} \tag{4.19}$$

Therefore, to study the behaviour of the above expression for the stress energy correlator, we need to evaluate the above derivatives. We also need to find the traces of the gamma matrices which can be evaluated using the known properties [12] of the gamma matrices.

4.4.1 Gamma mechanics

The expression (4.19) can be written as follows (using cyclic property of the traces)

$$\begin{aligned}
\langle \hat{t}_{0000}(x,y)\rangle &= \lim_{x'\to x}\lim_{y'\to y}\frac{H^4}{4}\left[\partial_0^{x'}\partial_0^{y'}+\partial_0^{x}\partial_0^{y}-\partial_0^{x'}\partial_0^{y}-\partial_0^{x}\partial_0^{y'}\right] \\
&\sum_{\varepsilon=\pm}\sum_{\delta=\pm} Tr\Bigg(\Gamma_0\left[i\Gamma^\lambda\partial_\lambda^{x'}+i\frac{a'_{x'}}{a_{x'}}\Gamma^0+a_{x'}m\right]\left[\frac{1+\varepsilon\Gamma^0}{2}\right] \\
&\Gamma_0\left[i\Gamma^\sigma\partial_\sigma^{y'}+i\frac{a'_{y'}}{a_{y'}}\Gamma^0+a_{y'}m\right]\left[\frac{1+\delta\Gamma^0}{2}\right]\Bigg)S_\varepsilon(x',y)S_\delta(y',x) \\
&= \lim_{x'\to x}\lim_{y'\to y}\frac{H^4}{4}\left[\partial_0^{x'}\partial_0^{y'}+\partial_0^{x}\partial_0^{y}-\partial_0^{x'}\partial_0^{y}-\partial_0^{x}\partial_0^{y'}\right] \\
&\sum_{\varepsilon=\pm}\sum_{\delta=\pm} Tr\Bigg(i^2\Gamma^\lambda M\Gamma^\sigma P\partial_\lambda^{x'}\partial_\sigma^{y'}+i^2\Gamma^\lambda M\Gamma^0 P\frac{a'_{y'}}{a_{y'}}\partial_\lambda^{x'}+ia_{y'}m\Gamma^\lambda MP\partial_\lambda^{x'} \\
&+i^2\frac{a'_{x'}}{a_{x'}}\Gamma^0 M\Gamma^\sigma P\partial_\sigma^{y'}+i^2\frac{a'_{x'}}{a_{x'}}\frac{a'_{y'}}{a_{y'}}\Gamma^0 M\Gamma^0 P+im\frac{a'_{x'}}{a_{x'}}a_{y'}\Gamma^0 MP \\
&+ia_{x'}mM\Gamma^\sigma P\partial_\sigma^{y'}+ia_{x'}m\frac{a'_{y'}}{a_{y'}}M\Gamma^0 P+a_{x'}a_{y'}m^2MP\Bigg)S_\varepsilon(x',y)S_\delta(y',x)\,,
\end{aligned} \tag{4.20}$$

where $M=\left[\frac{\Gamma^0+\varepsilon}{2}\right]$ and $P=\left[\frac{\Gamma^0+\delta}{2}\right]$.

If we now evaluate the traces of the gamma matrices in the above expression, we find that it reduces to [164]

$$\begin{aligned}
\langle \hat{t}_{0000}(x,y)\rangle &= -\lim_{x'\to x}\lim_{y'\to y}\frac{2H^4}{4}\left[\partial_0^{x'}\partial_0^{y'}+\partial_0^{x}\partial_0^{y}-\partial_0^{x'}\partial_0^{y}-\partial_0^{x}\partial_0^{y'}\right]\times \\
&\Bigg[\sum_{\varepsilon=\pm}\Bigg(\partial_0^{x'}\partial_0^{y'}+\left(\frac{a'_{x'}}{a_{x'}}-\varepsilon ima_{x'}\right)\partial_0^{y'}+\left(\frac{a'_{y'}}{a_{y'}}-\varepsilon ima_{y'}\right)\partial_0^{x'} \\
&+\left(\frac{a'_{y'}}{a_{y'}}-\varepsilon ima_{y'}\right)\left(\frac{a'_{x'}}{a_{x'}}-\varepsilon ima_{x'}\right)\Bigg)S_\varepsilon(x',y)S_\varepsilon(y',x) \\
&+\delta^{kl}\partial_k^{x'}\partial_l^{y'}\Big(S_+(x',y)S_-(y',x)+S_-(x',y)S_+(y',x)\Big)\Bigg]\,.
\end{aligned} \tag{4.21}$$

4.4.2 Stress energy correlator on equal time sheets

As in the scalar field case, we find the behaviour of the considered component of the stress energy correlator for spacetime points which lie on same time sheets i.e., for which $\eta = \eta_x = \eta_y$, and have finite spatial separation between them. However, it is important that we take all the derivatives before taking $\eta_x = \eta_y$ and in fact, all the limits appearing in the above expression should also be taken only after evaluating the derivatives first. In order to evaluate the derivatives in the above obtained expression of the stress energy correlator, we use the property of $S_{\pm}(x,y)$ that it is a function of x and y only through the de Sitter invariant distance i.e., $S_{\pm}(x,x') = S_{\pm}(Z(x,x'))$. Making use of formulae in Appendix C, the considered stress energy correlator component for points on equal time sheets is given by

$$\begin{aligned}
-\frac{2}{H^4}\langle \hat{t}_{0000}(x,y)\rangle \;=\;& \left(S''_+S'_- + (-\leftrightarrow +)\right)\left[-\frac{(\Delta\vec{x})^4}{8\eta^8} - \frac{(\Delta\vec{x})^2}{4\eta^6}\right] + \left(S''_+S''_-\right)\left[-\frac{(\Delta\vec{x})^6}{16\eta^{10}}\right] \\
&+ \left(S'''_+S'_- + (-\leftrightarrow +)\right)\left[\frac{(\Delta\vec{x})^6}{32\eta^{10}}\right] \\
&+ \sum_{\varepsilon=\pm}\left(1-\frac{\varepsilon i m}{H}\right)\Bigg[\frac{(S'_\varepsilon)^2}{4}\left[\frac{(\Delta\vec{x})^4}{2\eta^8} - \frac{(\Delta\vec{x})^2}{\eta^6}\right] + \frac{2}{\eta^4}S_\varepsilon S'_\varepsilon \\
&+ S_\varepsilon S''_\varepsilon\left[-\frac{3(\Delta\vec{x})^4}{8\eta^8} - \frac{(\Delta\vec{x})^2}{4\eta^6}\right] + \left(S_\varepsilon S'''_\varepsilon - S'_\varepsilon S''_\varepsilon\right)\frac{(\Delta\vec{x})^6}{32\eta^{10}}\Bigg] \\
&+ \left(1-\frac{\varepsilon i m}{H}\right)^2\Bigg[\frac{(S'_\varepsilon)^2}{4}\left[\frac{(\Delta\vec{x})^4}{2\eta^8}\right] + \frac{S_\varepsilon S_\varepsilon}{\eta^4} + S_\varepsilon S'_\varepsilon\left[\frac{(\Delta\vec{x})^2}{2\eta^6} + \frac{1}{\eta^4}\right] \\
&- S_\varepsilon S''_\varepsilon\frac{(\Delta\vec{x})^4}{8\eta^8}\Bigg] + \Bigg[\frac{(S'_\varepsilon)^2}{4}\left[\frac{(\Delta\vec{x})^4}{4\eta^8} - 2\frac{(\Delta\vec{x})^2}{\eta^6} + \frac{2}{\eta^4}\right] \\
&+ \frac{S'_\varepsilon S''_\varepsilon}{8}\frac{(\Delta\vec{x})^4}{4\eta^6}\left[\frac{2}{\eta^2} + \frac{(\Delta\vec{x})^2}{\eta^4}\right] + \frac{S''_\varepsilon S''_\varepsilon}{16}\frac{(\Delta\vec{x})^8}{8\eta^{12}} - \frac{S'_\varepsilon S'''_\varepsilon}{16}\frac{(\Delta\vec{x})^8}{8\eta^{12}}\Bigg].
\end{aligned} \tag{4.22}$$

As in the scalar field case, we want to study the behaviour of the stress energy correlator for the limit in which the scale factor goes to infinity which, for de Sitter case, corresponds to $\eta \to 0$ limit. We now recall that the $S'_\varepsilon s$ are Hypergeometric functions (refer (2.69)) and the derivatives of Hypergeometric functions are also Hypergeometric functions [165]. Taking these facts into account and using the asymptotic behaviour of Hypergeometric functions, the considered component of the stress energy correlator, in the $\eta \to 0$ limit, has the following leading order behaviour [164]

$$\langle \hat{t}_{0000}(x,y)\rangle = \frac{2H^4\eta^2}{\pi^3(\Delta\vec{x})^6}\frac{(1+\frac{m^2}{H^2})(\frac{m^3}{H^3})}{\sinh(\frac{2\pi m}{H})} + O(\eta^4). \tag{4.23}$$

There are a number of observations that one can make from the above expression. We see that the considered component of the stress energy correlator/noise kernel decays to 0 in the late time limit i.e., $\eta \to 0$. This decay of the correlations of stress energy operators, for spatially separated points, occurs independently of the mass of the field and hence we see that this decay is a universal feature for all spinor fields whether massive or light. We observe that these results for spinor fields stand in contrast with the results for scalar fields, obtained in the previous chapter. Where in this case, we always have decay in the stress energy correlations, the considered component of the stress energy correlator for scalar fields shows a transition from decaying to divergent behaviour with the variation of the mass of the scalar field (refer Chapter 3). To understand these results better, recall that any two spatially separated points with finite coordinate distance have negligible physical separation in the past (i.e., in the $\eta \to -\infty$ limit) because the scale factor goes to zero in the asymptotic past and hence the correlations between quantum fields for these spatially separated points is large in the asymptotic past. However, as time progresses, the scale factor (multiplying the coordinate distances to give the physical distances) increases and the physical separations between spatially separated points (with finite coordinate distances) increase. We expect that the correlations between stress energy operators (or any other quantities depending upon quantum field correlations) for spatially separated points should decrease as the physical distances between these points increase with time and this is, in fact, what we observe for stress energy correlations of arbitrarily massive spinor fields. This means that the quantum dynamics of spinor fields (whether massive or light) over de Sitter spacetime can not compensate for the decay in the correlations caused by the increasing physical separations between spatially separated points in the de Sitter spacetime and the stress energy correlations vanish in the $\eta \to 0$ limit. But the results of the previous chapter suggest that the quantum dynamics of light scalar fields, with $(m^2/H^2) \in [0,2]$, can indeed overcome the decay in the correlations caused by the increasing physical separations between spatially separated points and we have divergenct correlations between stress energy operators in these cases. We also observe that the considered component of the stress energy correlator/noise kernel goes inversely to the coordinate distances of the spatially separated points which is to say that it takes longer to have the same decay between points with less coordinate distances compared to the ones with large coordinate distances.

Since we know that the noise kernel is a bi-tensor quantity and hence frame dependent, we express our results also in terms of the invariant/ energy-energy correlator of the previous chapter. Recall that, in the conformal coordinates, the invariant correlator has the following form

$$\langle \hat{t}_{\mu\nu\alpha\beta}(x,y)\rangle t^{\mu}(x)t^{\nu}(x)t^{\alpha}(y)t^{\beta}(y) = \frac{\langle \hat{t}_{0000}(x,y)\rangle}{a^2(\eta_x)a^2(\eta_y)}\,. \tag{4.24}$$

Using the expression of the stress energy correlations obtained above for spatially separated points in the late time limit, we find that the leading order behaviour of the invariant correlator in the late time limit is [164]

$$\frac{2H^8\eta^6}{\pi^3(\Delta\vec{x})^6}\frac{(1+\frac{m^2}{H^2})(\frac{m^3}{H^3})}{\sinh(\frac{2\pi m}{H})}. \tag{4.25}$$

We observe that the qualitative behaviour of the invariant corrrelator is the same as that of the above considered component of the noise kernel i.e., we still have a universal decay for invariant correlations independent of the mass of the spinor field. The only change that we have for the invariant correlator compared to the noise kernel component is that we have an extra factor of $(H\eta)^4$ multiplying the noise kernel expression and because of which the late time decay of the invariant correlator is even faster.

4.5 Noise kernel for massless spinors in general FRW spacetimes

Now we turn our focus to the case of massless spinor fields in general FRW spacetimes and calculate the behaviour of the stress energy correlator and the invariant correlator for them. To perform the analysis for this case, we employ the relation between the Wightman functions of massless spinor fields in general FRW spacetimes with that of the corresponding massless spinor field in de Sitter spacetime that we discussed in section 4.2. As we discussed earlier in section 4.2, this equivalence is analogous to the equivalence between scalar fields in FRW and de Sitter spacetimes that we made use of in last chapter. According to the equivalence of spinor fields, we can relate a massless spinor field, ψ, in an FRW spacetime with scale factor, $a(\eta)$, to another massless spinor field, Ω, in de Sitter spacetime and the relation between them is $\psi(x) = \big((b(\eta))/(a(\eta))\big)^{\frac{3}{2}}\Omega(x)$ where $b(\eta) = -(1/(H\eta))$. The relations between the Wightman functions in these related settings are as follows

$$S_{ij}^{FRW}(x,y) = \left(\frac{b(\eta)}{a(\eta)}\right)^{\frac{3}{2}}\left(\frac{b(\eta')}{a(\eta')}\right)^{\frac{3}{2}} S_{ij}^{dS}(x,y) \tag{4.26}$$

and

$$R_{ji}^{FRW}(y,x) = \left(\frac{b(\eta)}{a(\eta)}\right)^{\frac{3}{2}}\left(\frac{b(\eta')}{a(\eta')}\right)^{\frac{3}{2}} R_{ji}^{dS}(y,x). \tag{4.27}$$

Substituting these relations in the formula (4.17), we obtain the following expression for the $(a=0,\ b=0,\ c=0,\ d=0)$ component of the stress energy correlator/ noise kernel for a massless spinor field in an

FRW spacetime (with scale factor $a_x \equiv a(\eta_x)$)

$$\begin{aligned}\langle \hat{t}^{FRW}_{0000}(x,y)\rangle &= \lim_{x'\to x}\lim_{y'\to y}\frac{a_x a_y}{4}\left[\partial_0^{x'}\partial_0^{y'}+\partial_0^{x}\partial_0^{y}-\partial_0^{x'}\partial_0^{y}-\partial_0^{x}\partial_0^{y'}\right]H^4\frac{(b_x b_{x'} b_y b_{y'})}{(a_x a_{x'} a_y a_{y'})^{\frac{3}{2}}}\\ &\qquad Tr\Bigg(\Gamma_0\left[i\Gamma^{\lambda}\partial_{\lambda}^{x'}+i\frac{b'_{x'}}{b_{x'}}\Gamma^0\right]\Big[\sum_{\varepsilon=\pm}S_{\varepsilon}(x',y)\frac{1+\varepsilon\Gamma^0}{2}\Big]\Gamma_0\\ &\qquad\qquad \left[i\Gamma^{\sigma}\partial_{\sigma}^{y'}+i\frac{b'_{y'}}{b_{y'}}\Gamma^0\right]\Big[\sum_{\varepsilon=\pm}S_{\varepsilon}(y',x)\frac{1+\varepsilon\Gamma^0}{2}\Big]\Bigg).\end{aligned} \tag{4.28}$$

Using the product rule of differentiation, one finds that the action of the derivative operators in the larger square brackets is such that the factor $H^4\left((b_x b_{x'} b_y b_{y'})/(a_x a_{x'} a_y a_{y'})^{\frac{3}{2}}\right)$ on the right of the brackets comes to the left of the brackets with the derivatives in the larger square brackets acting only on the left over trace term. Also, the factor $H^4\left((b_x b_{x'} b_y b_{y'})/(a_x a_{x'} a_y a_{y'})^{\frac{3}{2}}\right)$ multiplies the factor $a_x a_y$ to give us that the above considered component of the noise kernel is $\left((b_x b_y)^2/(a_x a_y)^2\right)$ times the noise kernel of massless spinor field in de Sitter spacetime. Using the fact that, for massless spinor fields in de Sitter spacetime, we have

$$S_+(Z(x,y)) = S_-(Z(x,y)) = \frac{1}{16\pi^2(1-Z(x,y))}, \tag{4.29}$$

the above expression for the stress energy correlator of massless spinor field in FRW spacetimes becomes [164]

$$\langle \hat{t}^{FRW}_{0000}(x,y)\rangle = \frac{b_x^4}{a_x^4}\frac{3H^4\eta^4}{2\pi^4(\Delta\vec{x})^8} = \frac{3}{2\pi^4(\Delta\vec{x})^8}a_x^{-4}. \tag{4.30}$$

From the above expression, we observe that the considered noise kernel component (corresponding to a massless spinor field in an FRW spacetime with scale factor $a_x = a(\eta_x)$) has a behaviour opposite to that of the scale factor. For spatially separated points with same time coordinates i.e., on constant time sheets, the correlations between stress energy operators for massless spinor field grow for contracting spacetimes whereas the stress energy correlations decay for expanding spacetimes. We also observe that any monotonic or non-monotonic behaviour in the scale factor is reflected in the behaviour of the stress energy correlations. For example, for a monotonically increasing scale factor corresponding to an expanding spacetime, the stress energy correlations between spatially separated points are always smaller on a given constant time sheet compared to their magnitude for all constant time slices that lie earlier than the given time sheet. Let us now consider the behaviour of the considered component of the stress energy correlator for power-law type FRW spacetimes i.e., $a(\eta) \propto \eta^{-q}$. We obtain the following expression for these spacetimes [164]

$$\langle \hat{t}^{FRW}_{0000}(x,y)\rangle = \frac{3}{2\pi^4(\Delta\vec{x})^8}(H\eta)^{4q}. \tag{4.31}$$

Here, as in the previous chapter for scalar field case, we discuss these results for different power-law type FRW spacetimes in terms of the corresponding equation of state parameter. As discussed in the previous chapter, for spacetimes with positive values of q i.e for $q \in (0,\infty)$, the late time limit (in which the scale factor goes to infinity) corresponds to the $\eta \to 0$ limit and the corresponding equation of state paramer, w, belongs to the range $(-\infty,-1/3)$. Phantom spacetimes driven by fluids with equation of state parameter, $w \in (-\infty,-1)$, and quintessence spacetimes driven by fluids with $w \in (-1,-1/3)$ belong to this class of power-law type FRW spacetimes. In fact, the present day dark-energy driven Universe belongs to this regime with $w = -1.03 \pm 0.03$ [23]. Spacetimes with negative values of q i.e. $q \in (-\infty,0)$, have the late time limit given by $\eta \to \infty$ and the corresponding equation of state parameter, $w \in (-1/3,\infty)$. This class of power-law type FRW spacetimes with negative q values include both the radiation dominated as well as matter dominated cases. For all power-law type FRW spacetimes, the considered stress energy correlator component vanishes in the late time limit as all of them are expanding spacetimes for which we have already seen that the correlations decay with the evolution of the spacetimes. Along similar lines as in the previous section, such an observation about the decay of correlations between stress energy operators (for spatially separated comoving points) with the evolution of expanding spacetimes imply that the quantum evolution of massless spinor fields in expanding FRW spacetimes is different than that for massless scalar fields in certain FRW spacetimes (see previous chapter) in that it can overcome the decaying effects of increasing physical separations between spatially separated comoving points in an expanding spacetime.

As is done for previously considered cases, for this case also, we look at the behaviour of the coordinate independent correlator introduced in the previous chapter and also discussed in the previous section i.e., the invariant correlator. For spatially separated points on equal time sheets, the invariant correlator behaves as follows

$$\frac{\langle \hat{t}^{FRW}_{0000}(x,y)\rangle}{a^2(\eta_x)a^2(\eta_y)} = a_x^{-8}\frac{3}{2\pi^4(\Delta\vec{x})^8}. \tag{4.32}$$

From the above expression, it is easily seen that the qualitative behaviour of the invariant correlator is same as the behaviour of the $(a=b=c=d=0)$ component of the stress energy correlator except for the fact that the scale factor has more negative power in the former case as compared to the latter. Particularly, we notice that the invariant correlator vanishes for expanding spacetimes whereas it diverges for contracting spacetimes.

We can compare the results obtained here for massless spinor fields in power-law type FRW spacetimes with the ones obtained in the previous chapter for massless scalar fields in the same spacetimes. On one hand, we have seen that, for massless spinor fields, there is always a decay of stress energy correlations in power-law type expanding spacetimes whereas, on the other hand, in case of minimally coupled massless scalar fields, we found that there are certain power-law type expanding FRW spacetimes for which the stress energy correlations are significant. For example, for spacetimes with $w \in (0, -1/3) \cup (-1/3, -1)$, there are significant second order quantum effects of massless scalar fields quantified by the noise kernel/ stress energy correlator. Thus, we conclude that, in cases where we have both massless spinor and scalar fields present in these power-law FRW spacetimes, we expect only the scalar field (but not the spinor field) to provide second order quantum corrections to first order semiclassical gravity analysis.

4.6 Summary

Here, we collect all the important results that we have derived in this chapter. Our focus, in this chapter, has been on studying the dynamics of massive spinor fields in de Sitter spacetime and massless spinor fields in general FRW spacetimes. Particularly, we have calculated the behaviour of the stress energy correlator for spatially separated points in the late time limit. We place the massive spinor fields considered in de Sitter spacetime in the fermionic Bunch-Davies vacuum and for massless spinor fields in FRW spacetimes, we have employed their conformal invariance to place them in the Bunch-Davies vacuum of the related massless spinor field of de Sitter spacetime. The main results of this chapter are as follows

- **Behaviour of the noise kernel for spinor fields in de Sitter spacetime:** First, we look at the behaviour of the $(a = b = c = d = 0)$ component of the stress energy correlator for arbitrarily massive spinor fields in de Sitter spacetime. We observe that the leading order behaviour of this component of the stress energy correlator, for spatially separated points on constant time slices, vanishes in the late time $\eta \to 0$ limit. We also observe that the decay in the stress energy correlations is a universal feature of these correlations for massive spinor fields in de Sitter spacetime as this decay occurs independently of the mass of the field. In that sense, the behaviour of the stress energy correlators for spinor fields is in contrast to that for scalar fields as there are scalar fields with certain mass range which have, in fact, divergent stress energy correlator (see previous chapter). The invariant correlator for arbitrarily massive spinor fields also decays in the late time limit (with even faster rate compared to the considered stress energy correlator component). The considered correlators also have the usual decay with the increasing coordinate distances between the spatially separated points.

- **Behaviour of the noise kernel for spinor fields in FRW spacetimes:** We also look at the behaviour of the $(a = b = c = d = 0)$ component of the stress energy correlator/noise kernel for massless spinor fields in FRW spacetimes. For this purpose, we have employed the equivalence between massless spinor fields in general FRW spacetimes with that of massless spinor field in de Sitter spacetime. We observe that the considered component of the stress energy correlator behaves opposite to the scale factor. For expanding spacetimes, the correlations decay whereas they grow for contracting spacetimes. This is true, particularly, for power-law expanding FRW spacetimes and hence, we find that these results for massless spinor fields is in contrast to those for massless scalar fields in the same spacetimes (as seen in the last chapter, there are certain power-law type expanding FRW spacetimes for which the noise kernel does not decay). The invariant correlator for massless spinor fields in FRW spacetimes also behaves opposite to the behaviour of the scale factor.

On the basis of these results, one can infer that, for massless spinor fields in expanding FRW spacetimes and arbitrarily massive spinor fields in de Sitter spacetime, the second order quantum effects (quantified by the stress energy correlator) do not provide any significant corrections to the results obtained using the first order quantum analysis based only on the quantum averages of the stress energy operator. Hence, in these scenarios, we can expect the predictions made from the first order analysis to hold true against quantum fluctuations. One important point to emphasize is that the above conclusions have been obtained assuming the Bunch-Davies like vacua for spinor fields and it would be interesting to perform similar analysis for other vacua as well.

Chapter 5

Response of derivatively coupled Unruh deWitt detectors in FRW spacetimes

As argued in chapter 2, one way to study the correlations of quantum fields in curved spacetime is through studying the response of UdW detectors which couple with quantum fields. In this chapter, we study the quantum fields in FRW spacetimes by coupling them with both conventional and derivatively coupled UdW detectors. In this analysis also, we make use of the FRW-de Sitter equivalence that we have talked about earlier. Particularly, the focus of this study is on the infrared divergence of the Bunch-Davies vacuum in the massless limit and also on the related infrared divergence which is inherited by massless scalar field in matter dominated spacetime.

5.1 Introduction

It can be shown [166, 167] that the UdW detectors can be used to model the interaction between electromagnetic waves and atoms. This provides an opportunity to verify the predictions obtained from UdW detector analyses through quantum optical setups like testing the validity of thermal response of UdW detector while in uniformly accelerating motion in flat spacetime [104, 105]. It should also be interesting to analyse the curvature effect of curved spacetimes on the response of UdW detectors [168]. In that regard, our Universe's expansion history provides a situation where there is curvature present and one can study the curvature effects of these FRW spacetimes on UdW detectors. Many works in the past [169–174], have taken up the analysis of quantum fields in FRW spacetimes through UdW coupling.

For example, response of UdW detector coupled with real quantum scalar fields in de Sitter spacetime have been studied in [169], whereas the case of complex scalar fields which are quadratically coupled with UdW detectors in de Sitter spacetime have been analysed in [173], [171] considers the case of scalar fields for conformal vacua in FRW spacetimes. The Wightman function of quantum fields in FRW spacetimes are well known to show infrared divergences [13, 65, 84, 123, 124, 134, 137]. In a study done in [175] for flat spacetimes, it has been shown that the divergences in correlation functions have the potential to strongly enhance the UdW responses so as to reveal small acceleration dependence. Since the quantum fields in FRW spacetimes possess infrared divergences, we expect that these divergences can also cause an enhancement in the response of UdW detectors. This motivates us to analyse the response of UdW detectors which are coupled to quantum fields in FRW spacetimes. We consider the response of both conventionally coupled and derivatively coupled UdW detectors by coupling them to quantum fields in FRW spacetimes. The motivation for studying derivatively coupled UdW detectors will be explained once we discuss conventional UdW detectors.
We divide the remaining chapter in three sections. In section 5.2, the case of conventionally coupled UdW detectors is considered. Particularly, we look at the finite time response rate of UdW detectors which are coupled to scalar fields in de Sitter, radiation-dominated and matter-dominated spacetimes. In section 5.3, a similar consideration with derivatively coupled UdW detectors has been carried out. In section 5.4, we collect all the important results derived in this chapter.

5.2 Conventional UdW detectors

This section considers the analysis of massless scalar fields in FRW spacetimes by coupling them with conventional UdW detectors. From chapter 2, we recall that the interaction Hamiltonian for conventional UdW detectors is given by equation (2.79) i.e.,

$$H_{int} = c\chi(\tau)\hat{\mu}(\tau)\hat{\phi}(x(\tau)), \tag{5.1}$$

where the meaning of different symbols is explained in chapter 2.

For the scenario in which the detector goes from a state $|0\rangle_D$ to another state $|\Omega\rangle_D$ (which have energies 0 and Ω, respectively) while the field goes from $|\psi\rangle$ to any arbitrary final state which we finally trace over, the transition probability, is given by Eq. (2.82) i.e.,

$$P_{0\to\Omega} = c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2 \iint d\tau_1 d\tau_2 \chi(\tau_1)\chi(\tau_2)e^{-i\Omega(\tau_1-\tau_2)}G(x(\tau_1),x(\tau_2)), \tag{5.2}$$

where $G(x(\tau_1),x(\tau_2)) = \langle\psi|\hat{\phi}(x(\tau_1))\hat{\phi}(x(\tau_2))|\psi\rangle$ is the Wightman function of the field for the state $|\psi\rangle$.

Considering the case of finite time uniform switching for the interval $(\tau_i,\ \tau_f)$, the above expression reduces to

$$P_{0\to\Omega} = c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2 \int_{\tau_i}^{\tau_f}\int_{\tau_i}^{\tau_f} d\tau_1 d\tau_2 e^{-i\Omega(\tau_1-\tau_2)}G(x(\tau_1),x(\tau_2))\,. \tag{5.3}$$

As already mentioned, we want to analyse how the response of UdW detectors is affected by the curvature of different FRW epochs of the Universe and the divergences present in the field correlations in these spacetimes. To perform this analysis, we specialize to the case of FRW spacetimes. We take UdW detectors to be moving along the comoving trajectories i.e., for which the spatial coordinates are fixed and the comoving or cosmic time is the proper time. We switch to the conformal coordinates i.e., $d\tau = a(\eta)d\eta$ with $a(\eta)$ being the scale factor of the FRW spacetime under investigation i.e., $ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2)$. In conformal coordinates, the above expression becomes

$$P_{0\to\Omega} = c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2 \int_{\eta_i}^{\eta_f}\int_{\eta_i}^{\eta_f} d\eta_1 d\eta_2 e^{-i\Omega(\tau(\eta_1)-\tau(\eta_2))}a(\eta_1)a(\eta_2)G(x(\eta_1),x(\eta_2))\,, \tag{5.4}$$

where η_i and η_f are the conformal coordinate values corresponding to τ_i and τ_f, respectively.
Let us introduce the following combinations of η_1 and η_2

$$\tilde{\eta} \equiv \frac{\eta_1+\eta_2}{2} \text{ and } \Delta\eta \equiv \eta_1 - \eta_2\,.$$

For $\eta_1,\eta_2 \in (\eta_i,\eta_f)$, $\tilde{\eta} \in (\eta_i,\eta_f)$. For any fixed $\tilde{\eta} \in (\eta_i,(\eta_i+\eta_f)/2)$, we have $\eta_2 \in (\eta_i, 2\tilde{\eta} - \eta_i)$ and $\Delta\eta \in (-2(\tilde{\eta}-\eta_i), 2(\tilde{\eta}-\eta_i))$. For $\tilde{\eta} \in ((\eta_i+\eta_f)/2,\eta_f)$, we have $\eta_2 \in (2\tilde{\eta}-\eta_f,\eta_f)$ and $\Delta\eta \in (-2(\eta_f-\tilde{\eta}), 2(\eta_f-\tilde{\eta}))$. Changing the variables to $(\tilde{\eta},\Delta\eta)$ in the equation (5.4), the rate of transition probability with respect to $\tilde{\eta}$, for $\tilde{\eta} \in (\eta_i,(\eta_i+\eta_f)/2)$, is found to be

$$\begin{aligned}\frac{1}{c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2}\frac{dP_{0\to\Omega}}{d\tilde{\eta}} &= \int_{-2(\tilde{\eta}-\eta_i)}^{2(\tilde{\eta}-\eta_i)} d(\Delta\eta)\; e^{-i\Omega(\tau(\tilde{\eta}+(\Delta\eta)/2)-\tau(\tilde{\eta}-(\Delta\eta)/2))} \\ &G\Big(x\big(\tilde{\eta}+(\Delta\eta)/2\big),x\big(\tilde{\eta}-(\Delta\eta)/2\big)\Big)a\big(\tilde{\eta}+(\Delta\eta)/2\big)a\big(\tilde{\eta}-(\Delta\eta)/2\big)\,. \end{aligned} \tag{5.5}$$

For $\tilde{\eta} \in ((\eta_i + \eta_f)/2, \eta_f)$, the rate has the following form

$$\frac{1}{c^2 |_D \langle \Omega | \hat{\mu}(0) | 0 \rangle_D |^2} \frac{dP_{0\to\Omega}}{d\tilde{\eta}} = \int_{-2(\eta_f - \tilde{\eta})}^{2(\eta_f - \tilde{\eta})} d(\Delta\eta)\; e^{-i\Omega(\tau(\tilde{\eta}+(\Delta\eta)/2)-\tau(\tilde{\eta}-(\Delta\eta)/2))}$$
$$G\Big(x(\tilde{\eta}+(\Delta\eta)/2), x(\tilde{\eta}-(\Delta\eta)/2)\Big) a(\tilde{\eta}+(\Delta\eta)/2) a(\tilde{\eta}-(\Delta\eta)/2)\,. \quad (5.6)$$

Let us now analyze the case of interest i.e., massless scalar fields in power-law type FRW spacetimes. To perform the calculations for this case, we employ the FRW-de Sitter equivalence which we discussed in chapter 3. The Wightman functions in the two settings are related by the Eq. (3.32) i.e.,

$$G^{FRW}(x_1, x_2) = (H\eta_1)^{q-1}(H\eta_2)^{q-1} G^{dS}(x_1, x_2)\,. \quad (5.7)$$

As in chapter 3, we take the de Sitter vacuum to be the Bunch-Davies vacuum of chapter 2 for which the Wightman function is given by Eq. (2.39) and we perform the calculations corresponding to it.
Formula for the mass of the scalar field in de Sitter spacetime which is equivalent to a massless scalar field in FRW spacetime with $a(\eta) = (H\eta)^{-q}$ i.e., $m^2 = H^2(1-q)(2+q)$, implies that the square of the mass is positive only for the cases in which $q \in [-2, 1)$. These are the spacetimes that we considered in Chapter 3 and in this chapter also, we focus only on them. Let us briefly look at the response rate for UdW detectors which remain operative for the full time range of these spacetimes. One can argue (see Appendix F) that the Ω and H dependences of the infinite time response rate with respect to $\tilde{\eta}$ is given by

$$\frac{1}{c^2 |_D \langle \Omega | \hat{\mu}(0) | 0 \rangle_D |^2} \frac{dP_{0\to\Omega}}{d\tilde{\eta}} \propto (\Omega H^{-q})^{\frac{1}{1-q}}\,. \quad (5.8)$$

This expression tells us that the response rate increases with increasing H for $q \in (-2, 0)$ whereas it decreases with increasing H for $q \in (0, 1)$. Since, for FRW spacetimes, the Ricci scalar, $R \propto H^{2q}$, we see that the Ricci scalar and the above response rate behaves opposite of each other as a function of H. We also observe that the response rate changes maximally with H either for $q = -2$ or $q = 1$ depending upon whether $H > 1$ or $H < 1$ respectively.

During the evolution history of our Universe, it has stayed in any particular FRW like phase only for a finite time, therefore we now take up the finite time response rates for the following cases. We again take the case in which UdW detectors move along comoving trajectories.

5.2.1 Nearly massless scalar fields in de Sitter spacetime

In this subsection, we look at the finite time response rate of a UdW detector when it is coupled to nearly massless scalar fields in de Sitter spacetime. For nearly massless fields, $\nu = 3/2 - \delta$ where $\delta << 1$. The Wightman function [65] for this case, expanded as a power series in δ, is

$$G^{dS}(Z(x,x')) = \left(\frac{H^2}{16\pi^2}\right)\left(\frac{2}{\delta} + \frac{4}{y} - 4 - 2ln(y) + 4ln2 + O(\delta)\right). \tag{5.9}$$

Thus, from the above expression, we notice the well known infrared divergence [84, 124, 125, 134] of massless scalar fields in de Sitter spacetime. Making use of (5.5), the UdW detector response rate, for $\tilde{\eta} \in (\eta_i, (\eta_i + \eta_f)/2)$, is

$$\begin{aligned}\frac{1}{c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2}\frac{dP^{dS}_{0\to\Omega}}{d\tilde{\eta}} &= \frac{1}{16\pi^2}\int_{-2(\tilde{\eta}-\eta_i)}^{2(\tilde{\eta}-\eta_i)} d(\Delta\eta)\left(\frac{\tilde{\eta}+(\Delta\eta)/2}{\tilde{\eta}-(\Delta\eta)/2}\right)^{\frac{i\Omega}{H}}\frac{1}{(\tilde{\eta}^2-(\Delta\eta)^2/4)}\\ &\left(\frac{2}{\delta} - \frac{4(\tilde{\eta}^2-(\Delta\eta)^2/4)}{(\Delta\eta - i\varepsilon)^2} - 4 - 2ln\left(-\frac{(\Delta\eta - i\varepsilon)^2}{(\tilde{\eta}^2-(\Delta\eta)^2/4)}\right)\right.\\ &\left.+4ln2 + O(\delta)\right). \end{aligned} \tag{5.10}$$

One can obtain a similar formula for the case in which $\tilde{\eta} \in ((\eta_i + \eta_f)/2, \eta_f)$ using Eq. (5.6). The above integrand is easily seen to have poles at $(\Delta\eta) = \pm 2\tilde{\eta}, i\varepsilon$ but the interval over which the above integral is performed does not contain the $\pm\tilde{\eta}$ poles. If we enclose the contour in the lower half plane of the $(\Delta\eta)$ complex plane (see Fig. (5.1)), then the value of the integral for any term in the Wightman function along the above real line segment is equal to the integral of that term along the curved part of the contour in the lower half plane, with the $i\varepsilon$ pole of the upper half plane making no contribution. The integral along the curved part of the above contour is a proper integral and hence is finite. Keeping these observations in mind, we find that the response rate has the following expansion in δ [176]

$$\begin{aligned}\frac{1}{c^2|\langle\Omega|\hat{\mu}(0)|0\rangle|^2}\frac{dP^{dS}_{0\to\Omega}}{d\tilde{\eta}} &= \frac{1}{\delta}\left(\frac{1}{8\pi^2}\int_{-2(\tilde{\eta}-\eta_i)}^{2(\tilde{\eta}-\eta_i)} d(\Delta\eta)\left(\frac{\tilde{\eta}+(\Delta\eta)/2}{\tilde{\eta}-(\Delta\eta)/2}\right)^{\frac{i\Omega}{H}}\frac{1}{(\tilde{\eta}^2-(\Delta\eta)^2/4)}\right)\\ &+O(\delta^0). \end{aligned} \tag{5.11}$$

Thus, we observe that the UdW response rate for the present case of nearly massless fields in de Sitter spacetime shows the same infrared (IR) divergence as is present in the Wightman function for this case. Therefore, the smaller the mass becomes, the faster the transitions take place within the internal states of UdW detectors.

There have been a number of past works which consider UdW detectors coupled with scalar fields in de

Sitter spacetime. For example, the infinite time UdW response rate has been analysed in [169] for scalar fields in de Sitter spacetime. [177–179] analyse the dynamics of scalar fields in de Sitter spacetime from open quantum systems framework with different detector trajectories and different field vacua. We now consider a similar analysis for other FRW spacetimes which model different epochs of universe's evolution history.

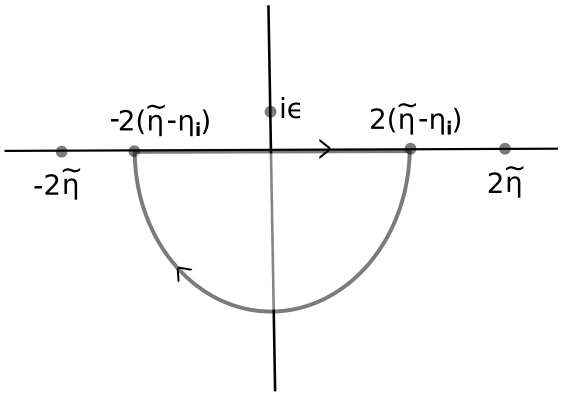


Figure 5.1: The chosen contour does not contain the poles inside it.

5.2.2 Massless scalar fields in radiation dominated spacetime

In this subsection, we consider a UdW detector which couples to a massless scalar field in radiation dominated universe. The radiation dominated spacetime phase of the Universe is believed to suceed the inflationary phase of the Universe. The scale factor for this case is $a(\eta) = (H\eta)$ i.e., $q = -1$. The mass of the scalar field in de Sitter spacetime corresponding to the massless field in radiation dominated case is $(m^2/H^2) = 2$ and the Wightman function is

$$G^{rad}(x(\eta_1), x(\eta_2)) = (H^2\eta_1\eta_2)^{-2}\frac{H^2}{4\pi^2 y(x(\eta_1), x(\eta_2))} = -(H^2\eta_1\eta_2)^{-1}\frac{1}{4\pi^2(\eta_1 - \eta_2 - i\varepsilon)^2}. \quad (5.12)$$

This case of massless scalar field in radiation dominated spacetime is also conformally related to a massless scalar field in flat spacetime (refer to Chapter 3 of [75]). For this case, the comoving time is related to the conformal time by the relation $(2Ht)^{\frac{1}{2}} = H\eta$. With the above Wightman function and the comoving time and conformal time relation, the transition probability for this case is found to be

$$P^{rad}_{0\to\Omega} = -\frac{c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2}{4\pi^2}\int_{\eta_i}^{\eta_f}\int_{\eta_i}^{\eta_f} d\eta_1 d\eta_2 e^{-\frac{i\Omega H}{2}(\eta_1^2-\eta_2^2)}\frac{1}{(\eta_1 - \eta_2 - i\varepsilon)^2}. \quad (5.13)$$

Using the expression (5.5), the UdW detector response rate with respect to $\tilde{\eta}$, for $\tilde{\eta} \in \big(\eta_i, (\eta_i+\eta_f)/2\big)$, is seen to be given by [176]

$$\frac{1}{c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2}\frac{dP^{rad}_{0\to\Omega}}{d\tilde{\eta}} = -\frac{1}{4\pi^2}\int_{-2(\tilde{\eta}-\eta_i)}^{2(\tilde{\eta}-\eta_i)} d(\Delta\eta)e^{-i\Omega H\tilde{\eta}(\Delta\eta)}\frac{1}{(\Delta\eta-i\varepsilon)^2}. \tag{5.14}$$

A similar formula is obtained for $\tilde{\eta} \in ((\eta_i+\eta_f)/2, \eta_f)$ if one uses the formula (5.6). The above response rate for massless scalar fields in radiation dominated spacetimes is easily seen to be similar to that of flat spacetime case except that in the present case one has $\Omega H\tilde{\eta}$ in place of Ω in the flat spacetime case. Making use of the same arguments as were used in the previous subsection, we enclose the contour in the lower half plane and thus the $i\varepsilon$ pole does not lie inside the contour. Hence, the above integral on the specified real line segment is equal to the integral of the above integrand along the curved part of the contour. This integral is a proper integral and hence is finite in value. Therefore, we can say that the UdW response rate for a massless field in radiation dominated spacetime is finite and it does not lead to any significant enchancement of transitions within the internal quantum states of the detector. Let us now turn to the case of massless scalar fields in nearly matter dominated spacetimes.

5.2.3 Massless scalar fields in nearly matter dominated spacetimes

Let us now couple UdW detectors to massless scalar fields in nearly matter-dominated spacetimes. For nearly matter dominated spacetimes i.e., for $q = -2+\delta$ where $\delta << 1$, the mass of the scalar fields in de Sitter spacetime corresponding to massless fields in these spacetimes is given by

$$\frac{m^2}{H^2} = (1-q)(2+q) = (3-\delta)\delta \approx 3\delta\,, \tag{5.15}$$

and it approaches zero as δ goes to zero. Thus, we see that as spacetimes approach the matter dominated spacetime limit the mass of the corresponding scalar field in de Sitter spacetime approaches zero and we expect that the infrared divergence of massless scalar fields in de Sitter spacetime is inherited by massless scalar fields in nearly matter dominated spacetimes and they manifest themselves in the UdW response rate for these cases. In fact, the Wightman function for massless fields in nearly matter dominated spacetimes, using Eq. (5.7) and Eq. (5.15), is seen to have the following expression

$$G^{matter}(x(\eta_1),x(\eta_2)) = (H^2\eta_1\eta_2)^{-3+\delta}\Big(\frac{H^2}{16\pi^2}\Big)\Big(\frac{2}{\delta}+\frac{4}{y}-4-2ln(y)+4ln2+O(\delta)\Big). \tag{5.16}$$

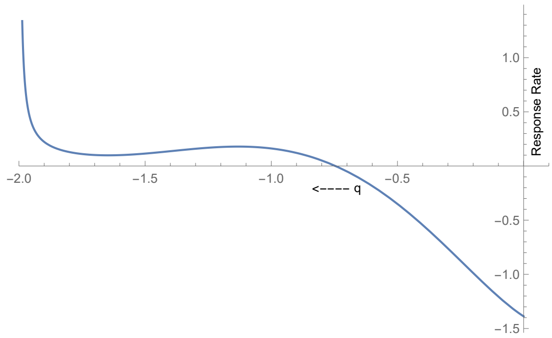
For this case, the comoving time is related to the conformal time by the relation $\left((3-\delta)Ht\right)^{\frac{1}{3-\delta}} = (H\eta)$. From expression (5.5), we find that the UdW detector response rate for the present case, for $\tilde{\eta} \in (\eta_i, (\eta_i+\eta_f)/2)$, is

$$\begin{aligned}\frac{1}{c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2}\frac{dP^{matter}_{0\to\Omega}}{d\tilde{\eta}} &= \frac{1}{16\pi^2}\int_{-2(\tilde{\eta}-\eta_i)}^{2(\tilde{\eta}-\eta_i)} d(\Delta\eta)e^{-\frac{i\Omega H^2(3\tilde{\eta}^2+(\Delta\eta)^2/(4))(\Delta\eta)}{3}}\frac{1}{(\tilde{\eta}^2-(\Delta\eta)^2/4)} \\ &\left(\frac{2}{\delta}-\frac{4(\tilde{\eta}^2-(\Delta\eta)^2/4)}{(\Delta\eta-i\varepsilon)^2}-4-2ln\Big(-\frac{(\Delta\eta-i\varepsilon)^2}{(\tilde{\eta}^2-(\Delta\eta)^2/4)}\Big)+4ln2\right. \\ &-\frac{2i\Omega H^2(\tilde{\eta}+(\Delta\eta)/2)^3}{3}\Big(\frac{1}{3}-log(H(\tilde{\eta}+(\Delta\eta)/2))\Big) \\ &+\frac{2i\Omega H^2(\tilde{\eta}-(\Delta\eta)/2)^3}{3}\Big(\frac{1}{3}-log(H(\tilde{\eta}-(\Delta\eta)/2))\Big) \\ &\left.+O(\delta)\right). \end{aligned} \tag{5.17}$$

As in the previous cases, the integral of any of the above terms can be argued to be finite. Hence, the expansion of the above expression in the $\delta \to 0$ limit is given by [176]

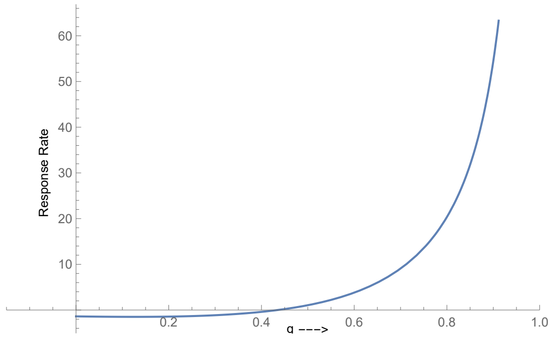
$$\begin{aligned}\frac{1}{c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2}\frac{dP^{matter}_{0\to\Omega}}{d\tilde{\eta}} &= \frac{1}{\delta}\left(\frac{1}{8\pi^2}\int_{-2(\tilde{\eta}-\eta_i)}^{2(\tilde{\eta}-\eta_i)} d(\Delta\eta)e^{-\frac{i\Omega H^2(3\tilde{\eta}^2+(\Delta\eta)^2/(4))(\Delta\eta)}{3}}\frac{1}{(\tilde{\eta}^2-(\Delta\eta)^2/4)}\right) \\ &+O(\delta^0). \end{aligned} \tag{5.18}$$

Thus, the UdW response rate for massless scalar fields in nearly matter dominated spacetimes has $1/\delta$ term as the most dominant term in the $\delta \to 0$ limit. From the above expression, we can conclude that the response rate becomes very fast for δ being close to zero. The origin of this behaviour of response rate for the present case of nearly matter dominated spacetimes is in the infrared divergence of the corresponding nearly massless scalar fields in de Sitter spacetime. From the formula $m^2 = H^2(1-q)(2+q)$, we see that the mass of the scalar field in de Sitter spacetime equivalent to massless scalar fields in FRW spacetimes approaches zero for both $q=-2$ and $q=1$ cases. Hence, for both these limits, the UdW response rate is expected to show divergent behaviour. To demonstrate this for the considered spacetimes i.e, for $q \in (-2,1)$, we numerically plot the formula (5.5) for the Wightman function (5.7) as a function of q taking specific values for η_i, η_f, $\tilde{\eta}$, Ω and H. Figure 5.2a shows the variation of response rate as a function of q for the range $(-2,0)$ while the variation of the response rate as a function of q for the range $(0,1)$ is shown in figure 5.2b. The figures show that the UdW response rate is finite for all q values except when q approaches the values -2 and 1. We saw above that the UdW response rate for massless scalar fields in de Sitter spacetime inherits the infrared divergence of its Wightman function. Any quantity which depends upon the Wightman function can be, in general, expected to suffer from



(a) For $q \in (-2, 0)$

(Taking $\eta_i = 1, \eta_f = 4, \tilde{\eta} = 2$, $\Omega = 1$ and $H = 1$)



(b) For $q \in (0, 1)$

(Taking $\eta_i = -3, \eta_f = 0, \tilde{\eta} = -2$, $\Omega = 1$ and $H = 1$)

Figure 5.2: Variation of the response rate for a conventionally coupled UdW detector as a function of q. (a) shows that the response rate diverges as q approaches -2 i.e., matter dominated spacetime and similarly (b) shows that the response rate diverges as q approaches 1 i.e., de Sitter spacetime.

the same infrared divergences as that of the Wightman function. The infrared divergence of massless scalar fields in de Sitter spacetime has been discussed at many places [84, 123, 124, 134, 180, 181]. In order to obtain physically meaningful IR finite results, many suggestions have been given [181, 182]. For example, one resolution is to consider vacua which do not enjoy the full de Sitter symmetry but which are IR finite for massless fields in de Sitter spacetime [13, 84, 125, 180]. Sometimes, it is also argued that only those operators which are IR finite should be considered physical. Such a line of argument is presented in [131] which states that only shift invariant operators, like the differences of the field operators and derivatives of the field operators etc., should be taken as truly physical observables as they are free from infrared divergences, as [131] shows, at least, for massless scalar fields in de Sitter spacetime. Similarly, the derivative operators in the stress energy tensor kill the time independent infrared divergence term of massless scalar fields in de Sitter spacetime in the evaluation of the stress energy expectation and hence it does not suffer from infrared divergence [90]. From these arguments, one would expect that for more 'physical' derivatively coupled UdW detectors, the UdW rates would not suffer from IR divergences. In fact, certain previous works [110, 113, 183] have considered derivatively coupled UdW detectors to deal with IR divergences. Keeping in mind this motivation, we now consider derivatively coupled UdW detectors coupled with massless fields in FRW spacetimes.

5.3 Derivatively coupled UdW detectors

This section considers the case of derivatively coupled UdW detectors. In case of these detectors, the interaction Hamiltonian is given as follows [110]

$$H_{int} = c\hat{\mu}(\tau)\chi(\tau)\dot{x}^{\sigma}\nabla_{\sigma}\hat{\phi}(x(\tau)) = c\hat{\mu}(\tau)\chi(\tau)\frac{d}{d\tau}\hat{\phi}(x(\tau))\,, \tag{5.19}$$

where we see that instead of coupling with the field operator, the detector couples with the derivative of the field with respect to the proper time along the classical trajectory of the detector i.e., $(d/d\tau)\hat{\phi}(x(\tau))$, and all other terms have the same meaning as in the previous section. The dot over $\dot{x}^{\sigma}$ denotes a derivative with respect to the proper time. Considering the case in which the detector starts in some state $|0\rangle_D$ with energy 0 and makes a transition to a state $|\Omega\rangle_D$ with energy Ω while the field starting from the state $|\psi\rangle$ is allowed to go to any arbitrary final state, the probability for this to take place, upto first order in perturbation theory, is

$$P_{0\to\Omega} = c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2\iint d\tau_1 d\tau_2 e^{-i\Omega(\tau_1-\tau_2)}\chi(\tau_1)\chi(\tau_2)\frac{d}{d\tau_1}\frac{d}{d\tau_2}G(x(\tau_1),x(\tau_2)). \tag{5.20}$$

Let us again specialize to the case of FRW spacetimes. Like in the previous section, the UdW detectors are taken to move along comoving trajectories for which the comoving time is the proper time for detectors. Thus, transforming to conformal coordinates i.e., $d\tau = a(\eta)d\eta$, the probability, for a detector that is uniformly operative for a finite time interval, can be expressed as

$$P_{0\to\Omega} = c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2\int_{\eta_i}^{\eta_f}\int_{\eta_i}^{\eta_f} d\eta_1 d\eta_2 e^{-i\Omega(\tau(\eta_1)-\tau(\eta_2))}\frac{d}{d\eta_1}\frac{d}{d\eta_2}G^{FRW}(x(\eta_1),x(\eta_2))\,. \tag{5.21}$$

Let us transform to the $(\tilde{\eta},\Delta\eta)$ coordinates of the previous section. In these coordinates, the rate of transition with respect to $\tilde{\eta}$, for $\tilde{\eta}\in(\eta_i,(\eta_i+\eta_f)/2)$, is

$$\begin{aligned}\frac{1}{c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2}\frac{dP_{0\to\Omega}}{d\tilde{\eta}} &= \int_{-2(\tilde{\eta}-\eta_i)}^{2(\tilde{\eta}-\eta_i)} d(\Delta\eta)\; e^{-i\Omega(\tau(\tilde{\eta}+(\Delta\eta)/2)-\tau(\tilde{\eta}-(\Delta\eta)/2))} \\ &\quad \Big[\Big(\frac{d}{d\eta_1}\frac{d}{d\eta_2}G^{FRW}\Big)\Big(x\big(\tilde{\eta}+(\Delta\eta)/2\big),x\big(\tilde{\eta}-(\Delta\eta)/2\big)\Big)\Big]\,. \end{aligned} \tag{5.22}$$

For $\tilde{\eta} \in ((\eta_i+\eta_f)/2, \eta_f)$, the rate has the following form

$$\frac{1}{c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2}\frac{dP_{0\to\Omega}}{d\tilde{\eta}} = \int_{-2(\eta_f-\tilde{\eta})}^{2(\eta_f-\tilde{\eta})} d(\Delta\eta)e^{-i\Omega(\tau(\tilde{\eta}+(\Delta\eta)/2)-\tau(\tilde{\eta}-(\Delta\eta)/2))} \left[\Big(\frac{d}{d\eta_1}\frac{d}{d\eta_2}G^{FRW}\Big)\Big(x\big(\tilde{\eta}+(\Delta\eta)/2\big),x\big(\tilde{\eta}-(\Delta\eta)/2\big)\Big)\right]. \quad (5.23)$$

To express the term $\left[\Big((d/d\eta_1)(d/d\eta_2)G^{FRW}\Big)\Big(x\big(\tilde{\eta}+(\Delta\eta)/2\big),x\big(\tilde{\eta}-(\Delta\eta)/2\big)\Big)\right]$ in the square brackets, as a function of $\tilde{\eta}$ and $\Delta\eta$, we first evaluate the derivatives of the Wightman function with respect to η_1 and η_2 and only after that we transform the resultant expression in $(\tilde{\eta},\Delta\eta)$ coordinates.
Using the Wightman function (5.7) for massless scalar fields in FRW spacetimes, we see that (refer to Appendix G)

$$\frac{d}{d\eta_1}\frac{d}{d\eta_2}G^{FRW}(x(\eta_1),x(\eta_2)) = (H^2\eta_1\eta_2)^{q-1}\Bigg[(q-1)^2\frac{G^{dS}}{\eta_1\eta_2}+(q-1)\frac{dG^{dS}}{dy}\Big(\frac{(\eta_1-\eta_2-i\varepsilon)(-2i\varepsilon)}{\eta_1^2\eta_2^2}\Big) + \frac{d^2G^{dS}}{dy^2}\frac{y((\eta_1+\eta_2)^2+\varepsilon^2)}{\eta_1^2\eta_2^2}+\frac{dG^{dS}}{dy}\frac{(\eta_1^2+\eta_2^2+\varepsilon^2)}{\eta_1^2\eta_2^2}\Bigg]. \quad (5.24)$$

Substituting the above expression for the derivatives of the Wightman function in the response rate formula (5.22), we investigate the behavior of the rate for derivatively coupled UdW detectors. We compare the results obtained for the present derivatively coupled case to the case of conventional coupling of the previous section.
As in the previous section, let us first discuss how the infinite time response rate for these detectors depend on the energy gap, Ω, between the detector states and on the parameter H which appears in the expression of the scale factor ($a(\eta)=(H\eta)^{-q}$) of the FRW spacetime. In Appendix F, we argue that the Ω and H dependence of the infinite time rate for these detectors in FRW spacetimes is given by

$$\frac{1}{c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2}\frac{dP_{0\to\Omega}}{d\tilde{\eta}} \propto \Omega^2(\Omega H^{-q})^{\frac{1}{1-q}}. \quad (5.25)$$

Therefore, compared to the infinite time rate for conventional UdW detectors, the Ω dependence has an extra factor of 2 in the exponent. But the H dependence is the same for the present case as for the case of conventional UdW detectors considered in the previous section. Particularly, we see that, for $q \in (0,1)$, the exponent of H is negative and hence the rate decreases as H increases. Whereas for $q \in (-2,0)$, the exponent of H is positive and hence the rate increases as H increases. Recalling that, for the considered FRW spacetimes, the Ricci scalar, $R \propto H^{2q}$, it is seen that the behaviours of the Ricci scalar and the infinite time rate with H are opposite of each other.

Let us now consider the analysis of finite time response rate of derivatively coupled UdW detectors for the cases which were considered in the previous section. For these detectors, we find that the infrared divergence of massless scalar fields in de Sitter spacetime does not contribute to the response rate whereas for massless scalar fields in nearly matter dominated spacetimes, the corresponding infrared divergence contributes to the response rate.

5.3.1 Nearly massless scalar fields in de Sitter spacetime

In this subsection, the analysis is carried out for nearly massless scalar fields in de Sitter spacetime which couple to derivatively coupled UdW detectors. Because the infrared divergence for this case has no spacetime dependence (see equation (5.9)) and it is the derivatives of the Wightman function which appears in the expression of the rate i.e., equation (5.22), we find that the rate for nearly massless fields in de Sitter spacetime does not share the infrared divergence of the Wightman function for this case. To see it explicitly, let us substitute the expression (5.24) in the formula for the rate i.e., (5.22), taking $q=1$ and the expression (5.9) for G^{dS}. We obtain that the rate, for $\tilde{\eta} \in (\eta_i, (\eta_i+\eta_f)/2)$, is given by [176]

$$\frac{1}{c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2}\frac{dP^{dS}_{0\to\Omega}}{d\tilde{\eta}} = \frac{H^2}{4\pi^2}\int_{-2(\tilde{\eta}-\eta_i)}^{2(\tilde{\eta}-\eta_i)} d(\Delta\eta)\left(\frac{\tilde{\eta}+(\Delta\eta)/2}{\tilde{\eta}-(\Delta\eta)/2}\right)^{\frac{i\Omega}{H}}\frac{1}{(\Delta\eta-i\varepsilon)^4}\Big(6(\tilde{\eta}^2-(\Delta\eta)^2/4) +2\varepsilon^2+2i\varepsilon(\Delta\eta)\Big)+O(\delta)\,. \tag{5.26}$$

From the above expression, we see that the rate for the present case does not contain infrared divergence of the massless fields in de Sitter spacetime. Hence, the rate of transitions for derivatively coupled UdW detectors among its internal quantum levels does not diverge as the mass of the field approaches zero. The behaviour of the rate for the present case can be demonstrated by numerically plotting its expression (5.26). In fig. 5.3a, we plot the variation of the rate as a function of Ω taking $\delta=0.001$ with the values of other parameters specified in the figure. Negative values for Ω in the plot, refer to the cases in which the detector de-excites to a lower energy state from a higher energy state whereas positive values of Ω refer to the cases in which the detector makes a transition from a lower energy state to a higher energy state. For more 'physical' derivative operators, the infrared divergence of massless scalar fields in de Sitter spacetime is expected not to make its appearance in the response rate corresponding to the derivative coupling [131]. These infrared divergences, however, may still appear for certain FRW spacetimes even with these 'physical' derivative couplings.

5.3.2 Massless scalar fields in radiation dominated spacetime

Now we consider the behavior of derivatively coupled UdW detectors interacting with massless scalar fields in radiation-dominated spacetime i.e., $q=-1$ case. Taking $q=-1$ in (5.24), one finds that

$$\begin{aligned}\frac{d}{d\eta_1}\frac{d}{d\eta_2}G^{rad}(x(\eta),x(\eta')) &= (H^2\eta_1\eta_2)^{-2}\Bigg[-4\frac{H^2}{4\pi^2(\eta_1-\eta_2-i\varepsilon)^2}-\frac{4H^2 i\varepsilon(\eta_1-\eta_2-i\varepsilon)}{4\pi^2(\eta_1-\eta_2-i\varepsilon)^4}\\ &+\frac{2H^2((\eta_1+\eta_2)^2+\varepsilon^2)}{4\pi^2(\eta_1-\eta_2-i\varepsilon)^4}-\frac{H^2(\eta_1^2+\eta_2^2+\varepsilon^2)}{4\pi^2(\eta_1-\eta_2-i\varepsilon)^4}\Bigg]. \end{aligned} \tag{5.27}$$

Thus, the rate with respect to $\tilde{\eta}$, for $\tilde{\eta}\in(\eta_i,(\eta_i+\eta_f)/2)$, is [176]

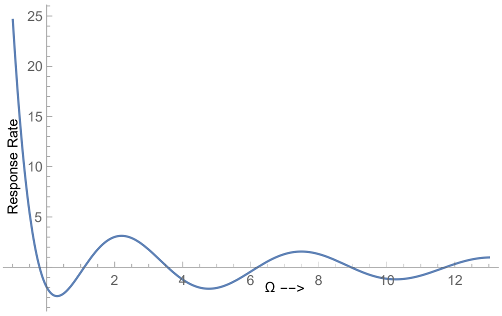
$$\begin{aligned}\frac{1}{c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2}\frac{dP^{rad}_{0\to\Omega}}{d\tilde{\eta}} &= \int_{-2(\tilde{\eta}-\eta_i)}^{2(\tilde{\eta}-\eta_i)} d(\Delta\eta)\frac{e^{-i\Omega H\tilde{\eta}(\Delta\eta)}}{H^2\pi^2}\frac{1}{\left(\tilde{\eta}^2-(\Delta\eta)^2/4\right)^2}\Bigg[-\frac{1}{(\Delta\eta-i\varepsilon)^2}-\frac{i\varepsilon}{(\Delta\eta-i\varepsilon)^3}\\ &+\frac{(4\tilde{\eta}^2+\varepsilon^2)}{2(\Delta\eta-i\varepsilon)^4}-\frac{(2\tilde{\eta}^2+(\Delta\eta)^2/2+\varepsilon^2)}{4(\Delta\eta-i\varepsilon)^4}\Bigg]. \end{aligned} \tag{5.28}$$

Employing the same arguments as have been used in the previous section, the above expression for the rate is seen to be finite in value. Therefore, we conclude that, for the present case, the rate of transitions for a derivatively coupled UdW detector among its internal quantum levels are finite just as the rate of transitions are finite for conventional UdW detectors coupled with massless fields in radiation dominated spacetime. We plot the above expression for the rate as a function of Ω in figure 5.3b. From figures 5.3a and 5.3b, we see that the de-excitation rate for both de Sitter and radiation dominated cases are more pronounced than the excitation rates.

5.3.3 Massless scalar fields in nearly matter dominated spacetimes

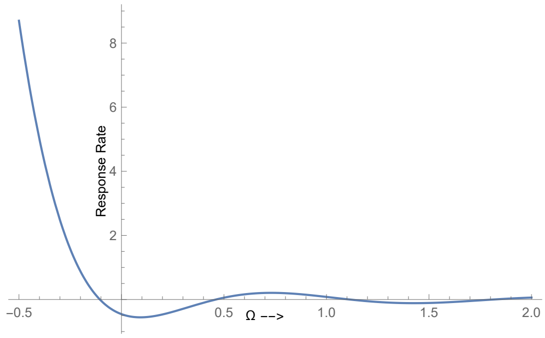
This subsection considers the case of derivatively coupled UdW detectors which interact with massless scalar fields in nearly matter-dominated spacetimes. For this case, the infrared divergent term that the Wightman function (5.16) inherits from the corresponding nearly massless scalar fields in de Sitter spacetime has spacetime-dependent factors multiplying it. Thus, under the action of the derivatives appearing in the response rate expression (5.22) or (5.23), this term survives. Using the relation that the conformal and comoving times are related by

$$t=\frac{H^2\eta^3 e^{-\delta(ln(H\eta))}}{3-\delta}=\frac{H^2\eta^3}{3}\left(1+\frac{\delta}{3}-\delta\ ln(H\eta)\right)+O(\delta^2), \tag{5.29}$$



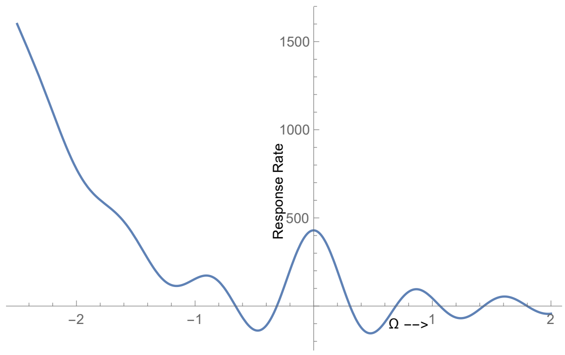
(a) For a nearly massless field in de Sitter spacetime

(Taking $\eta_i = -3, \eta_f = 0, \tilde{\eta} = -2$, $H = 1$ and $\delta = 0.001$)



(b) For a massless field in radiation dominated spacetime

(Taking $\eta_i = 1, \eta_f = 4, \tilde{\eta} = 2$ and $H = 1$)



(c) For a massless field in a nearly matter dominated spacetime

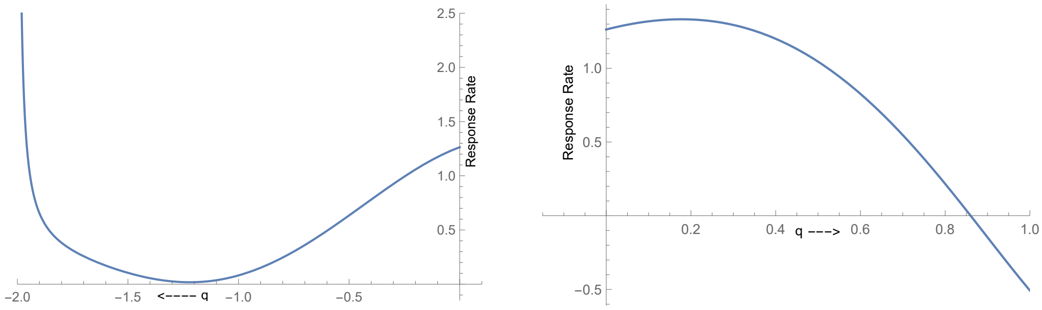
(Taking $\eta_i = 1, \eta_f = 4, \tilde{\eta} = 2$, $H = 1$ and $\delta = 0.001$)

Figure 5.3: Variation of the response rate for a derivatively coupled UdW detector as a function of Ω.

and the Wightman function (5.16) in the response rate formula (5.22), one obtains that the rate, for $\tilde{\eta} \in (\eta_i, (\eta_i + \eta_f)/2)$, has the following form [176]

$$\frac{1}{c^2 |_D \langle \Omega | \hat{\mu}(0) | 0 \rangle_D |^2} \frac{dP^{matter}_{0\to\Omega}}{d\tilde{\eta}} = \frac{1}{\delta} \left(\frac{9}{8H^4 \pi^2} \int_{-2(\tilde{\eta}-\eta_i)}^{2(\tilde{\eta}-\eta_i)} d(\Delta\eta) e^{-\frac{i\Omega H^2 (3\tilde{\eta}^2 + (\Delta\eta)^2/(4))(\Delta\eta)}{3}} \frac{1}{(\tilde{\eta}^2 - (\Delta\eta)^2/4)^4} \right)$$
$$+ O(\delta^0) \,. \tag{5.30}$$

Thus, we conclude that in the limit $\delta \to 0$, the leading order term is the $(1/\delta)$ term and this results in very rapid transitions within the internal quantum states of the detector. Therefore, the present case UdW rate shows the infrared divergence of the considered massless scalar fields in nearly matter dominated spacetimes. The behaviour of detectors for the present case is unlike the behaviour of detectors for the case of nearly massless scalar fields in de Sitter spacetime where the derivatively coupled UdW detectors do not manifest the infrared divergence of the field.



(a) For $q \in (-2, 0)$

(Taking $\eta_i = 1, \eta_f = 4, \tilde{\eta} = 2$, $\Omega = 1$ and $H = 1$)

(b) For $q \in (0, 1)$

(Taking $\eta_i = -3, \eta_f = 0, \tilde{\eta} = -2$, $\Omega = 1$ and $H = 1$)

Figure 5.4: Variation of the response rate for a derivatively coupled UdW detector as a function of q. (a) shows that the response rate diverges as q approaches -2 i.e., derivatively coupled UdW detectors manifest infrared divergence for matter dominated spacetime. However, (b) shows that the response rate is finite for all $q \in (0, 1)$ i.e., the infrared divergence of de Sitter spacetime disappears for derivatively coupled detectors.

Figure 5.3c shows the variation of the rate for the present case as a function of Ω. The values of the other parameters which appear in the rate expression are given in the caption of the figure. From the figure, we see that, for small values of Ω, the rate shows similar behaviour for both excitations and de-excitations. This can be argued to be expected from the above expression where the leading order term is invariant under the change of sign of Ω and in the limit $\delta \to 0$, this term is the one which would decide the rate. In the case of non-zero but small values of δ, the other subdominant terms grow for larger Ω to make the vacuum de-excitations take over the excitations and break the $\Omega \to -\Omega$ symmetry. This behaviour of the present case is not seen for the previously considered cases of de Sitter and radiation dominated spacetimes where there was no symmetry between de-excitation and excitation rates. In order to demonstrate the behaviour of the derivatively coupled UdW detectors interacting with massless scalar fields in different FRW spacetimes, we plot the corresponding response rates in figure 5.4 as a function of q. As argued above, we see that the response rate diverges as $q \to -2$ whereas it remains finite as $q \to 1$. This is different from the behaviour of the response rate for conventionally coupled UdW detectors where it diverges as q approaches both -2 and 1.

5.4 Summary

In this chapter, we have looked at the correlations of quantum fields in FRW spacetimes by coupling them to UdW detectors. The cases of both conventionally and derivatively coupled UdW detectors have been considered. We have employed the FRW-de Sitter equivalence that we discussed in Chapter 3. Using this equivalence we place the massless fields in FRW spacetimes in the Bunch-Davies like vacua of the corresponding massive scalar fields in de Sitter spacetime. The main results obtained in this chapter are as follows

1. **Conventional UdW detectors :** First, we consider the case of conventionally coupled UdW detectors interacting with massless scalar fields in FRW spacetimes. It is found that, for nearly massless scalar fields in de Sitter spacetime, the infrared divergent term present in the Wightman function manifests itself at the response rate level and leads to very rapid transitions within the detector states as the mass of the field is taken to smaller and smaller values. We also consider the case of massless scalar fields in nearly matter dominated spacetimes in which the Wightman function inherits the infrared divergence of the corresponding nearly massless scalar fields in de Sitter spacetime. The response rate for massless scalar fields in nearly matter dominated spacetimes also shows these infrared divergences. We analyse the response rate of UdW detectors for some other FRW spacetimes also but the mentioned divergence occurs for the de Sitter and matter dominated spacetimes.

2. **Derivatively coupled UdW detectors :** Next, we take up the case of derivatively coupled UdW detectors interacting with massless scalar fields in FRW spacetimes and nearly massless scalar fields in de Sitter spacetime. In this case, because the derivatives of the Wightman function decide the rate, the spacetime independent infrared divergent term of the de Sitter case vanishes under the action of derivatives and does not contribute to the transition rates of the detector. In the case of massless scalar fields in nearly matter dominated spacetimes, the infrared divergent term is spacetime dependent and does not vanish under the action of derivatives. Hence, the response rate for this case gets contribution from the infrared divergent term and the rate becomes faster and faster as the spacetimes approach the matter dominated limit. Among the considered FRW spacetimes in the chapter, it is shown that it is only for the case of matter dominated spacetime that the response rate shows infrared divergences.

Analysis performed in this chapter has been mostly formal but in the next chapter, we consider the applicability of the results obtained in this chapter to the case of coupling of atoms with metric perturbations over FRW spacetimes, which can be shown to harbour same kind of vacuum correlator structure as that of a scalar field.

Chapter 6

Atoms in FRW spacetimes

In this chapter, we look at the dynamics of atoms in FRW spacetimes with and without metric perturbations over them. We consider the atoms to be moving along comoving trajectories and write the interaction between the atoms and the spacetime curvature by building Fermi normal coordinates about the comoving trajectories of atoms. We show that the coupling of atoms with tensor perturbations over FRW spacetimes takes the form of a generalized UdW detector and apply the results obtained in the previous chapter to this situation. We also discuss the potential observational signatures of this analysis.

6.1 Atoms in curved spacetime

Let us consider the dynamics of atoms in curved spacetimes. Following the treatment given in [184], let us assume that the center of mass of an atom follows a classical time-like trajectory in spacetime. The internal structure of the atom is governed by considering that the electron follows the Dirac equation in the presence of the electromagnetic potential of the nucleus. Thus, the internal quantum space of the atom is decided by

$$i\nabla_0\psi = \Big(-(g^{00})^{-1}\gamma^0 m + i(g^{00})^{-1}\gamma^0\gamma^i\nabla_i\Big)\psi\,, \tag{6.1}$$

where ψ is a four component Dirac spinor and $\gamma^\mu = e^\mu_a\Gamma^a$ are the curved spacetime gamma matrices that relate to flat spacetime gamma matrices, Γ^a, via tetrad basis e^μ_a. The flat space gamma matrices, Γ^a, satisfy the algebra that $\{\Gamma^a,\Gamma^b\} = -2\eta^{ab}$ and the tetrad basis relate the metric of the spacetime with the

Minkowski metric as $e^u_a e^v_b g_{\mu\nu} = \eta_{ab}$. The covariant derivatives are given by

$$\nabla_\mu = \partial_\mu - \frac{1}{8}\omega^{ab}_\mu[\Gamma_a, \Gamma_b] - iqA_\mu\,, \tag{6.2}$$

where $\omega^{ab}_\mu = e^a_\lambda e^{\tau b}\Gamma^\lambda_{\tau\mu} - e^{\tau b}\partial_\mu e^a_\tau$ are components of the spin connection. The electromagnetic four-potential, A_μ, is determined by solving the curved spacetime Maxwell's equations in the presence of a point source at the nucleus.

Now we build Fermi normal coordinates (FNCs) around the 'central' timelike geodesic that the center of mass of the atom follows. The construction of the FNCs is shown in figure 6.1. The points which lie on the central timelike geodesic are taken to have zero spatial coordinates and the time component for these points is taken to be the proper time for them along the central geodesic. For a point, say P, which does not lie on the central geodesic (as shown in the figure 6.1), we consider the unique space-like geodesic, Γ, passing from P which intersects the central geodesic orthogonally at point, say G, then the point P is assigned the time coordinate of the point G. To assign spatial coordinates to point P, we consider the tangent vector, v^i, to Γ at point G and take the spatial coordinates to be $x^i = v^i s$ where s is the proper time along Γ from P to G.

One can show that the different components of the metric written upto 2nd order in FNCs have the

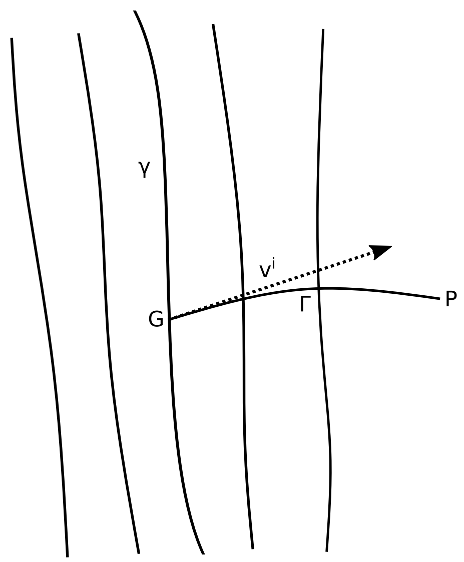


Figure 6.1: This figure captures the construction of Fermi normal coordinates. Here, γ is some time-like geodesic about which we construct FNCs. P is some point in the spacetime which intersects γ orthogonally at point G via the unique spacelike geodesic Γ. v^i are the components of the tangent vector to Γ at point G.

following expansion

$$
\begin{aligned}
g_{00} &= -1 - R_{0l0m}x^l x^m , & (6.3)\\
g_{0i} &= -\frac{2}{3}R_{0lim}x^l x^m , & (6.4)\\
g_{ij} &= \delta_{ij} - \frac{1}{3}R_{iljm}x^l x^m , & (6.5)\\
g &= -1 + \frac{1}{3}(R_{lm} - 2R_{0l0m})x^l x^m . & (6.6)
\end{aligned}
$$

Other quantities like the inverse metric components, the tetrad bases, and the Christoffel connections, etc. can also be similarly written up to second order in FNCs. For more details, refer to [184, 185].
Substituting the above given expansion of spacetime metric in the curved spacetime Dirac equation and Maxwell's equation, it can be shown that the Dirac equation has the following form

$$i\partial_t \psi = \Big(-i\alpha^i \partial_i + m\beta - \frac{\zeta}{r} + H_I \Big)\psi , \tag{6.7}$$

where β, α^i are the Dirac matrices. Here H_I denotes the perturbation to the flat spacetime Dirac equation in the central Coulomb potential $\frac{\zeta}{r}$ by the curvature induced terms. Here $\zeta = Ze^2$ where e is the electron's charge and Z is the number of protons in the nucleus. The full expression for the interaction Hamiltonian, H_I, is given in [184].
Taking the non-relativistic limit of the above equation, we obtain the following Schrodinger equation form [184]

$$\Big(i\frac{\partial}{\partial t} - m\Big)\psi = \Big(-\frac{1}{2m}\nabla^2 - \frac{\zeta}{r} + \frac{1}{2}mR_{0l0m}x^l x^m \Big)\psi . \tag{6.8}$$

In the above equation, the term containing R_{0l0m} represents the corrections to the flat spacetime Schrodinger equation (with the central Coulomb potential of the nucleus) by the curvature of the spacetime. Unlike ψ in the Dirac equation, ψ in the above equation is a one-component function of space and time. The above introduced curvature induced corrections are obtained by working only upto 2nd order in FNCs. Here R_{0l0m} are the Riemann tensor components and are to be evaluated for the points on central geodesic in FNCs. The Riemann components in FNCs are related to those in an arbitrary coordinate system, $R^{arbitrary}_{\mu\nu\gamma\delta}$, by

$$R^{FNC}_{abcd} = R^{arbitrary}_{\mu\nu\gamma\delta}\vec{e}^{\,\mu}_a \vec{e}^{\,\nu}_b \vec{e}^{\,\gamma}_c \vec{e}^{\,\delta}_d , \tag{6.9}$$

where $\vec{e}^{\,\mu}_a$ are a set of orthonormal basis parallel transported along the central timelike geodesic. The vector field $\vec{e}^{\,\mu}_0$ represents the tangent vector field to the central geodesic.

In subsequent sections, we make use of the formalism presented in this section to study the curvature effects of the unperturbed and perturbed FRW spacetimes on the atoms moving in these spacetimes.

6.2 FRW spacetimes with no perturbation

Let us take the case of an atom in flat FRW spacetimes with scale factor $a(\eta)$ and consider that the center of mass of the atom moves along comoving trajectories for which the spatial coordinates are fixed i.e., $x^{\mu}(t) = (\eta(t), c^{i})$. Hence, we obtain the following tangent vector field for comoving trajectories in FRW spacetimes

$$\frac{dx^{\mu}}{dt} = \left(\frac{1}{a}, 0\right). \tag{6.10}$$

One can take the following set of orthonormal basis vectors which are parallel transported along the considered comoving geodesics

$$\vec{e}_0^{\,\mu} = \frac{1}{a}(1,0,0,0), \;\; \vec{e}_1^{\,\mu} = \frac{1}{a}(0,1,0,0), \;\; \vec{e}_2^{\,\mu} = \frac{1}{a}(0,0,1,0), \;\; \vec{e}_3^{\,\mu} = \frac{1}{a}(0,0,0,1). \tag{6.11}$$

Thus, the relation between the Riemann tensor in the chosen conformal coordinates and FNCs, using $R^{FNC}_{0l0m} = R^{Con}_{\mu\nu\gamma\delta}\vec{e}_0^{\,\mu}\vec{e}_l^{\,\nu}\vec{e}_0^{\,\gamma}\vec{e}_m^{\,\delta}$, is found to be given by $R^{FNC}_{0l0m} = R^{Con}_{0l0m}/a^4$. Substituting $R^{Con}_{0l0m} = -\delta_{lm}(aa'' - a'^2)$ in this relation, we see that

$$R^{FNC}_{0l0m} = -\delta_{lm}\frac{1}{a^4}(aa'' - a'^2), \tag{6.12}$$

where $'$ represents a derivative with respect to conformal time, η,. The interaction Hamiltonian in Eq. (6.8) becomes

$$H_I = -\frac{m}{2}\frac{\ddot{a}}{a}r^2, \tag{6.13}$$

where $\dot{}$ represents a derivative with respect to comoving time coordinate, t.

If we consider the case in which the atom makes a transition from some atomic state ψ_{nlm} to $\psi_{n'l'm'}$ under the cosmological expansion of FRW spacetimes, then the probability for this to happen, up to first order in perturbation theory, is given by

$$P_{\psi_{nlm}\to\psi_{n'l'm'}} = \frac{m^2}{4}|\langle\psi_{n'l'm'}|r^2|\psi_{nlm}\rangle|^2 \int_{\eta_i}^{\eta_f} d\eta_1 \int_{\eta_i}^{\eta_f} d\eta_2 e^{-i\Omega(t(\eta_1)-t(\eta_2))}\frac{1}{a_1 a_2}\left(\frac{a_1''}{a_1} - \frac{a_1'^2}{a_1^2}\right)\left(\frac{a_2''}{a_2} - \frac{a_2'^2}{a_2^2}\right), \tag{6.14}$$

where $\Omega = E_{n'l'} - E_{nl}$ is the difference in the energies of the considered atomic states.

It is clear from the form of the interaction Hamiltonian that the above transition probability is zero for those cases in which the considered atomic states (between which the atom makes the transition) have different spherical harmonics. To make it precise, let us consider

$$\langle \psi_{n'l'm'}|\, r^2 \,|\psi_{nlm}\rangle = \int dr\, r^4 R^*_{n'l'}(r) R_{nl}(r) \int\int \sin\theta\, d\theta\, d\phi\, Y^{m'*}_{l'}(\theta,\phi)\, Y^m_l(\theta,\phi)\,. \tag{6.15}$$

From the above integral, we see that the interaction Hamiltonian makes a contribution only in the radial integral part but not in the angular integrals. Thus, for states with different spherical harmonics, the above integral vanishes because of the orthonormality of the spherical harmonics (allowed transitions for the $\hat{x}^i\hat{x}^k$ term i.e., the selection rules are given in Appendix H). This result is not surprising as the FRW expansion of the Universe has homogeneous and isotropic spatial slices at all times and thus the expansion respects the spherical symmetry of the spatial slices. In fact, taking $a(\eta) = (H\eta)^{-q}$, one finds that the above expression for transition probability, between states with same spherical harmonics, is given by [176]

$$P_{\psi_{nlm}\to\psi_{n'lm}} = \frac{m^2H^4q^2}{4}\Big|\int dr\, r^4 R^*_{n'l}(r) R_{nl}(r)\Big|^2 \int_{\eta_i}^{\eta_f} d\eta_1 \int_{\eta_i}^{\eta_f} d\eta_2 e^{-i\Omega(t(\eta_1)-t(\eta_2))}(H^2\eta_1\eta_2)^{q-2}\,. \tag{6.16}$$

From the above formula, one can then find the rate of transitions with respect to $\tilde{\eta}$ by going to $(\tilde{\eta},\Delta\eta)$ coordinates. But the main point is that the transitions take place only between states with same spherical harmonics and hence there is no change of angular momenta that is caused by FRW spacetimes with no metric perturbations. Other thing to notice from the above formula is that as the overlap between the radial wavefunctions of the states (which are participating in the transition) decreases, the probability of transition between them also decreases. We also notice that to obtain this result, nothing quantum has been assumed for the background spacetime and hence the considered transitions are the result of the purely classical expansion of the FRW spacetimes. Therefore, by considering quantized metric perturbations over FRW backgrounds if one finds that they can lead to transitions between states with different spherical harmonics, then observing such transitions would provide a hint for the quantum nature of the perturbations over FRW backgrounds.

6.3 FRW spacetimes with perturbations

In this section, we consider FRW spacetimes with metric perturbations over them i.e.,

$$ds^2 = a^2(\eta)(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu\,. \tag{6.17}$$

The perturbation in the above metric leads to perturbations in the comoving trajectories of the previous section. Considering the above metric, one finds that (refer to [186]) the set of parallel transported orthonormal basis considered in the previous section has the following perturbations upto first order in h i.e.,

$$\vec{e}_0^{\mu} = \frac{1}{a}(1+\frac{h_{00}}{2}, V_{0i}), \tag{6.18}$$
$$\vec{e}_i^{\mu} = \frac{1}{a}(V_{0i}+h_{0i}, \delta_i^j - \frac{h_i^j}{2} + \frac{1}{2}\varepsilon_i^{jk}\omega_k), \tag{6.19}$$

where V_{0i}, ω_k along with $h_{\mu\nu}$ denote deviations from the comoving geodesics of the unperturbed FRW spacetimes of the previous section. The quantities V_{0i}, ω_k are given by

$$V_{0i}' + \frac{a'}{a}V_{0i} = \frac{1}{2}\partial_i h_{00} - h_{0i}' - \frac{a'}{a}h_{0i}, \tag{6.20}$$
$$\omega_k' = -\frac{1}{2}\varepsilon_k^{ij}(\partial_i h_{oj} - \partial_j h_{oi}). \tag{6.21}$$

Instead of studying all scalar, vector and tensor perturbations, we specialize to the case of gravitational waves i.e., we take $h_{00} = h_{0i} = 0$ and only h_{ij} non-zero. The tensor perturbations, h_{ij}, satisfy $h_{ij}\delta^{ij} = 0$ and $\delta^{ki}\partial_k h_{ij} = 0$. For the case of tensor perturbations, the quantities $V_{0i} = \omega_k = 0$ and the perturbed comoving geodesics in perturbed FRW spacetimes have the following form

$$\vec{e}_0^{\mu} = \frac{1}{a}(1,0), \tag{6.22}$$
$$\vec{e}_i^{\mu} = \frac{1}{a}(0, \delta_i^j - \frac{h_i^j}{2}). \tag{6.23}$$

The above expressions tell us that the tangent vector field to the considered comoving geodesics is not affected upto first order in h. This implies that for comoving geodesics, upto first order in h, the spatial coordinates remain fixed and the proper time is just the same as the cosmic time i.e., $x^{\mu}(t) = (\eta(t), c^i)$.

Thus, using the formula Eq. (6.9) and the above derived set of orthonormal basis to the comoving geodesics in the perturbed FRW spacetimes, we relate the Reimann tensor coponents in FNCs in terms of those in conformal coordinate system as follows

$$R^{FNC}_{abcd} = R^{Con}_{\mu\nu\gamma\delta}\vec{e}_a^{\mu}\vec{e}_b^{\nu}\vec{e}_c^{\gamma}\vec{e}_d^{\delta}. \tag{6.24}$$

For the components of the Riemann tensor that appear in the interaction Hamiltonian Eq. (6.8) are given by

$$
\begin{aligned}
R^{FNC}_{0l0m} &= R^{Con}_{\mu\nu\gamma\delta}\vec{e}^{\mu}_{0}\vec{e}^{\nu}_{l}\vec{e}^{\gamma}_{0}\vec{e}^{\delta}_{m} \\
&= \frac{1}{a^4}R^{Con}_{0k0p}\left(\delta^k_l - \frac{h^k_l}{2}\right)\left(\delta^p_m - \frac{h^p_m}{2}\right).
\end{aligned} \tag{6.25}
$$

The Riemann tensor components that we need, have the following form in conformal coordinates (refer [7, 10])

$$
R^{Con}_{0l0m} = -\delta_{lm}(aa'' - a'^2) - (aa'' - a'^2)h_{lm} - \frac{aa'}{2}h'_{lm} - \frac{a^2}{2}h''_{lm}. \tag{6.26}
$$

Substituting the above expression in Eq. (6.25), we obtain that the relevant Riemann tensor components in the FNCs, upto first order in h, are given by

$$
\begin{aligned}
R^{FNC}_{0l0m} &= \frac{1}{a^4}\Big(-\delta_{kp}(aa'' - a'^2) - (aa'' - a'^2)h_{kp} - \frac{aa'}{2}h'_{kp} - \frac{a^2}{2}h''_{kp}\Big)\Big(\delta^k_l - \frac{h^k_l}{2}\Big)\Big(\delta^p_m - \frac{h^p_m}{2}\Big) \\
&= \frac{1}{a^4}\Big(-\delta_{lm}(aa'' - a'^2) - \frac{aa'}{2}h'_{lm} - \frac{a^2}{2}h''_{lm}\Big) + O(h^2).
\end{aligned} \tag{6.27}
$$

Therefore, using the above expression, the interaction Hamiltonian becomes

$$
H_I = \frac{m}{2}\frac{1}{a^4}\Big(-\delta_{lm}(aa'' - a'^2) - \frac{aa'}{2}h'_{lm} - \frac{a^2}{2}h''_{lm}\Big)x^l x^m \tag{6.28}
$$

$$
= \frac{m}{2}\Big(-\delta_{lm}\frac{\ddot{a}}{a} - \frac{\dot{a}}{a}\dot{h}_{lm} - \frac{1}{2}\ddot{h}_{lm}\Big)x^l x^m = \frac{m}{2}H_{lm}x^l x^m. \tag{6.29}
$$

As already mentioned above, $'$ represents derivative with respect to conformal time, η, and $\dot{}$ represents derivative with respect to comoving time, t. Here,

$$
H_{lm} = \Big(-\delta_{lm}\frac{\ddot{a}}{a} - \frac{\dot{a}}{a}\dot{h}_{lm} - \frac{1}{2}\ddot{h}_{lm}\Big). \tag{6.30}
$$

Let us look at the probability for the above interaction term to cause the atom to make a transition from ψ_{nlm} to $\psi_{n'l'm'}$ and for the field to make a transition from an initial vacuum state to all possible final states i.e., we trace over all the allowed final states of the field. For the above interaction Hamiltonian, the transition probability, upto first order in perturbation theory, is given by [176]

$$
\begin{aligned}
P_{\psi_{nlm}\to\psi_{n'l'm'}} = \frac{m^2}{4}&\langle\psi_{n'l'm'}|\hat{x}^i\hat{x}^j|\psi_{nlm}\rangle^* \langle\psi_{n'l'm'}|\hat{x}^p\hat{x}^k|\psi_{nlm}\rangle \\
&\int_{\eta_i}^{\eta_f} d\eta_1 \int_{\eta_i}^{\eta_f} d\eta_2 e^{-i\Omega(t(\eta_1)-t(\eta_2))}a(\eta_1)a(\eta_2)\langle 0|\hat{H}_{ij}(\vec{c},\eta_1)\hat{H}_{pk}(\vec{c},\eta_2)|0\rangle\,,
\end{aligned} \tag{6.31}
$$

where $\Omega = E_{n'l'} - E_{nl}$ is the difference in the energies of the considered atomic states and the fixed spatial coordinates for the comoving trajectory of the atom are denoted by $\vec{c}$. It is clear that in case there are no perturbations over FRW backgrounds i.e., for $h_{\mu\nu} = 0$, the interaction Hamiltonian reduces to the case of the previous section. Though, without perturbations, the only allowed transitions are between states with the same spherical harmonics (as seen in the previous section), we expect that the introduction of perturbations would also lead to transitions between states with different spherical harmonics.

6.3.1 Tensor perturbation induced transitions

As seen in chapter 1, any component of the gravitational waves or the tensor perturbations in FRW spacetimes satisfy the same equation of motion as satisfied by a massless scalar field in FRW background i.e.,

$$h''_{lm} + 2\frac{a'}{a}h'_{lm} - \nabla^2 h_{lm} = 0\,. \tag{6.32}$$

Because of the symmetry, transverse and traceless properties of the tensor perturbations, there are only two independent polarization states of gravitational waves in an FRW spacetime. Thus, the dynamics of gravitational waves in an FRW background are equivalent to that of two massless scalar fields in that FRW spacetime.

Using the equation of motion for the components of the tensor perturbations i.e., Eq. (6.32), the H_{lm} can be written in the following form

$$H_{lm} = -\frac{\delta_{lm}}{a^4}(aa'' - a'^2) + \frac{1}{a^4}\left(\frac{aa'}{2}\frac{\partial}{\partial\eta} - \frac{a^2}{2}\nabla^2_{\vec{c}}\right)h_{lm}(\eta, \vec{c})\,. \tag{6.33}$$

As seen in chapter 1, the quantized tensor perturbations have the following expansion

$$\hat{h}_{ij}(\vec{c}, \eta) = \sum_{\lambda=+,\times}\int d^3\vec{q}\; e_{ij}(\hat{q}, \lambda)\left(e^{i\vec{q}.\vec{c}}h_q(\eta)\hat{b}_{\vec{q},\lambda} + e^{-i\vec{q}.\vec{c}}h^*_q(\eta)\hat{b}^\dagger_{\vec{q},\lambda}\right), \tag{6.34}$$

where $\vec{c}$ is the constant spatial vector for the considered comoving trajectories and all other symbols have their usual meaning (see chapter 1). The $e_{ij}(\vec{q}, \lambda)'s$ satisfy Eq. (1.20) [5] i.e.,

$$\sum_{\lambda=+,\times} e_{ij}(\hat{q}, \lambda)e_{kl}(\hat{q}, \lambda) = \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl} + \delta_{ij}\hat{q}_k\hat{q}_l + \delta_{kl}\hat{q}_i\hat{q}_j$$
$$- \delta_{ik}\hat{q}_j\hat{q}_l - \delta_{il}\hat{q}_j\hat{q}_k - \delta_{jk}\hat{q}_i\hat{q}_l - \delta_{jl}\hat{q}_i\hat{q}_k + \hat{q}_i\hat{q}_j\hat{q}_k\hat{q}_l\,. \tag{6.35}$$

The time evolution part of the mode functions i.e., $h_q(\eta)$, satisfy

$$h_q^{''}(\eta)+2\frac{a'}{a}h_q^{'}(\eta)+q^2h_q(\eta)=0\,, \tag{6.36}$$

and hence we see that it is independent of the polarization state and direction of the wave vector of mode functions.

For the rest of the chapter, we specialize to those transitions that take place between states with different spherical harmonics i.e., $(l,m)\neq(l',m')$. For these transitions, the δ_{lm} term of Eq. (6.33) i.e., H_{lm}, drops out. Using Eq. (6.33) and the point splitting technique, we can write the two point correlation between $\hat{H}_{ij}$'s as follows

$$\begin{aligned}&\langle 0|\,\hat{H}_{ij}(\vec{c}_1,\eta_1)\hat{H}_{pk}(\vec{c}_1,\eta_2)\,|0\rangle\\ &\quad=\lim_{\vec{c}_2\to\vec{c}_1}\frac{1}{a_1^4}\Big(\frac{a_1a_1^{'}}{2}\frac{\partial}{\partial\eta_1}-\frac{a_1^2}{2}\nabla^2_{\vec{c}_1}\Big)\frac{1}{a_2^4}\Big(\frac{a_2a_2^{'}}{2}\frac{\partial}{\partial\eta_2}-\frac{a_2^2}{2}\nabla^2_{\vec{c}_2}\Big)\langle 0|\,\hat{h}_{ij}(\vec{c}_1,\eta_1)\hat{h}_{pk}(\vec{c}_2,\eta_2)\,|0\rangle\,. \end{aligned}\tag{6.37}$$

Using Eq. (6.34) for h_{ij} and the commutation relations between creation and annihilation operators, the two point correlation between h_{ij}'s appearing in the above expression is seen to be given by

$$\begin{aligned}\langle 0|\,\hat{h}_{ij}(\vec{c}_1,\eta_1)\hat{h}_{pk}(\vec{c}_2,\eta_2)\,|0\rangle &= \sum_{\lambda=+,\times}\int d^3\vec{q}\;e_{ij}(\hat{q},\lambda)e_{pk}(\hat{q},\lambda)e^{i\vec{q}.(\vec{c}_1-\vec{c}_2)}h_q(\eta_1)h_q^*(\eta_2)\\ &= \int d^3\vec{q}\,\Big(\sum_{\lambda=+,\times}e_{ij}(\hat{q},\lambda)e_{pk}(\hat{q},\lambda)\Big)e^{i\vec{q}.(\vec{c}_1-\vec{c}_2)}h_q(\eta_1)h_q^*(\eta_2)\,. \end{aligned}\tag{6.38}$$

We make use of the relation (6.35) in the above integral and move every q_i contribution from (6.35) outside of the above integral by replacing every such factor with a partial derivative with respect to spatial coordinates. Finally, we obtain that the two point function of tensor perturbations can be expressed as a sum of products of different Kronecker delta's and spatial partial derivatives acting on the two point correlation of some scalar field i.e.,

$$\begin{aligned}\langle 0|\,\hat{h}_{ij}(\vec{c}_1,\eta_1)\hat{h}_{pk}(\vec{c}_2,\eta_2)\,|0\rangle &= \Big(\delta_{ip}\delta_{jk}+\delta_{ik}\delta_{jp}-\delta_{ij}\delta_{pk}+\delta_{ij}\frac{\partial_{\vec{c}_{1p}}\partial_{\vec{c}_{1k}}}{\nabla^2_{\vec{c}_1}}+\delta_{pk}\frac{\partial_{\vec{c}_{1i}}\partial_{\vec{c}_{1j}}}{\nabla^2_{\vec{c}_1}}-\delta_{ip}\frac{\partial_{\vec{c}_{1j}}\partial_{\vec{c}_{1k}}}{\nabla^2_{\vec{c}_1}}\\ &-\delta_{ik}\frac{\partial_{\vec{c}_{1j}}\partial_{\vec{c}_{1p}}}{\nabla^2_{\vec{c}_1}}-\delta_{jp}\frac{\partial_{\vec{c}_{1i}}\partial_{\vec{c}_{1k}}}{\nabla^2_{\vec{c}_1}}-\delta_{jk}\frac{\partial_{\vec{c}_{1i}}\partial_{\vec{c}_{1p}}}{\nabla^2_{\vec{c}_1}}+\frac{\partial_{\vec{c}_{1i}}\partial_{\vec{c}_{1j}}\partial_{\vec{c}_{1p}}\partial_{\vec{c}_{1k}}}{\nabla^2_{\vec{c}_1}\nabla^2_{\vec{c}_1}}\Big)\int d^3\vec{q}e^{i\vec{q}.(\vec{c}_1-\vec{c}_2)}h_q(\eta_1)h_q^*(\eta_2)\,. \end{aligned}\tag{6.39}$$

From Eqs. (6.31), (6.37) and (6.39), we see that the interaction of atoms with gravitational waves takes the form of a generalized derivatively coupled UdW detector where we now also have spatial derivatives along with time derivatives. Here instead of having two levels for the UdW detector, we have many (in fact, infinite) quantum levels corresponding to the atomic states of the hydrogen atom and in place of quantum scalar fields, we have quantized tensor perturbations. Combining these equations, one obtains that the transition probability for the atom to make transitions between states with different angular momentum quantum numbers is given by [176]

$$P_{\psi_{nlm}\to\psi_{n'l'm'}} = \frac{m^2}{4}\langle\psi_{n'l'm'}|\hat{x}^i\hat{x}^j|\psi_{nlm}\rangle^*\langle\psi_{n'l'm'}|\hat{x}^p\hat{x}^k|\psi_{nlm}\rangle \lim_{\vec{c}_1\to\vec{c}_2}\int_{\eta_i}^{\eta_f} d\eta_1\int_{\eta_i}^{\eta_f} d\eta_2 e^{-i\Omega(t(\eta_1)-t(\eta_2))}$$
$$\frac{1}{a_1^3}\left(\frac{a_1a_1^{'}}{2}\frac{\partial}{\partial\eta_1}-\frac{a_1^2}{2}\nabla^2_{\vec{c}_1}\right)\frac{1}{a_2^3}\left(\frac{a_2a_2^{'}}{2}\frac{\partial}{\partial\eta_2}-\frac{a_2^2}{2}\nabla^2_{\vec{c}_2}\right)\left(\delta_{ip}\delta_{jk}+\delta_{ik}\delta_{jp}-\delta_{ij}\delta_{pk}+\delta_{ij}\frac{\partial_{\vec{c}_{1p}}\partial_{\vec{c}_{1k}}}{\nabla^2_{\vec{c}_1}}+\delta_{pk}\frac{\partial_{\vec{c}_{1i}}\partial_{\vec{c}_{1j}}}{\nabla^2_{\vec{c}_1}}\right.$$
$$\left.-\delta_{ip}\frac{\partial_{\vec{c}_{1j}}\partial_{\vec{c}_{1k}}}{\nabla^2_{\vec{c}_1}}-\delta_{ik}\frac{\partial_{\vec{c}_{1j}}\partial_{\vec{c}_{1p}}}{\nabla^2_{\vec{c}_1}}-\delta_{jp}\frac{\partial_{\vec{c}_{1i}}\partial_{\vec{c}_{1k}}}{\nabla^2_{\vec{c}_1}}-\delta_{jk}\frac{\partial_{\vec{c}_{1i}}\partial_{\vec{c}_{1p}}}{\nabla^2_{\vec{c}_1}}+\frac{\partial_{\vec{c}_{1i}}\partial_{\vec{c}_{1j}}\partial_{\vec{c}_{1p}}\partial_{\vec{c}_{1k}}}{\nabla^2_{\vec{c}_1}\nabla^2_{\vec{c}_1}}\right)\int d^3\vec{q}e^{i\vec{q}.(\vec{c}_1-\vec{c}_2)}h_q(\eta_1)h_q^*(\eta_2)\,. \tag{6.40}$$

The above expression leads to a number of important results. For example, as we have considered only those transitions which involve a non-trivial change of spherical harmonics, the classical FRW expansion considered in the previous section does not contribute to the present case, and hence, the considered transitions are the results of the metric perturbations only. To make further remarks, one needs to specify the state in which the quantized metric perturbations are placed. Since the equation satisfied by $h_q(\eta)$ is the same as that of time dependent part of massless scalar field mode functions in FRW backgrounds, we can take the integral in the above expression corresponding to different scalar field vacua. As an example, we can take the above integral to be the Wightman function Eq. (5.7) considered in the previous chapter. As we saw in the last chapter that the derivatively coupled UdW detectors experience very rapid transitions within their internal quantum space while interacting with massless scalar fields in nearly matter dominated spacetimes, we expect that the atoms should also experience these rapid transitions when they pass through nearly matter dominated phases of the Universe because of their interaction with quantized tensor perturbations. In fact, if we consider the case in which the atom is in nearly matter dominated backgrounds and we take the above integral to be given by Eq. (5.16) i.e.,

$$G^{matter}(x(\eta_1),x(\eta_2)) \;=\; (H^2\eta_1\eta_2)^{-3+\delta}\left(\frac{H^2}{16\pi^2}\right)\left(\frac{2}{\delta}+\frac{4}{y}-4-2ln(y)+4ln2+O(\delta)\right),$$

then we see that the $1/\delta$ term has time dependent conformal factors multiplying it and hence it does not vanish under the derivatives present in the transition probability expression given above. Hence, in the limit $\delta\to 0$, the rate becomes very large and we expect that these transitions within the atomic states during nearly matter dominated spacetimes should have potentially important observational signatures.

6.4 Summary

In this chapter, we have looked at the dynamics of atoms in FRW backgrounds with and without perturbations over them. By working in the leading order in FNCs built around the classical time-like geodesic of the center of mass of the atom, the interaction term between the atom and the curvature of the background spacetime has been written for FRW spacetimes with and without metric perturbations. We show that the classical expansion of the FRW backgrounds with no metric perturbations can cause transitions only between those atomic states which have the same spherical harmonics. This result is expected as the FRW expansion of the Universe respects the homogeneity and isotropy of the spatial slices of the spacetime. Also, these transitions are purely classical in the sense that nothing quantum mechanical has been assumed about the background spacetime while deriving the stated conclusion. We also arrive at the coupling of atoms with tensor perturbations over FRW spacetimes and find that the coupling takes the form of a generalized derivatively coupled UdW detector. Since the tensor perturbations over an FRW spacetime are equivalent to two massless scalar fields in the same FRW spacetime, the vacuum correlations of quantized tensor perturbations have the same structure as that of massless scalar fields in FRW spacetimes. This fact implies that the vacuum correlations of tensor perturbations during nearly matter dominated phases acquire the infrared divergences of the nearly massless scalar fields in de Sitter spacetime (as discussed in previous chapter). Thus, by considering quantized tensor perturbations and the equivalence of tensor perturbations with two massless scalar fields in FRW background, we argue that the results obtained in the previous chapter regarding the derivatively coupled UdW detector can be carried over to the coupling of atoms with quantized tensor perturbations. Particularly, atoms interacting with gravitational waves during nearly matter dominated spacetimes are expected to lead to very rapid transitions within their internal quantum space. We expect that these rapid transitions within the atomic states of an atom caused by quantized tensor perturbations (gravitons), have the tendency to leave significant imprint on the CMB while they pass through nearly matter dominated phase of the Universe. Compared to the case of FRW spacetimes with no perturbations over them, we find that the tensor perturbations can also cause transitions even between states which have different spherical harmonics. Another important aspect is that the transitions caused by the tensor perturbations are quantum in nature as we have considered the quantized tensor perturbations over FRW spacetimes.

Chapter 7

Conclusion

Quantum field theory in curved spacetimes is an important framework that tries to bring together notions of quantum theory and relativity and explores the implications that follow from such an interplay. Applications of quantum field theory in curved spacetime during the early inflationary phase of the Universe have provided predictions that fit well with observations like the temperature anisotropies in the cosmic microwave background (CMB) etc [23]. Thus, quantum effects during the evolution of the Universe play an important role, particularly during the early inflationary (near de Sitter) phase. After passing through the inflationary phase, the universe also underwent radiation dominated phase which was then followed by matter dominated phase. In this thesis, we have investigated quantum fields during different epochs of the Universe's evolution. We have considered not only de Sitter spacetime but also other FRW spacetimes (including radiation and matter dominated phases) to see whether quantum effects during these later phases of the Universe evolution can become important or not. We have investigated quantum fields in FRW backgrounds through the behaviour of corresponding noise kernels and by coupling them with Unruh deWitt (UdW) detectors.

In chapter 3, we have studied the quantum effects of scalar fields evolving on fixed classical backgrounds encapsulated in the behaviour of the noise kernel of the stochastic gravity paradigm [101]. In semiclassical gravity, one is interested in effects of the expectation of the stress energy operator of the field under consideration whereas the noise kernel captures the two point correlations of stress energy operators. Considering massive scalar fields in de Sitter spacetime and placing them in the Bunch-Davies vacuum, we have looked at the behaviour of the noise kernel for spacetime points which have same time coordinates but a finite spatial distance. It is shown that the noise kernel for this case, in the late time limit i.e., the limit in which the scale factor becomes very large, shows a transition from vanishing to divergent behavior as the parameter ν $\left(=\sqrt{(9/4)-(m^2/H^2)}\right)$ is changed in the range $[0,3/2]$ with the transition taking place at $\nu = 1/2$. Similarly, to study the behaviour of the noise kernel for massless scalar fields in power-law FRW spacetimes, we employed an equivalence that relates a massless scalar field in a power-law FRW spacetime with a massive scalar field in de Sitter spacetime. Using this equivalence, we expressed the Wightman function of the massless field in the considered FRW spacetime in terms of time dependent conformal factors and the Wightman function of the corresponding massive field in de Sitter which is taken to be corresponding to the Bunch-Davies one. Again considering the case of spacetime points which have the same time coordinates and finite spatial distance, we find that, for power-law FRW spacetimes with $q \in [0,1)$, the noise kernel saturates to constant values in the late time limits of those spacetimes. The most interesting behaviour is obtained for massless fields in nearly matter dominated spacetimes. For these cases, it is found that, for sufficiently late time limit, the noise kernel becomes larger and larger as the spacetimes approach closer and closer to the matter dominated limit. Since the matter dominated case corresponds to later phase of the Universe's evolution, it can be expected that the quantum effects can become very large during these phases. In fact, the cases for which the noise kernel becomes significantly large, we expect that, for these cases, second-order quantum effects have the potential to modify the conclusions drawn from the first-order semiclassical gravity.

In chapter 4, we have investigated the dynamics of spinor fields in de Sitter and FRW spacetimes by studying the behaviour of the corresponding noise kernel. For arbitrarily massive spinor fields in de Sitter spacetime, the analysis has been carried out by placing them in the fermionic Bunch-Davies vacuum which is defined analogously to how one defines the scalar field Bunch-Davies vacuum. Like in the case of scalar fields, the behaviour of the noise kernel for the present case has been studied for spacetime points which have same time coordinates and have finite spatial distance between them. It is shown in this thesis that the noise kernel for massive spinor fields in de Sitter spacetime decays in the late time limit. Thus, the correlations between stress energy operators of spinor matter decay as the de Sitter spacetime expands. This decay of correlations for spinor fields, for spatially separated points on constant time sheets, has been shown to occur irrespective of the mass of the spinor field i.e., the fact that how massive or light the spinor field is, does not save it from the eventual decay of the correlations. This behaviour of the noise kernel is in contrast to the behaviour of the noise kernel for scalar fields where for light scalar fields, the noise kernel can attain very large values in late time limits. This thesis has also looked at how the stress energy correlations of massless spinor fields in general

FRW spacetimes behave. For these cases, we have used the conformal invariance of the massless spinor fields and placed the fields in the Poincare vacuum of the corresponding massless spinor field of the flat spacetime. Since massless spinor fields are conformal, their correlations in FRW spacetimes follow a similar structure as that of massless spinors in flat spacetime along with some time dependent conformal factors and in fact, the noise kernel for FRW spacetimes is also found to be conformally related to the flat spacetime noise kernel. One would expect similar such results for electromagnetic fields as well which are also conformal. It is shown that the noise kernel of a massless spinor field in an FRW spacetime, for spatially separated spacetime points on constant time sheets, behaves opposite to the behaviour of the scale factor. This implies that the correlations, between (massless) spinor matter located at spatially separated points on constant time sheets, decay during the expanding phases of the universe while they grow for contracting spacetime metrics. Thus, we have found that the second order quantum effects of spinor fields do not play any significant role during the entire (expanding) evolution history of the Universe. Therefore, with both scalar and spinor fields present in expanding FRW backgrounds, it is only the scalar fields which are expected to contribute second order quantum effects and hence potentially modify the predictions obtained solely from first order semiclassical gravity analyses.

In chapter 5, we studied the correlations of scalar fields by coupling them with Unruh deWitt (UdW) detectors. Unruh deWitt detectors record the correlations of quantum fields in any particular state by undergoing transitions within its internal quantum space while following classical trajectories in spacetime. First, we considered the case of conventional UdW coupling where the operator causing transitions within the UdW detector's internal quantum space couples with the field through a monopole coupling. For the case in which the detector makes a transition from some state $|0\rangle$ to another state $|\Omega\rangle$ with energies 0 and Ω, respectively and the field starts in vacuum while is allowed to go any arbitrary state, the transition probability depends upon the Wightman function of the field in the considered spacetime. With this setting, we studied the case of massless scalar fields in FRW spacetimes as well as nearly massless scalar fields in de Sitter spacetime. We again employed the equivalence between massless fields in FRW spacetimes with massive fields in de Sitter spacetime that was also used for analyzing the noise kernel for these fields. The detectors were taken to follow comoving trajectories and the scalar fields were placed in the Bunch-Davies like vacua just as in the noise kernel analysis. For nearly massless scalar fields in de Sitter spacetime or massless scalar fields in nearly matter dominated spacetimes, the Wightman functions have a term of infrared origin which dominates over any other term. It was shown that the transition probability, in these cases, are dominated by these dominating terms of infrared origin and they lead to very fast transition rates. We also considered the case of derivatively coupled Unruh deWitt detectors where the detector couples to the derivative of the field with respect to the proper time along the detector's trajectory. For these detectors, the transition probability depends upon the derivatives of the Wightman function of the field under consideration. It was shown that the dominating infrared term vanishes in the de Sitter Universe because the infrared term is spacetime independent in this case and goes to zero under the action of derivatives. But, for massless scalar fields in nearly matter dominated case, there are time dependent conformal factors multiplying the infrared divergent term and it contributes to the transition probability even in this derivatively coupled case. Thus, we expect that if there are physical systems which couple with background scalar fields in a derivatively coupled UdW manner during matter dominated phases of the Universe, then the background field should lead to very rapid transitions within the internal states of the physical systems.

In chapter 6, we have explored the dynamics of atoms in unperturbed FRW backgrounds as well as perturbed FRW backgrounds i.e., with metric perturbations over FRW spacetimes. Particularly, the tensor perturbations over an FRW spacetime behave like two massless scalar fields in the same FRW spacetime and hence this analysis again provides a scenario where one hopes to see the effects of correlations of quantum scalar fields in FRW spacetimes. For this analysis, we have taken the center of mass of the atom to move along classical trajectories, which we take to be comoving trajectories, and the internal structure of the atom is taken to be decided by the quantum mechanics of a particle in the presence of the electromagnetic potential of the nucleus in curved spacetime. Building Fermi normal coordinates (FNCs) about the timelike trajectory of the atom, one can show that, in curved spacetimce to the lowest order in FNCs, the internal structure of the atom is governed by the flat spacetime Schrodinger equation in the central electrostatic potential of the nucleus and interaction term between the atom and the curvature of the spacetime is given by $(m/2)R_{0l0p}\hat{x}^l\hat{x}^p$ [184] where m is the mass of the electron in the atom, e is the charge of the electron and R_{0l0p} are the Riemann tensor components expressing the curvature induced corrections to the flat spacetime Schrodinger equation with the central Coulomb potential of the nucleus. We showed that for unperturbed FRW spacetimes, the interaction term can cause transitions of electrons only within those atomic states which have same spherical harmonics as one would expect because of the spatial homogeneity and isotropy of the FRW backgrounds. We then considered the case of perturbed FRW backgrounds with metric perturbations i.e., $ds^2 = a^2(\eta)(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu$ where a is the scaling factor of the background FRW spacetime. For this case, it has been shown in the thesis that the interaction term between curvature of the spacetime and the atom takes the form of a generalized derivatively coupled Unruh deWitt detector where the internal transitions in the atom are induced by the position operators and background fields are the metric perturbations over the FRW spacetimes. Thus, one can carry the results obtained in the previous study to this case. Particularly, with this interaction Hamiltonian and specializing to the case of only tensor perturbations, we have shown that the rate of transitions within the atomic states of the atom when it passes through nearly matter dominated phases of the Universe, becomes extremely large. We expect that these extremely rapid transitions within atoms during the nearly matter dominated phases of the Universe would leave their characteristic imprints on CMB whose presence in observations should provide a hint of quantum nature of gravity.

Thus, these studies have shown that the quantum effects in cosmological contexts are not only important for early inflationary (near de Sitter) phase but they can become important also during the late time epochs (particularly matter dominated era) of the Universe. We conclude this from the large noise kernel for massless scalar fields in nearly matter dominated spacetimes as well as rapid transitions within the derivatively coupled UdW detectors e.g., atoms, caused by massless fields in nearly matter dominated spacetimes. Though these results have been derived by placing quantum fields in FRW spacetimes in the Bunch-Davies like vacua, we expect that for other well-behaved normalizable states also these results are not going to change significantly as such states also have the Bunch-Davies like character along with their own characteristic features [42, 187, 188]. In any case, it is important to perform the analysis carried out here (for Bunch-Davies like vacua) to other states as well. Hence, performing these analyses for non-vacuous states as well as other vacua is a direction that we think is worth exploring. The investigation of the potential consequences of the rapid transition rates within atoms caused by their coupling with quantized tensor perturbations while they pass through nearly matter dominated phases of the Universe is something that, we expect, may provide a new window for testing quantum nature of gravity. It should be particularly interesting to explore its signatures on cosmological data like the cosmic microwave background. Another interesting possibility that one can explore in this setting of coupling of atoms with quantized tensor perturbations is that of entanglement generation between spatially separated atoms via the tensor perturbations. Such an analysis would fit in the general paradigm of testing quantum nature of gravity through entanglement generation between spatially separated quantum systems, mainly in quantum optical settings [189–192]. The analysis that should be followed for this scenario is, though, more akin to what is done in [193, 194] but with the derivatively coupled UdW like interaction term that we have for atoms interacting with tensor perturbations. The generation of entanglement between spatially separated quantum systems is possible because of the non-trivial quantum correlations of quantum fields between different spacetime points.

One important quantum feature, other than the ones considered in this work, that one can investigate for quantum fields in FRW spacetimes is that of quantum entanglement. It should be interesting and potentially important to investigate quantum information theoretic notions in cosmological contexts and whether they will have any characteristic features in the cosmological data that one can look for. For example, one can analyze the effects of the existence of entanglement between modes of scalar and tensor perturbations or between the modes of metric perturbations and the electromagnetic field. These analyses would require checking whether the presence of the mentioned entanglement leaves the otherwise well-verified features of the data unaffected as well as at the same time leaving their own imprints whose presence in the data would confirm the presence of the considered entanglement. Another important quantum consideration for cosmological contexts is to look for mechanisms that lead the quantum fluctuations during the inflation to decohere to give rise to the observed classical CMB spectrum. One mechanism that could be a possible explanation for this quantum to classical transition is based on the fact that it is only the super horizon modes during the inflation that are observationally available today not the sub horizon modes. Therefore, we can treat the quantum system consisting only of the super horizon modes as an open quantum system interacting with the environment consisting of the sub horizon modes and study the consequences of the evolution of this system. This setting provides a background for potentially important studies. For example, we can consider the evolution of the reduced density matrices of super horizon scalar and tensor modes by solving for their corresponding equations obtained by tracing over the unobserved sub-horizon modes. We obtain Lindblad terms leading to non-unitary evolution only if we include non-linear interaction terms which couple the sub and super horizon modes [195–197]. Such an analysis would tell us how the off diagonal terms in the reduced density matrices vanish and lead to the decoherence of the quantum system of super horizon modes. One can also analyse how a partial decoherence would manifest itself in the CMB data and provide the signatures of the quantum origin of our universe.

Another line of investigation that may further enrich our understanding of the quantum workings of the Universe is to explore the spectrum of tensor modes sourced by primordial magnetic fields. In order to explain large scale magnetic fields observed in our universe, many primordial origin proposals [51–55] have been put forth. Among these models, certain inflationary models with particular types of conformal invariance breaking terms and parity-violating terms for the Maxwell field seem to provide an explanation for the observationally relevant values for magnetic fields. Other than explaining the observed values, the Maxwell field can act as a source for primordial scalar and tensor mode fluctuations. Therefore, this allows for the possibility of testing the viability of these models of primordial origin for magnetic fields by comparing the theoretical predictions for Maxwell field induced characteristics in the scalar and tensor spectra with the upcoming data.

Appendix A

Christoffel connections and geometrical quantities for FRW spacetimes

In this appendix, we provide expressions of Christoffel connections, Riemann tensor, Ricci tensor, Ricci scalar and Einstein tensor for FRW spacetimes in both cosmic and conformal time coordinates

A.1 In conformal time coordinates

In conformal time coordinates, the FRW metric is given by:

$$ds^2 = a^2(\eta)(-d\eta^2 + \tilde{g}_{ij}dx^i dx^j) \tag{A.1}$$

i.e.,

$$g_{\mu\nu} = a^2(\eta) \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & \tilde{g}_{ij} & \\ 0 & & & \end{bmatrix}. \tag{A.2}$$

With this, we find that the non-zero Christoffel connections are

$$\Gamma^0_{00} = \frac{a'}{a}, \quad \Gamma^0_{kl} = \tilde{g}_{kl}\frac{a'}{a}, \quad \Gamma^k_{0l} = \delta^k_l \frac{a'}{a}, \quad \Gamma^i_{jk} = \tilde{\Gamma}^i_{jk}, \tag{A.3}$$

where $\tilde{\Gamma}^i_{jk}$ are the Christoffel connections for the 3-metric $\tilde{g}_{ij}$ and prime, $'$, denotes a derivative with respect to η. Using the above expressions for Christoffel connections, we see that the only non-zero components of the Riemann tensor are

$$R^0_{k0i} = \left(\frac{a''}{a} - \frac{a'^2}{a^2}\right)\tilde{g}_{ki}, \; R^k_{00i} = \left(\frac{a''}{a} - \frac{a'^2}{a^2}\right)\delta^k_i, \; R^i_{jkl} = \tilde{R}^i_{jkl} + \frac{a'^2}{a^2}(\delta^i_k\tilde{g}_{jl} - \delta^i_l\tilde{g}_{jk}) \quad \text{(A.4)}$$

where $\tilde{R}^i_{jkl}\big(= k(\delta^i_k\tilde{g}_{jl} - \delta^i_l\tilde{g}_{jk})$ for maximally symmetric spaces and k corresponds to different types of maximally symmetric spaces.$\big)$ are the components of the Riemann tensor for the 3-metric $\tilde{g}_{ij}$. Similarly, the components of the Ricci tensor are given by :

$$R_{00} = -3\left(\frac{a''}{a} - \frac{a'^2}{a^2}\right), \quad R_{0i} = 0, \quad R_{ij} = \left(\frac{a''}{a} + \frac{a'^2}{a^2} + 2k\right)\tilde{g}_{ij}. \quad \text{(A.5)}$$

This provides the expression for Ricci scalar which is given by:

$$R = 6\left(\frac{a''}{a^3} + \frac{k}{a^2}\right) \quad \text{(A.6)}$$

which readily implies that the components of the Einstein tensor are the following:

$$G_{00} = 3\left(\frac{a'^2}{a^2} + k\right), \quad G_{0i} = 0, \quad G_{ij} = \left(-2\frac{a''}{a} + \frac{a'^2}{a^2} - k\right)\tilde{g}_{ij}. \quad \text{(A.7)}$$

A.2 In cosmic time coordinates

In cosmic time coordinates, the FRW metric is given by:

$$ds^2 = -dt^2 + a^2(t)\tilde{g}_{ij}dx^i dx^j \quad \text{(A.8)}$$

i.e.,

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & a^2(t)\tilde{g}_{ij} & \\ 0 & & & \end{bmatrix}. \quad \text{(A.9)}$$

With this, we find that the non-zero Christoffel connections are

$$\Gamma^0_{kl} \;=\; \tilde{g}_{kl}\dot{a}a, \quad \Gamma^k_{0l} \;=\; \delta^k_l\frac{\dot{a}}{a}, \quad \Gamma^i_{jk} \;=\; \tilde{\Gamma}^i_{jk}, \tag{A.10}$$

where $\tilde{\Gamma}^i_{jk}$ are the Christoffel connections for the 3-metric $\tilde{g}_{ij}$ and a dot over scaling factor, a(t), denotes a derivative with respect to cosmic time, t. Using the above expressions for Christoffel connections, we see that the only non-zero components of the Riemann tensor are

$$R^0_{k0i} \;=\; a\ddot{a}\tilde{g}_{ki}, \; R^k_{00i} \;=\; \frac{\ddot{a}}{a}\delta^k_i, \; R^i_{jkl} \;=\; \tilde{R}^i_{jkl} \;+\; \dot{a}^2(\delta^i_k\tilde{g}_{jl} \;-\; \delta^i_l\tilde{g}_{jk}) \tag{A.11}$$

where $\tilde{R}^i_{jkl}$ is the same as in the above section. Similarly, the components of the Ricci tensor are given by :

$$R_{00} \;=\; -3\frac{\ddot{a}}{a}, \quad R_{0i} \;=\; 0, \quad R_{ij} \;=\; \big(a\ddot{a} \;+\; 2\dot{a}^2 \;+\; 2k\big)\tilde{g}_{ij}. \tag{A.12}$$

This provides the following expression for Ricci scalar:

$$R = 6\big(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\big) \tag{A.13}$$

using which, we find that the components of the Einstein tensor are given as follows:

$$G_{00} \;=\; 3\big(\frac{\dot{a}^2}{a^2} \;+\; \frac{k}{a^2}\big), \quad G_{0i} \;=\; 0, \quad G_{ij} \;=\; -\big(2a\ddot{a} \;+\; \dot{a}^2 \;+\; k\big)\tilde{g}_{ij}. \tag{A.14}$$

Appendix B

Some basic properties of the Hankel functions

In this appendix, we collect some important properties of the Hankel functions that have been used in this thesis. The following differential equation, called Bessel's equation,

$$\frac{d^2 f(z)}{d^2 z} + \frac{1}{z}\frac{df(z)}{dz} + \big(1 - \frac{\nu^2}{z^2}\big) f(z) = 0\,, \tag{B.1}$$

has two linearly independent solutions, $J_\nu(z)$ and $Y_\nu(z)$, called the Bessel functions. Hankel functions are related to the Bessel functions by the relations $H^{(1)}_\nu(z) = J_\nu(z) + iY_\nu(z)$ and $H^{(2)}_\nu(z) = J_\nu(z) - iY_\nu(z)$ which implies that the Hankel functions are also solutions of the Bessel's equation. We also observe that

$$\big(H^{(1)}_\nu(z)\big)^* = H^{(2)}_{\nu^*}(z^*) \tag{B.2}$$

using the fact that the Bessel functions satisfy similar properties.
Derivative of the Hankel functions w.r.t. z is given by:

$$\frac{d}{dz} H^{(1,2)}_\nu(z) = H^{(1,2)}_{\nu-1}(z) - \frac{\nu}{z} H^{(1,2)}_\nu(z) \tag{B.3}$$

We also make use of the following large argument expansion of the Hankel functions quite often in this thesis.

$$H_\nu^{(1)}(z) \;=\; \sqrt{\frac{2}{\pi z}}e^{i(z-\frac{\pi}{2}\nu-\frac{\pi}{4})}\Big[1+O\Big(\frac{1}{z}\Big)\Big]\,, \tag{B.4}$$

$$H_\nu^{(2)}(z) \;=\; \sqrt{\frac{2}{\pi z}}e^{-i(z-\frac{\pi}{2}\nu-\frac{\pi}{4})}\Big[1+O\Big(\frac{1}{z}\Big)\Big]\,, \tag{B.5}$$

$$for\;|z|\to\infty,\;\; Re(\nu)>-\frac{1}{2}\;\, and\;|arg(z)|<\pi\,.$$

Certain other properties that have been used in the main text, but may not have been necessarily referred to, are

$$H_{-\nu}^{(1)}(z) \;=\; e^{i\pi\nu}H_\nu^{(1)}(z)\,, \tag{B.6}$$

$$H_{-\nu}^{(2)}(z) \;=\; e^{-i\pi\nu}H_\nu^{(2)}(z)\,, \tag{B.7}$$

$$H_\nu^{(1)}(e^{i\pi}z) \;=\; -H_{-\nu}^{(2)}(z)=-e^{-i\pi\nu}H_\nu^{(2)}(z)\,, \tag{B.8}$$

$$H_\nu^{(2)}(e^{-i\pi}z) \;=\; -H_{-\nu}^{(1)}(z)=-e^{i\pi\nu}H_\nu^{(1)}(z)\,. \tag{B.9}$$

Above given properties of the Hankel functions have been taken from [86, 89]. For more, refer to same.

Appendix C

Basic computation on constant time surface

Here we enlist some important formulae that are used in evaluating stress energy correlator/noise kernel.

$$Z(x',y) = 1+\frac{(\eta_{x'}-\eta_y)^2-(\vec{x}'-\vec{y})^2}{2\eta_{x'}\eta_y} \tag{C.1}$$

$$\frac{\partial Z(x',y)}{\partial x'^{\mu}} = \left[\frac{\Delta s^2}{2\eta_{x'}^2\eta_y}\delta_{\mu 0}-\frac{(x'-y)_\mu}{\eta_{x'}\eta_y}\right] \tag{C.2}$$

$$\frac{\partial Z(x',y)}{\partial y^{\nu}} = \left[\frac{\Delta s^2}{2\eta_{x'}\eta_y^2}\delta_{\nu 0}+\frac{(x'-y)_\nu}{\eta_{x'}\eta_y}\right] \tag{C.3}$$

$$\frac{\partial^2 Z(x',y)}{\partial x'^{\nu}\partial x'^{\mu}} = \left[-\frac{\Delta s^2}{\eta_{x'}^3\eta_y}\delta_{\mu 0}\delta_{\nu 0}+\frac{(x'-y)_\nu}{\eta_{x'}^2\eta_y}\delta_{\mu 0}+\frac{(x'-y)_\mu}{\eta_{x'}^2\eta_y}\delta_{\nu 0}-\frac{\eta_{\mu\nu}}{\eta_{x'}\eta_y}\right] \tag{C.4}$$

$$\frac{\partial^2 Z(x',y)}{\partial y^{\nu}\partial y^{\mu}} = \left[-\frac{\Delta s^2}{\eta_{x'}\eta_y^3}\delta_{\mu 0}\delta_{\nu 0}-\frac{(x'-y)_\nu}{\eta_{x'}\eta_y^2}\delta_{\mu 0}-\frac{(x'-y)_\mu}{\eta_{x'}\eta_y^2}\delta_{\nu 0}-\frac{\eta_{\mu\nu}}{\eta_{x'}\eta_y}\right] \tag{C.5}$$

$$\frac{\partial^2 Z(x',y)}{\partial y^{\nu}\partial x'^{\mu}} = \left[-\frac{\Delta s^2}{2\eta_{x'}^2\eta_y^2}\delta_{\mu 0}\delta_{\nu 0}-\frac{(x'-y)_\nu}{\eta_{x'}^2\eta_y}\delta_{\mu 0}+\frac{(x'-y)_\mu}{\eta_{x'}\eta_y^2}\delta_{\nu 0}+\frac{\eta_{\mu\nu}}{\eta_{x'}\eta_y}\right] \tag{C.6}$$

$$\begin{aligned}\frac{\partial^3 Z(x',y)}{\partial y^{\rho}\partial x'^{\nu}\partial x'^{\mu}} &= \left[\frac{\Delta s^2}{\eta_{x'}^3\eta_y^2}\delta_{\mu 0}\delta_{\nu 0}\delta_{\rho 0}-\frac{(x'-y)_\nu}{\eta_{x'}^2\eta_y^2}\delta_{\mu 0}\delta_{\rho 0}-\frac{(x'-y)_\mu}{\eta_{x'}^2\eta_y^2}\delta_{\nu 0}\delta_{\rho 0}\right.\\ &\left.+\frac{\eta_{\mu\nu}}{\eta_{x'}\eta_y^2}\delta_{\rho 0}-\frac{\eta_{\mu\rho}}{\eta_{x'}^2\eta_y}\delta_{\nu 0}-\frac{\eta_{\nu\rho}}{\eta_{x'}^2\eta_y}\delta_{\mu 0}+2\frac{(x'-y)_\rho}{\eta_{x'}^3\eta_y}\delta_{\mu 0}\delta_{\nu 0}\right]\end{aligned} \tag{C.7}$$

$$\frac{\partial^3 Z(x',y)}{\partial x'^{\rho}\partial y^{\nu}\partial y^{\mu}} = \left[\frac{\Delta s^2}{\eta_{x'}^2\eta_y^3}\delta_{\mu 0}\delta_{\nu 0}\delta_{\rho 0}+\frac{(x'-y)_\nu}{\eta_{x'}^2\eta_y^2}\delta_{\mu 0}\delta_{\rho 0}+\frac{(x'-y)_\mu}{\eta_{x'}^2\eta_y^2}\delta_{\nu 0}\delta_{\rho 0}\right.$$

$$+\frac{\eta_{\mu\nu}}{\eta_{x'}^2\eta_y}\delta_{\rho 0}-\frac{\eta_{\mu\rho}}{\eta_{x'}\eta_y^2}\delta_{\nu 0}-\frac{\eta_{\nu\rho}}{\eta_{x'}\eta_y^2}\delta_{\mu 0}-2\frac{(x'-y)_\rho}{\eta_{x'}\eta_y^3}\delta_{\mu 0}\delta_{\nu 0}\Big] \tag{C.8}$$

In the above expressions, $\Delta s^2 = -(\eta_{x'}-\eta_y)^2+(\vec{x}'-\vec{y})^2$. With the above formulae and using the fact that, for de-Sitter invariant vacuum, $G(x',y)=G(Z(x',y))$, we obtain

$$\nabla_{y^\mu}G = G'\Big[\frac{(x'-y)_\mu}{\eta_{x'}\eta_y}+\frac{\Delta s^2}{2\eta_{x'}\eta_y^2}\delta_{\mu 0}\Big], \tag{C.9}$$

$$\nabla_{x'^\nu}G = G'\Big[-\frac{(x'-y)_\nu}{\eta_{x'}\eta_y}+\frac{\Delta s^2}{2\eta_{x'}^2\eta_y}\delta_{\nu 0}\Big], \tag{C.10}$$

$$\begin{aligned}\nabla_{x'^\nu}\nabla_{y^\mu}G &= G''\Big[\frac{(x'-y)_\mu}{\eta_{x'}\eta_y}+\frac{\Delta s^2}{2\eta_{x'}\eta_y^2}\delta_{\mu 0}\Big]\Big[-\frac{(x'-y)_\nu}{\eta_{x'}\eta_y}+\frac{\Delta s^2}{2\eta_{x'}^2\eta_y}\delta_{\nu 0}\Big]\\ &+ G'\Big[\frac{\eta_{\mu\nu}}{\eta_{x'}\eta_y}-\frac{(x'-y)_\mu}{\eta_x^2\eta_y}\delta_{\nu 0}+\frac{(x'-y)_\nu}{\eta_{x'}\eta_y^2}\delta_{\mu 0}-\frac{\Delta s^2}{2\eta_{x'}^2\eta_y^2}\delta_{\nu 0}\delta_{\mu 0}\Big].\end{aligned} \tag{C.11}$$

In the above expressions, a single prime ($'$) on G represents a single derivative with respect to Z and similarly, two primes represent a double derivative with respect to Z. On constant time sheets i.e., $\eta_{x'}=\eta_y$, we have

$$Z(x',y) = 1-\frac{(\Delta\vec{x}')^2}{2\eta_{x'}^2}, \tag{C.12}$$

$$\nabla_{y^i}G = G'\Big[\frac{(x'-y)_i}{\eta_{x'}^2}\Big], \tag{C.13}$$

$$\nabla_{x'^i}G = G'\Big[-\frac{(x'-y)_i}{\eta_{x'}^2}\Big], \tag{C.14}$$

$$\nabla_{y^0}G = G'\Big[\frac{(\vec{x}'-\vec{y})^2}{2\eta_{x'}^3}\Big], \tag{C.15}$$

$$\nabla_{x'^0}G = G'\Big[\frac{(\vec{x}'-\vec{y})^2}{2\eta_{x'}^3}\Big], \tag{C.16}$$

$$\nabla_{x'^i}\nabla_{y^j}G = G''\Big[-\frac{(x'-y)_i(x'-y)_j}{\eta_{x'}^4}\Big]+G'\Big[\frac{\delta_{ij}}{\eta_{x'}^2}\Big], \tag{C.17}$$

$$\nabla_{x'^0}\nabla_{y^j}G = G''\Big[\frac{(x'-y)_j(\vec{x}'-\vec{y})^2}{2\eta_{x'}^5}\Big]-G'\Big[\frac{(x'-y)_j}{\eta_{x'}^3}\Big], \tag{C.18}$$

$$\nabla_{x'^i}\nabla_{y^0}G = G''\Big[-\frac{(x'-y)_i(\vec{x}'-\vec{y})^2}{2\eta_{x'}^5}\Big]+G'\Big[\frac{(x'-y)_i}{\eta_{x'}^3}\Big], \tag{C.19}$$

$$\nabla_{x'^0}\nabla_{y^0}G = G''\Big[\frac{(\vec{x}'-\vec{y})^4}{4\eta_{x'}^6}\Big]+G'\Big[-\frac{1}{\eta_{x'}^2}-\frac{(\vec{x}'-\vec{y})^2}{2\eta_{x'}^4}\Big]. \tag{C.20}$$

Appendix D

Power counting for noise kernel

D.0.1 Minimal coupling

In this appendix, we present a power counting argument to find out for what values of ν the noise kernel for de Sitter space-time i.e., the equation (3.12), diverges as $\eta \to 0$ (late time universe). If we look at the first term in the equation (3.12) i.e.,

$$(G'')^2 \left[\frac{(\Delta\vec{x})^6}{4\eta^{10}} + \frac{(\Delta\vec{x})^8}{32\eta^{12}} + \frac{(\Delta\vec{x})^4}{2\eta^8} \right], \tag{D.1}$$

we see that the most divergent term in the square brackets is $O(\eta^{-12})$. So, if we can find the values of ν for which the least power of η in $(G'')^2$ is < 12, then we have found the range of ν for which this term diverges.

Since the Wightman function and its derivatives are functions of $((1+Z)/2)\big(= 1 - ((\Delta\vec{x})^2/(4\eta^2))\big)$ [1], we must look at the series expansion of the Wightman function and its derivative at large values of their arguments in the $\eta \to 0$ limit. If we look at the following series expansion of ${}_2F_1(a,b,c,z)$ [87] (valid for large $|z|$ and $a-b \notin \mathbb{Z}$)[2]:

$$\begin{aligned} {}_2F_1(a,b,c,z) &= \frac{\Gamma(b-a)\Gamma(c)(-z)^{-a}}{\Gamma(b)\Gamma(c-a)} \sum_{k=0}^{\infty} \frac{a_k(a-c+1)_k z^{-k}}{k!(a-b+1)_k} \\ &\quad + \frac{\Gamma(a-b)\Gamma(c)(-z)^{-b}}{\Gamma(a)\Gamma(c-b)} \sum_{k=0}^{\infty} \frac{b_k(b-c+1)_k z^{-k}}{k!(-a+b+1)_k}, \end{aligned} \tag{D.2}$$

[1] See equations (3.13), (3.14), (3.15).

[2] In our case, $a-b = 2\nu$ which is not an integer for every value of ν in the range $[0, \frac{3}{2}]$ except for $\nu = 0, \frac{1}{2}, 1, \frac{3}{2}$. But we have already considered these cases separately in the section 3.

and keep in mind equation (3.15), we find that the least power of η in $(G'')^2$ is $14-4\nu$. Therefore, the above term diverges for $\nu > 1/2$. A similar analysis with the other terms in the equation (3.12) tells us that the equation (3.12) diverges for $\nu > 1/2$.

These arguments can be applied to the general components of the noise kernel. In fact, looking at the least powers of η in the formulae listed in the Appendix C for different covariant derivatives of Wightman function on constant time sheets and the equation (3.11), we see that

$\langle \hat{t}_{ab}(\eta,\vec{x})\hat{t}_{cd}(\eta,\vec{x'}) >_{dS}$	Leading order behaviour in η
$a=0,b=0,c=0,d=0$	$O(\eta^{2-4\nu})$
$a=0,b=0,c=0,d=l$	$O(\eta^{3-4\nu})$
$a=0,b=j,c=0,d=0$	$O(\eta^{3-4\nu})$
$a=0,b=j,c=0,d=l$	$O(\eta^{4-4\nu})$
$a=i,b=j,c=0,d=0$	$O(\eta^{2-4\nu})$
$a=0,b=0,c=k,d=l$	$O(\eta^{2-4\nu})$
$a=i,b=j,c=k,d=0$	$O(\eta^{3-4\nu})$
$a=0,b=j,c=k,d=l$	$O(\eta^{3-4\nu})$
$a=i,b=i,c=k,d=l$ and $k\neq l$	$O(\eta^{4-4\nu})$
$a=i,b=j,c=k,d=k$ and $i\neq j$	$O(\eta^{4-4\nu})$
$a=i,b=i,c=k,d=k$	$O(\eta^{2-4\nu})$
$a=i,b=j,c=k,d=l$ and $i\neq j,k\neq l$	$O(\eta^{6-4\nu})$

D.0.2 Non-minimal coupling

Below is given the expression of the noise kernel for the non-minimally coupled massive scalar field on de Sitter space-time in terms of the Wightman function and its covariant derivatives. First, we substituted the expression (3.27) for stress-energy tensor in the definition of the noise kernel (2.78). Since the definition of the noise kernel contains the vacuum expectation of product of two stress-energy tensors and each stress-energy operator contains two field operators, we would get the vacuum expectation of product of four field operators. Then, we can use the Wick theorem to express this vacuum expectation as the product of two Wightman function and obtain the equation (3.30). We obtain the below given expression by substituting the expressions (3.28), (3.29) for $P_{ab}(x,y)$ and $M_{ab}(x,y)$ in equation(3.30)

$$\langle t^{nm}_{ab}(x)t^{nm}_{cd}(x')\rangle = \Big[(1-2\xi)^2\big(\nabla_b\nabla'_cG(x,x')\nabla_a\nabla'_dG(x,x')+\nabla_b\nabla'_dG(x,x')\nabla_a\nabla'_cG(x,x')\big)$$

$$-(1-4\xi)(1-2\xi)\eta_{cd}\eta^{\rho\sigma}\nabla_a\nabla'_\rho G(x,x')\nabla_b\nabla'_\sigma G(x,x')-\frac{(m^2+6H^2\xi)(1-2\xi)}{H^2\eta'^2}\eta_{cd}\nabla_aG(x,x')\nabla_bG(x,x')$$

$$-(1-4\xi)(1-2\xi)\eta_{ab}\eta^{\gamma\delta}\nabla_\gamma\nabla'_cG(x,x')\nabla_\delta\nabla'_dG(x,x')+\frac{(1-4\xi)^2}{2}\eta_{ab}\eta^{\gamma\delta}\eta_{cd}\eta^{\rho\sigma}\nabla_\gamma\nabla'_\rho G(x,x')\nabla_\delta\nabla'_\sigma G(x,x')$$

$$
+\frac{(m^2+6H^2\xi)(1-4\xi)}{2H^2\eta'^2}\eta_{ab}\eta^{\gamma\delta}\eta_{cd}\nabla_\gamma G(x,x')\nabla_\delta G(x,x')-\frac{(m^2+6H^2\xi)(1-2\xi)}{H^2\eta^2}\eta_{ab}\nabla'_c G(x,x')\nabla'_d G(x,x')
$$

$$
+\frac{(m^2+6H^2\xi)(1-4\xi)}{2H^2\eta^2}\eta_{ab}\eta_{cd}\eta^{\rho\sigma}\nabla'_\rho G(x,x')\nabla'_\sigma G(x,x')+\frac{1}{2H^4\eta^2\eta'^2}(6H^2\xi+m^2)^2\eta_{ab}\eta_{cd}G^2\Bigg]
$$

$$
+2\xi\Bigg[2\eta_{cd}(1-2\xi)\big(\nabla_{(a}G\nabla_{b)}\Box' G\big)-2(1-2\xi)\big(\nabla_{(a}G\nabla_{b)}\nabla'_{(c}\nabla'_{d)}G\big)-\frac{(6H^2\xi+m^2)}{(H\eta)^2}\eta_{ab}\eta_{cd}G\Box' G
$$

$$
-(1-4\xi)\eta_{ab}\eta_{cd}(\eta^{rs}\nabla_s G\nabla_r\Box' G)+(1-4\xi)\eta_{ab}(\eta^{rs}\nabla_s G\nabla_r\nabla'_{(c}\nabla'_{d)}G)+\frac{(6H^2\xi+m^2)}{(H\eta)^2}\eta_{ab}G\nabla'_{(c}\nabla'_{d)}G\Bigg]
$$

$$
+2\xi\Bigg[2\eta_{ab}(1-2\xi)\big(\Box\nabla'_{(c}G\nabla'_{d)}G\big)-2(1-2\xi)\big(\nabla_{(a}\nabla_{b)}\nabla'_{(c}G\nabla'_{d)}G\big)-\frac{(6H^2\xi+m^2)}{(H\eta')^2}\eta_{ab}\eta_{cd}G\Box G
$$

$$
-(1-4\xi)\eta_{ab}\eta_{cd}(\eta^{rs}\Box\nabla'_s G\nabla'_r G)+(1-4\xi)\eta_{cd}(\eta^{mn}\nabla_{(a}\nabla_{b)}\nabla'_n G\nabla'_m G)+\frac{(6H^2\xi+m^2)}{(H\eta')^2}\eta_{cd}G\nabla_{(a}\nabla_{b)}G\Bigg]
$$

$$
+4\xi^2\Bigg[\eta_{ab}\eta_{cd}\big(\Box' G\Box G+G\Box\Box' G\big)-\eta_{ab}\big(\nabla'_{(c}\nabla'_{d)}G\Box G+G\Box\nabla'_{(c}\nabla'_{d)}G\big)
$$

$$
-\eta_{cd}\big(\nabla_{(a}\nabla_{b)}G\Box' G+G\nabla_{(a}\nabla_{b)}\Box' G\big)+\big(\nabla_{(a}\nabla_{b)}G\nabla'_{(c}\nabla'_{d)}G+G\nabla_{(a}\nabla_{b)}\nabla'_{(c}\nabla'_{d)}G\big)\Bigg]. \quad \text{(D.3)}
$$

The first square bracket contains the $P_{ab}P_{cd}$ term, the second and the third square brackets contain the $P_{ab}M_{cd}$ and $M_{ab}P_{cd}$ terms respectively. Whereas the fourth square bracket contains the $M_{ab}M_{cd}$ term. We can use the same power counting analysis as is done for the minimal coupling section of this appendix and study the behaviour of divergences for noise kernel as a function of mass and the coupling constant ξ.

Appendix E

Divergence in noise kernel for $\omega \in (-1,0)$ driven universe

Looking at the equation (85) of [65], we see that the Wightman function for massless scalar field in Friedmann space-times is given by :

$$G(x,x') = \frac{\beta^2(\eta\eta')^{q-1}}{8\pi^2}\int_0^\infty ds \frac{s^{\frac{1}{2}-\nu}}{(s^2-2Zs+1)^{\frac{3}{2}}}\,. \tag{E.1}$$

Case $q < -2$:

For this case, we have $\nu = q+1/2 < -3/2$. Now, consider the integral for large s values i.e.,

$$\begin{aligned} G(x,x') &= \frac{\beta^2(\eta\eta')^{q-1}}{8\pi^2}\Big[finite\ term + \int_N^\infty ds\, s^{-\frac{5}{2}-\nu}\Big(1-2\frac{Z}{s}+\frac{1}{s^2}\Big)^{-\frac{3}{2}}\Big] \\ &= \frac{\beta^2(\eta\eta')^{q-1}}{8\pi^2}\Big[finite\ term + \int_N^\infty ds\, s^{-\frac{5}{2}-\nu}\Big(1-\frac{3}{2}\Big(-2\frac{Z}{s}+\frac{1}{s^2}\Big)+\frac{3*5}{2*2*2}\Big(-2\frac{Z}{s}+\frac{1}{s^2}\Big)^2+...\Big)\Big] \\ &= \frac{\beta^2(\eta\eta')^{q-1}}{8\pi^2}\Big[finite\ term + \Big(\frac{s^{-\frac{3}{2}-\nu}}{-\frac{3}{2}-\nu}+3Z\frac{s^{-\frac{5}{2}-\nu}}{-\frac{5}{2}-\nu}-\frac{3}{2}\frac{s^{-\frac{7}{2}-\nu}}{-\frac{7}{2}-\nu}+(lower\ powers\ of\ s)\Big)\Big|_N^\infty\Big]\,. \end{aligned} \tag{E.2}$$

In the above expression N is used to divide the integration range $(0,\infty)$ to $(0,N)\cup(N,\infty)$ and the finite term above corresponds to the part of the integral with the integration range $(0,N)$. Since $Z = 1+\frac{(\eta-\eta')^2-(\Delta\vec{x})^2}{2\eta\eta'}$, we see that the highest collective power of η and η' is $-3+2\nu$ and one such highest power term is multiplying an η and η' independent and always diverging term $s^{-\frac{3}{2}-\nu}|_N^\infty$ in the expression for

Wightman function. This implies that the behaviour of noise kernel, in this case, is similar to the $q=-2$ (or correspondingly to $\nu=-\frac{3}{2}$ case). In fact, the leading order divergent term is

$$\langle t_{00}(\eta,\vec{x})t_{00}(\eta,\vec{x'})\rangle_{P.L.}=\frac{H^{4q}(q-1)^4}{128\pi^4\eta^{8-4q}\varepsilon^2}\,, \tag{E.3}$$

where $q=\nu-\frac{1}{2}$ and $\frac{1}{\varepsilon}=\frac{s^{-\frac{3}{2}-\nu}|_N^{\infty}}{\frac{3}{2}+\nu}$.

Case $q>1$:

For this case, we have $\nu=q+1/2>3/2$. Now, consider the integral for small s values i.e.,

$$\begin{aligned} G(x,x') &= \frac{\beta^2(\eta\eta')^{q-1}}{8\pi^2}\Big[finite\,term+\int_0^{\varepsilon}ds\, s^{\frac{1}{2}-\nu}\Big(1-2Zs+s^2\Big)^{-\frac{3}{2}}\Big] \\ &= \frac{\beta^2(\eta\eta')^{q-1}}{8\pi^2}\Big[finite\,term+\int_0^{\varepsilon}ds\, s^{\frac{1}{2}-\nu}\Big(1-\frac{3}{2}\Big(-2Zs+s^2\Big)+\frac{3*5}{2*2*2}\Big(-2Zs+s^2\Big)^2+...\Big)\Big] \\ &= \frac{\beta^2(\eta\eta')^{q-1}}{8\pi^2}\Big[finite\,term+\Big(\frac{s^{\frac{3}{2}-\nu}}{\frac{3}{2}-\nu}+3Z\frac{s^{\frac{5}{2}-\nu}}{\frac{5}{2}-\nu}-\frac{3}{2}\frac{s^{\frac{7}{2}-\nu}}{\frac{7}{2}-\nu}+(higher\ powers\ of\ s)\Big)\Big|_0^{\varepsilon}\Big]. \end{aligned} \tag{E.4}$$

In the above expression ε is used to divide the integration range $(0,\infty)$ to $(0,\varepsilon)\cup(\varepsilon,\infty)$ and the finite term above corresponds to the part of the integral with the integration range (ε,∞). From the above expression, we find that the term $s^{3/2-\nu}|_0^{\varepsilon}$ is most divergent for the considered case of $q>1$ and hence correspondingly for $\nu>3/2$. This always divergent term comes with the time dependent factors $(\eta\eta')^{(q-1)}$ and hence is also present in the noise kernel expressions as this survives the time derivatives present in the noise kernel expression. Thus, for the considered case of $q>1$, the behaviour of the noise kernel is similar to the noise kernel behaviour for the case of $q=-2$.

Appendix F

Ω and H dependence of infinite time response rate

In this appendix, we consider the infinite time response rate of conventionally and derivatively coupled UdW detectors in FRW spacetimes i.e., $a(\eta) = (H\eta)^{-q}$ with $q \in (-2,1)$. By making use of the dimensional analysis, we try to find out the dependence of the response rate on the energy gap between the detector's states as well as on the parameter H.

F.0.1 Conventionally coupled Unruh deWitt detector

Let us consider the spacetimes with $q \in (-2,0)$ for which the cosmic time is related to the conformal time by the relation $t = \frac{H^{-q}\eta^{1-q}}{1-q}$ and $t \in (0,\infty)$ for $\eta \in (0,\infty)$. Using these relations and the formulae (5.4),(5.7), we find that the transition probability is given by

$$P_{0\to\Omega} = c^2 |_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2 \int_0^\infty \int_0^\infty d\eta_1 d\eta_2 e^{i\frac{\Omega H^{-q}}{(1-q)}(\eta_1^{1-q}-\eta_2^{1-q})} (H^2\eta_1\eta_2)^{-1} G^{dS}(y(x(\eta_1),x(\eta_2)))\,. \quad \text{(F.1)}$$

Now defining a new variable $z = \Omega H^{-q}\eta^{1-q}$, we can pull out all the Ω and H dependence out of the integral

$$\frac{P_{0\to\Omega}}{c^2|_D\langle\Omega|\hat{\mu}(0)|0\rangle_D|^2} = \int_0^\infty \int_0^\infty \frac{dz_1 dz_2}{(1-q)^2} \frac{(z_1z_2)^{\frac{q}{1-q}}}{(\Omega H^{-q})^{\frac{2}{1-q}}} e^{i\frac{(z_1-z_2)}{(1-q)}} \frac{(\Omega H^{-q})^{\frac{2}{1-q}}}{H^2(z_1z_2)^{\frac{1}{1-q}}} G^{dS}(y(x(\eta_1),x(\eta_2)))$$

$$= \int_0^\infty \int_0^\infty \frac{dz_1 dz_2}{(1-q)^2} e^{-i\frac{(z_1-z_2)}{(1-q)}} \frac{\Gamma\left(\frac{3}{2}+\nu\right)\Gamma\left(\frac{3}{2}-\nu\right)}{16\pi^2 (z_1 z_2)} {}_2F_1\left(\frac{3}{2}+\nu, \frac{3}{2}-\nu, 2, 1-\frac{y}{4}\right) \tag{F.2}$$

where $y = -\frac{\left(z_1^{\frac{1}{1-q}} - z_2^{\frac{1}{1-q}} - i\varepsilon(\Omega H^{-q})^{\frac{1}{1-q}}\right)^2}{(z_1 z_2)^{\frac{1}{1-q}}}$ for comoving observers. Since the only Ω and H dependences in the integral are through the term $\varepsilon(\Omega H^{-q})^{\frac{1}{1-q}}$ which go to zero in the $\varepsilon \to 0$ limit, we find that the above integral does not depend upon Ω and H. However, the rate, say with respect to $\tilde{\eta} = \frac{\eta_1+\eta_2}{2}$ or some other linear combinations of η_1 and η_2 has the Ω and H dependence of the following type

$$\frac{1}{c^2 |_D \langle \Omega | \hat{\mu}(0) | 0 \rangle_D |^2} \frac{dP_{0\to\Omega}}{d\tilde{\eta}} \propto (\Omega H^{-q})^{\frac{1}{1-q}} \tag{F.3}$$

Similarly, for spacetimes with $q \in (0,1)$, the response rate again has the same Ω and H dependences.

F.0.2 Derivatively coupled Unruh deWitt detector

Let us again consider the spacetimes with $q \in (-2,0)$. In order to find the Ω and H dependences of response rate for derivatively coupled cases, we make use of the following formulae for the infinite time transition probability for derivatively coupled UdW detectors

$$P_{0\to\Omega} = c^2 |_D \langle \Omega | \hat{\mu}(0) | 0 \rangle_D |^2 \int_0^\infty \int_0^\infty d\eta_1 d\eta_2 e^{-i\Omega(\tau(\eta_1)-\tau(\eta_2))} \frac{d}{d\eta_1} \frac{d}{d\eta_2} G(x(\eta_1), x(\eta_2)) \,. \tag{F.4}$$

Performing the same steps as in the previous subsection and using the formulae (5.24) for double derivatives of the Wightman function for FRW spacetimes, one obtains that the response rate for derivatively coupled UdW detectors has the following Ω and H dependences

$$\frac{1}{c^2 |_D \langle \Omega | \hat{\mu}(0) | 0 \rangle_D |^2} \frac{dP_{0\to\Omega}}{d\tilde{\eta}} \propto \Omega^2 (\Omega H^{-q})^{\frac{1}{1-q}} \tag{F.5}$$

We obtain the same Ω and H dependences of the response rate for spacetimes with $q \in (0,1)$

Appendix G

Derivatives of the Wightman function

In this appendix, we express the double time derivative (appearing in the expression (5.21) of the response rate for derivatively coupled UdW detectors) of the FRW Wightman function in terms of the derivatives of the de Sitter Wightman function by using the relation (5.7) between the Wightman functions in the two settings i.e.,

$$G^{FRW}(x_1,x_2) = (H^2\eta_1\eta_2)^{q-1}G^{dS}(y(x_1,x_2))\,. \tag{G.1}$$

Using the product rule of differentiation, we have

$$\begin{aligned}\frac{d}{d\eta_1}\frac{d}{d\eta_2}G^{FRW}(x(\eta_1),x(\eta_2)) \;=\; & (H^2\eta_1\eta_2)^{q-1}\Big[(q-1)^2\frac{G^{dS}}{\eta_1\eta_2}+(q-1)\frac{dG^{dS}}{dy}\Big(\frac{1}{\eta_1}\frac{dy}{d\eta_2}+\frac{1}{\eta_2}\frac{dy}{d\eta_1}\Big)\\ & +\Big(\frac{d^2G^{dS}}{dy^2}\frac{dy}{d\eta_1}\frac{dy}{d\eta_2}+\frac{dG^{dS}}{dy}\frac{d^2y}{d\eta_1 d\eta_2}\Big)\Big]\,. \end{aligned}\tag{G.2}$$

In conformal coordinates, the de Sitter invariant distance is given by

$$y = \frac{-(\eta_1-\eta_2-i\varepsilon)^2+(\Delta\vec{x})^2}{\eta_1\eta_2}\,. \tag{G.3}$$

For comoving observers, we have

$$\frac{dy}{d\eta_1} = \frac{(\eta_1-\eta_2-i\varepsilon)(-\eta_1-\eta_2-i\varepsilon)}{\eta_1^2\eta_2} \tag{G.4}$$

$$\frac{dy}{d\eta_2} = \frac{(\eta_1-\eta_2-i\varepsilon)(\eta_1+\eta_2-i\varepsilon)}{\eta_1\eta_2^2} \tag{G.5}$$

$$\frac{d^2y}{d\eta_1 d\eta_2} = \frac{-(\eta_1-\eta_2-i\varepsilon)(\eta_1+\eta_2-i\varepsilon)+2(\eta_1-i\varepsilon)\eta_1}{\eta_1^2\eta_2^2} \tag{G.6}$$

$$\frac{dy}{d\eta_1}\frac{dy}{d\eta_2} = \frac{y((\eta_1+\eta_2)^2+\varepsilon^2)}{\eta_1^2\eta_2^2}. \tag{G.7}$$

Using these expressions, we see that

$$\begin{aligned}\frac{d}{d\eta_1}\frac{d}{d\eta_2}G^{FRW}(x(\eta_1),x(\eta_2)) &= (H^2\eta_1\eta_2)^{q-1}\Bigg[(q-1)^2\frac{G^{dS}}{\eta_1\eta_2}+(q-1)\frac{dG^{dS}}{dy}\Big(\frac{(\eta_1-\eta_2-i\varepsilon)(-2i\varepsilon)}{\eta_1^2\eta_2^2}\Big)\\ &\quad+\frac{d^2G^{dS}}{dy^2}\frac{y((\eta_1+\eta_2)^2+\varepsilon^2)}{\eta_1^2\eta_2^2}+\frac{dG^{dS}}{dy}\frac{(\eta_1^2+\eta_2^2+\varepsilon^2)}{\eta_1^2\eta_2^2}\Bigg].\end{aligned} \tag{G.8}$$

Appendix H

Flat spacetime hydrogen atom and selection rules

We can solve for the energy eigenfunctions of the unperturbed flat spacetime hydrogen atom Hamiltonian. We find that they are given by

$$\psi_{nlm} = R_{nl}(r) Y_l^m(\theta, \phi) \tag{H.1}$$

where $Y_l^m(\theta, \phi)$ are the spherical harmonics and $R_{nl}(r)$ are the radial part of the eigenfunctions. $R_{nl}(r)$ are given by

$$R_{nl}(r) = -\sqrt{\left(\frac{2}{na_0}\right)^3 \frac{(n-l-1)!}{2n((n+l)!)^3}} e^{-\frac{r}{na_0}} \left(\frac{2r}{na_0}\right)^l L_{n+l}^{2l+1}\left(\frac{2r}{na_0}\right) \tag{H.2}$$

where L_{n+l}^{2l+1} are associated Laguerre polynomials and $a_0 = \frac{1}{me^2}$. Here n, l and m are just the hydrogen atom quantum numbers.

Using the orthonormality of spherical harmonics and the properties of addition of angular momenta, we find that the selections rules for the transitions

$$< n', l', m' | x^i | n, l, m >$$

are as given in table 1.

Using the above selection rules, we can find the selection rules for the transitions of the form

$$< n', l', m' | x^i x^p | n, l, m >$$

x,y		z	
Δl	Δm	Δl	Δm
± 1	± 1	± 1	0

Table H.1: Selection rules for transitions of the type $< n',l',m'|x^i|n,l,m >$.

which are given in the table 2.

x^2,y^2,xy,yx		xz,yz		z^2	
Δl	Δm	Δl	Δm	Δl	Δm
-2,0,2	-2,0,2	-2,0,2	-1,1	-2,0,2	0

Table H.2: Selection rules for transitions of the type $< n',l',m'|x^i x^p|n,l,m >$.

Bibliography

[1] Alan H. Guth. The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems. *Phys. Rev. D*, 23:347–356, 1981.

[2] Andrei D. Linde. A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems. *Phys. Lett. B*, 108:389–393, 1982.

[3] Charles L Bennett, Davin Larson, Janet L Weiland, N Jarosik, G Hinshaw, N Odegard, KM Smith, RS Hill, B Gold, M Halpern, et al. Nine-year wilkinson microwave anisotropy probe (wmap) observations: final maps and results. *The Astrophysical Journal Supplement Series*, 208(2):20, 2013.

[4] P. A. R. Ade et al. Planck 2013 results. XV. CMB power spectra and likelihood. *Astron. Astrophys.*, 571:A15, 2014.

[5] S. Weinberg. *Cosmology*. Cosmology. Oxford University Press, Oxford, 2008.

[6] Antonio Riotto. Inflation and the theory of cosmological perturbations. *ICTP Lect. Notes Ser.*, 14:317–413, 2003.

[7] N. Bartolo, E. Komatsu, Sabino Matarrese, and A. Riotto. Non-Gaussianity from inflation: Theory and observations. *Phys. Rept.*, 402:103–266, 2004.

[8] Daniel Baumann. Inflation. In *Physics of the large and the small, TASI 09, proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics, Boulder, Colorado, USA, 1-26 June 2009*, pages 523–686, 2011.

[9] Matthias Blau. Lecture Notes on General Relativity.

[10] Viatcheslav F. Mukhanov, H. A. Feldman, and Robert H. Brandenberger. Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions. *Phys. Rept.*, 215:203–333, 1992.

[11] T. S. Bunch and P. C. W. Davies. Quantum Field Theory in de Sitter Space: Renormalization by Point Splitting. *Proc. Roy. Soc. Lond.*, A360:117–134, 1978.

[12] Leonard E. Parker and D. Toms. *Quantum Field Theory in Curved Spacetime*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2009.

[13] Bruce Allen. Vacuum States in de Sitter Space. *Phys. Rev.*, D32:3136, 1985.

[14] Aditya Aravind, Dustin Lorshbough, and Sonia Paban. Non-Gaussianity from Excited Initial Inflationary States. *JHEP*, 07:076, 2013.

[15] Jerome Martin, Alain Riazuelo, and Mairi Sakellariadou. Nonvacuum initial states for cosmological perturbations of quantum mechanical origin. *Phys. Rev. D*, 61:083518, 2000.

[16] Jonathan Ganc and Eiichiro Komatsu. Scale-dependent bias of galaxies and mu-type distortion of the cosmic microwave background spectrum from single-field inflation with a modified initial state. *Phys. Rev. D*, 86:023518, 2012.

[17] Jerome Martin, Christophe Ringeval, and Vincent Vennin. Encyclopædia Inflationaris. *Phys. Dark Univ.*, 5-6:75–235, 2014.

[18] Jerome Martin. Cosmic Inflation: Trick or Treat? 2 2019.

[19] N. Aghanim et al. Planck 2018 results. V. CMB power spectra and likelihoods. *Astron. Astrophys.*, 641:A5, 2020.

[20] James M. Bardeen. Gauge Invariant Cosmological Perturbations. *Phys. Rev. D*, 22:1882–1905, 1980.

[21] Hideo Kodama and Misao Sasaki. Cosmological Perturbation Theory. *Prog. Theor. Phys. Suppl.*, 78:1–166, 1984.

[22] Jerome Martin. Inflation and precision cosmology. *Braz. J. Phys.*, 34:1307–1321, 2004.

[23] N. Aghanim et al. Planck 2018 results. VI. Cosmological parameters. *Astron. Astrophys.*, 641:A6, 2020. [Erratum: Astron.Astrophys. 652, C4 (2021)].

[24] Matias Zaldarriaga and Uros Seljak. An all sky analysis of polarization in the microwave background. *Phys. Rev. D*, 55:1830–1840, 1997.

[25] Marc Kamionkowski, Arthur Kosowsky, and Albert Stebbins. Statistics of cosmic microwave background polarization. *Phys. Rev. D*, 55:7368–7388, 1997.

[26] Kevork N. Abazajian et al. CMB-S4 Science Book, First Edition. 10 2016.

[27] Jerome Martin and Robert H. Brandenberger. The TransPlanckian problem of inflationary cosmology. *Phys. Rev. D*, 63:123501, 2001.

[28] Robert H. Brandenberger and Jerome Martin. Trans-Planckian Issues for Inflationary Cosmology. *Class. Quant. Grav.*, 30:113001, 2013.

[29] Robert H. Brandenberger and Jerome Martin. The Robustness of inflation to changes in super-Planck scale physics. *Mod. Phys. Lett. A*, 16:999–1006, 2001.

[30] W. G. Unruh. Sonic analog of black holes and the effects of high frequencies on black hole evaporation. *Phys. Rev. D*, 51:2827–2838, 1995.

[31] Jens C. Niemeyer. Inflation with a Planck scale frequency cutoff. *Phys. Rev. D*, 63:123502, 2001.

[32] Jens C. Niemeyer and Renaud Parentani. Transplanckian dispersion and scale invariance of inflationary perturbations. *Phys. Rev. D*, 64:101301, 2001.

[33] S. Shankaranarayanan. Is there an imprint of Planck scale physics on inflationary cosmology? *Class. Quant. Grav.*, 20:75–84, 2003.

[34] Achim Kempf. Mode generating mechanism in inflation with cutoff. *Phys. Rev. D*, 63:083514, 2001.

[35] Achim Kempf and Jens C. Niemeyer. Perturbation spectrum in inflation with cutoff. *Phys. Rev. D*, 64:103501, 2001.

[36] S. F. Hassan and Martin S. Sloth. TransPlanckian effects in inflationary cosmology and the modified uncertainty principle. *Nucl. Phys. B*, 674:434–458, 2003.

[37] Richard Easther, Brian R Greene, William H Kinney, and Gary Shiu. Inflation as a probe of short distance physics. *Physical Review D*, 64(10):103502, 2001.

[38] Ulf H. Danielsson. A Note on inflation and transPlanckian physics. *Phys. Rev. D*, 66:023511, 2002.

[39] Ulf H. Danielsson. Transplanckian energy production and slow roll inflation. *Phys. Rev. D*, 71:023516, 2005.

[40] C. P. Burgess, James M. Cline, F. Lemieux, and R. Holman. Are inflationary predictions sensitive to very high-energy physics? *JHEP*, 02:048, 2003.

[41] Paul R. Anderson, Carmen Molina-Paris, and Emil Mottola. Short distance and initial state effects in inflation: Stress tensor and decoherence. *Phys. Rev. D*, 72:043515, 2005.

[42] Sandipan Kundu. Inflation with General Initial Conditions for Scalar Perturbations. *JCAP*, 02:005, 2012.

[43] Kaushik Bhattacharya, Subhendra Mohanty, and Raghavan Rangarajan. Temperature of the inflaton and duration of inflation from WMAP data. *Phys. Rev. Lett.*, 96:121302, 2006.

[44] R. Holman and Andrew J. Tolley. Enhanced Non-Gaussianity from Excited Initial States. *JCAP*, 05:001, 2008.

[45] Pieter Daniel Meerburg, Jan Pieter van der Schaar, and Pier Stefano Corasaniti. Signatures of Initial State Modifications on Bispectrum Statistics. *JCAP*, 05:018, 2009.

[46] Ivan Agullo and Leonard Parker. Non-gaussianities and the Stimulated creation of quanta in the inflationary universe. *Phys. Rev. D*, 83:063526, 2011.

[47] Nishant Agarwal, R. Holman, Andrew J. Tolley, and Jennifer Lin. Effective field theory and non-Gaussianity from general inflationary states. *JHEP*, 05:085, 2013.

[48] Shiro Hirai. Initial condition of scalar perturbation in inflation. *Class. Quant. Grav.*, 20:1673–1684, 2003.

[49] Paul R. Anderson, Eric D. Carlson, Taylor M. Ordines, and Bradley Hicks. Semiclassical predictions regarding a preinflationary era and its effects on the power spectrum. *Phys. Rev. D*, 102(6):063528, 2020.

[50] Taylor M. Ordines and Eric D. Carlson. Argument for the radiation-dominated behavior of matter fields in the preinflationary era. *Phys. Rev. D*, 106(10):103537, 2022.

[51] Michael S. Turner and Lawrence M. Widrow. Inflation Produced, Large Scale Magnetic Fields. *Phys. Rev. D*, 37:2743, 1988.

[52] Bharat Ratra. Cosmological'seed'magnetic field from inflation. *The Astrophysical Journal*, 391:L1–L4, 1992.

[53] Chiara Caprini and Lorenzo Sorbo. Adding helicity to inflationary magnetogenesis. *JCAP*, 10:056, 2014.

[54] Kandaswamy Subramanian. Magnetic fields in the early universe. *Astron. Nachr.*, 331:110–120, 2010.

[55] Jerome Martin and Jun'ichi Yokoyama. Generation of Large-Scale Magnetic Fields in Single-Field Inflation. *JCAP*, 01:025, 2008.

[56] Sagarika Tripathy, Debika Chowdhury, Rajeev Kumar Jain, and L. Sriramkumar. Challenges in the choice of the nonconformal coupling function in inflationary magnetogenesis. *Phys. Rev. D*, 105(6):063519, 2022.

[57] Vittoria Demozzi, Viatcheslav Mukhanov, and Hector Rubinstein. Magnetic fields from inflation? *JCAP*, 08:025, 2009.

[58] Juan Martin Maldacena. Non-Gaussian features of primordial fluctuations in single field inflationary models. *JHEP*, 05:013, 2003.

[59] Xingang Chen, Min-xin Huang, Shamit Kachru, and Gary Shiu. Observational signatures and non-Gaussianities of general single field inflation. *JCAP*, 01:002, 2007.

[60] Andreas Albrecht, Pedro Ferreira, Michael Joyce, and Tomislav Prokopec. Inflation and squeezed quantum states. *Phys. Rev. D*, 50:4807–4820, 1994.

[61] Claus Kiefer and David Polarski. Why do cosmological perturbations look classical to us? *Adv. Sci. Lett.*, 2:164–173, 2009.

[62] David Polarski and Alexei A. Starobinsky. Semiclassicality and decoherence of cosmological perturbations. *Class. Quant. Grav.*, 13:377–392, 1996.

[63] J. Lesgourgues, David Polarski, and Alexei A. Starobinsky. Quantum to classical transition of cosmological perturbations for nonvacuum initial states. *Nucl. Phys. B*, 497:479–510, 1997.

[64] Michael E. Peskin and Daniel V. Schroeder. *An Introduction to quantum field theory*. Addison-Wesley, Reading, USA, 1995.

[65] Kinjalk Lochan, Karthik Rajeev, Amit Vikram, and T. Padmanabhan. Quantum correlators in Friedmann spacetimes: The omnipresent de Sitter spacetime and the invariant vacuum noise. *Phys. Rev. D*, 98(10):105015, 2018.

[66] Steven Weinberg. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*. John Wiley and Sons, New York, 1972.

[67] T.Padmanabhan. *Gravitation: Foundations and Frontiers*. Cambridge University Press, Cambridge, UK, 2010.

[68] Edwin Hubble. A relation between distance and radial velocity among extra-galactic nebulae. *Proceedings of the National Academy of Sciences*, 15(3):168–173, 1929.

[69] Kevin Cahill. Flat space, dark energy, and the cosmic microwave background. *European Journal of Physics*, 41(3):035603, 2020.

[70] Kazuharu Bamba, Salvatore Capozziello, Shin'ichi Nojiri, and Sergei D. Odintsov. Dark energy cosmology: the equivalent description via different theoretical models and cosmography tests. *Astrophys. Space Sci.*, 342:155–228, 2012.

[71] Archana Sangwan, Ankan Mukherjee, and H. K. Jassal. Reconstructing the dark energy potential. *JCAP*, 01:018, 2018.

[72] Marcus Spradlin, Andrew Strominger, and Anastasia Volovich. Les Houches lectures on de Sitter space. In *Unity from duality: Gravity, gauge theory and strings. Proceedings, NATO Advanced Study Institute, Euro Summer School, 76th session, Les Houches, France, July 30-August 31, 2001*, pages 423–453, 2001.

[73] S. W. Hawking and G. F. R. Ellis. *The Large Scale Structure of Space-Time*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2011.

[74] Guillem Perez-Nadal, Albert Roura, and Enric Verdaguer. Stress tensor fluctuations in de Sitter spacetime. *JCAP*, 1005:036, 2010.

[75] N.D. Birrell and P.C.W. Davies. Quantum Fields in Curved Space. *Cambridge Monogr.Math.Phys.*, 1982.

[76] L. H. Ford. Quantum field theory in curved space-time. In *Particles and fields. Proceedings, 9th Jorge Andre Swieca Summer School, Campos do Jordao, Brazil, February 16-28, 1997*, pages 345–388, 1997.

[77] Stephen A. Fulling. Nonuniqueness of canonical field quantization in Riemannian space-time. *Phys. Rev.*, D7:2850–2862, 1973.

[78] S. W. Hawking. Particle Creation by Black Holes. *Commun. Math. Phys.*, 43:199–220, 1975. [,167(1975)].

[79] P. C. W. Davies. Scalar particle production in Schwarzschild and Rindler metrics. *J. Phys.*, A8:609–616, 1975.

[80] Julian S. Schwinger. On gauge invariance and vacuum polarization. *Phys. Rev.*, 82:664–679, 1951.

[81] Thomas D. Cohen and David A. McGady. The Schwinger mechanism revisited. *Phys. Rev. D*, 78:036008, 2008.

[82] N. A. Chernikov and E. A. Tagirov. Quantum theory of scalar fields in de Sitter space-time. *Ann. Inst. H. Poincare Phys. Theor.*, A9:109, 1968.

[83] P. Candelas and D. J. Raine. General Relativistic Quantum Field Theory-An Exactly Soluble Model. *Phys. Rev.*, D12:965–974, 1975.

[84] Bruce Allen and Antoine Folacci. The Massless Minimally Coupled Scalar Field in De Sitter Space. *Phys. Rev.*, D35:3771, 1987.

[85] Paul R. Anderson and Emil Mottola. Instability of global de Sitter space to particle creation. *Phys. Rev.*, D89:104038, 2014.

[86] NIST Digital Library of Mathematical Functions. http://dlmf.nist.gov/, Release 1.0.17 of 2017-12-22. F. W. J. Olver, A. B. Olde Daalhuis, D. W. Lozier, B. I. Schneider, R. F. Boisvert, C. W. Clark, B. R. Miller, and B. V. Saunders, eds.

[87] Milton Abramowitz and Irene A. Stegun. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. Dover, New York, ninth dover printing, tenth gpo printing edition, 1964.

[88] G. N. Watson. A Treatise on the Theory of Bessel Functions (2nd ed.) . *Cambridge University Press*, 1996.

[89] I. S. Gradshteyn and I. M. Ryzhik. *Table of integrals, series, and products*. Elsevier/Academic Press, Amsterdam, seventh edition, 2007.

[90] L. H. Ford. Quantum Instability of De Sitter Space-time. *Phys. Rev.*, D31:710, 1985.

[91] E. Mottola. Particle Creation in de Sitter Space. *Phys. Rev.*, D31:754, 1985.

[92] Hael Collins, R. Holman, and Matthew R. Martin. The Fate of the alpha vacuum. *Phys. Rev. D*, 68:124012, 2003.

[93] Jurjen F. Koksma and Tomislav Prokopec. Fermion Propagator in Cosmological Spaces with Constant Deceleration. *Class. Quant. Grav.*, 26:125003, 2009.

[94] Hael Collins. Fermionic alpha-vacua. *Phys. Rev. D*, 71:024002, 2005.

[95] P. C. W. Davies. On the origin of black hole evaporation radiation. *Proc. R. Soc. Lond. A*, 351:129–132, 1976.

[96] L. Parker. Particle creation in expanding universes. *Phys. Rev. Lett.*, 21:562–564, 1968.

[97] Leonard Parker. Quantized fields and particle creation in expanding universes. 1. *Phys. Rev.*, 183:1057–1068, 1969.

[98] L. Parker. Quantized fields and particle creation in expanding universes. 2. *Phys. Rev.*, D3:346–356, 1971. [Erratum: Phys. Rev.D3,2546(1971)].

[99] Robert H. Brandenberger. Quantum Field Theory Methods and Inflationary Universe Models. *Rev. Mod. Phys.*, 57:1, 1985.

[100] Leonard Parker and Alpan Raval. Nonperturbative effects of vacuum energy on the recent expansion of the universe. *Phys. Rev.*, D60:063512, 1999. [Erratum: Phys. Rev.D67,029901(2003)].

[101] B. L. Hu and E. Verdaguer. Stochastic Gravity: Theory and Applications. *Living Rev. Rel.*, 11:3, 2008.

[102] Nicholas G. Phillips and B. L. Hu. Noise kernel in stochastic gravity and stress energy bitensor of quantum fields in curved space-times. *Phys. Rev.*, D63:104001, 2001.

[103] Valter Moretti. Direct zeta function approach and renormalization of one loop stress tensors in curved space-times. *Phys. Rev. D*, 56:7797–7819, 1997.

[104] W.G. Unruh. Notes on black hole evaporation. *Phys.Rev.*, D14:870, 1976.

[105] Luis C. B. Crispino, Atsushi Higuchi, and George E. A. Matsas. The Unruh effect and its applications. *Rev. Mod. Phys.*, 80:787–838, 2008.

[106] Ivette Fuentes-Schuller and Robert B. Mann. Alice falls into a black hole: Entanglement in non-inertial frames. *Phys. Rev. Lett.*, 95:120404, 2005.

[107] Shih-Yuin Lin and B. L. Hu. Entanglement creation between two causally disconnected objects. *Phys. Rev. D*, 81:045019, 2010.

[108] Subhajit Barman and Bibhas Ranjan Majhi. Radiative process of two entangled uniformly accelerated atoms in a thermal bath: a possible case of anti-Unruh event. *JHEP*, 03:245, 2021.

[109] Subhajit Barman, Bibhas Ranjan Majhi, and L. Sriramkumar. Radiative processes of single and entangled detectors on circular trajectories in $(2+1)$ dimensional Minkowski spacetime. 5 2022.

[110] Benito A. Juárez-Aubry and Jorma Louko. Onset and decay of the 1 + 1 Hawking-Unruh effect: what the derivative-coupling detector saw. *Class. Quant. Grav.*, 31(24):245007, 2014.

[111] Jorma Louko. Unruh-DeWitt detector response across a Rindler firewall is finite. *JHEP*, 09:142, 2014.

[112] Eduardo Martin-Martinez and Jorma Louko. Particle detectors and the zero mode of a quantum field. *Phys. Rev. D*, 90(2):024015, 2014.

[113] Erickson Tjoa and Robert B. Mann. Unruh-DeWitt detector in dimensionally-reduced static spherically symmetric spacetimes. *JHEP*, 03:014, 2022.

[114] B. F. Svaiter and N. F. Svaiter. Inertial and noninertial particle detectors and vacuum fluctuations. *Phys. Rev. D*, 46:5267–5277, 1992. [Erratum: Phys.Rev.D 47, 4802 (1993)].

[115] Eric G. Brown, Eduardo Martin-Martinez, Nicolas C. Menicucci, and Robert B. Mann. Detectors for probing relativistic quantum physics beyond perturbation theory. *Phys. Rev. D*, 87:084062, 2013.

[116] Shih-Yuin Lin and B. L. Hu. Backreaction and the Unruh effect: New insights from exact solutions of uniformly accelerated detectors. *Phys. Rev. D*, 76:064008, 2007.

[117] Sebastian Schlicht. Considerations on the Unruh effect: Causality and regularization. *Class. Quant. Grav.*, 21:4647–4660, 2004.

[118] Jorma Louko and Alejandro Satz. How often does the Unruh-DeWitt detector click? Regularisation by a spatial profile. *Class. Quant. Grav.*, 23:6321–6344, 2006.

[119] Antony R. Lee and Ivette Fuentes. Spatially extended Unruh-DeWitt detectors for relativistic quantum information. *Phys. Rev. D*, 89(8):085041, 2014.

[120] Eduardo Martín-Martínez, T. Rick Perche, and Bruno de S. L. Torres. General Relativistic Quantum Optics: Finite-size particle detector models in curved spacetimes. *Phys. Rev. D*, 101(4):045017, 2020.

[121] Y. Akrami et al. Planck 2018 results. X. Constraints on inflation. 2018.

[122] E. T. Akhmedov. Lecture notes on interacting quantum fields in de Sitter space. *Int. J. Mod. Phys.*, D23:1430001, 2014.

[123] Ignatios Antoniadis, J. Iliopoulos, and T. N. Tomaras. Quantum Instability of De Sitter Space. *Phys. Rev. Lett.*, 56:1319, 1986.

[124] D. Polarski. Infrared divergences in de Sitter space. *Phys. Rev.*, D43:1892–1895, 1991.

[125] Klaus Kirsten and Jaume Garriga. Massless minimally coupled fields in de Sitter space: O(4) symmetric states versus de Sitter invariant vacuum. *Phys. Rev.*, D48:567–577, 1993.

[126] Bharat Ratra. Restoration of Spontaneously Broken Continuous Symmetries in de Sitter Space-Time. *Phys. Rev.*, D31:1931–1955, 1985.

[127] A. D. Dolgov, M. B. Einhorn, and Valentin I. Zakharov. The Vacuum of de Sitter space. *Acta Phys. Polon.*, B26:65–90, 1995.

[128] M. V. Takook. Covariant two point function for minimally coupled scalar field in de Sitter space-time. *Mod. Phys. Lett.*, A16:1691–1698, 2001.

[129] Andrew J. Tolley and Neil Turok. Quantization of the massless minimally coupled scalar field and the dS / CFT correspondence. 2001.

[130] Bjorn Garbrecht. Ultraviolet Regularisation in de Sitter Space. *Phys. Rev.*, D74:043507, 2006.

[131] Don N. Page and Xing Wu. Massless Scalar Field Vacuum in de Sitter Spacetime. *JCAP*, 1211:051, 2012.

[132] Takahiro Tanaka and Yuko Urakawa. Loops in inflationary correlation functions. *Class. Quant. Grav.*, 30:233001, 2013.

[133] R. P. Woodard. de Sitter breaking in field theory. In *Deserfest: A celebration of the life and works of Stanley Deser. Proceedings, Meeting, Ann Arbor, USA, April 3-5, 2004*, pages 339–351, 2004.

[134] S. P. Miao, N. C. Tsamis, and R. P. Woodard. De Sitter Breaking through Infrared Divergences. *J. Math. Phys.*, 51:072503, 2010.

[135] C. Wetterich. Cosmic fluctuations from a quantum effective action. *Phys. Rev.*, D92(8):083507, 2015.

[136] C. Wetterich. Can observations look back to the beginning of inflation? *Phys. Lett.*, B754:109–113, 2016.

[137] L. H. Ford and L. Parker. Infrared Divergences in a Class of Robertson-Walker Universes. *Phys. Rev. D*, 16:245–250, 1977.

[138] S. A. Fulling. Aspects of Quantum Field Theory in Curved Space-time. *London Math. Soc. Student Texts*, 17:1–315, 1989.

[139] J. S. Dowker and Raymond Critchley. Effective Lagrangian and Energy Momentum Tensor in de Sitter Space. *Phys. Rev.*, D13:3224, 1976.

[140] Markus B. Fröb. Fully renormalized stress tensor correlator in flat space. *Phys. Rev.*, D88:045011, 2013.

[141] Stefan Hollands and Robert M. Wald. Quantum fields in curved spacetime. *Phys. Rept.*, 574:1–35, 2015.

[142] Jason D. Bates, H. T. Cho, Paul R. Anderson, and B. L. Hu. Noise kernel near the horizon of de Sitter space. *Class. Quant. Grav.*, 31:025015, 2014.

[143] H. Osborn and G. M. Shore. Correlation functions of the energy momentum tensor on spaces of constant curvature. *Nucl. Phys. B*, 571:287–357, 2000.

[144] H. T. Cho and B. L. Hu. Stress-energy Tensor Correlators of a Quantum Field in Euclidean R^N and AdS^N spaces via the generalized zeta-function method. *Phys. Rev. D*, 84:044032, 2011.

[145] Ankit Dhanuka and Kinjalk Lochan. Stress energy correlator in de Sitter spacetime: Its conformal masking or growth in connected Friedmann universes. *Phys. Rev. D*, 102(8):085009, 2020.

[146] Dmitry Krotov and Alexander M. Polyakov. Infrared Sensitivity of Unstable Vacua. *Nucl. Phys.*, B849:410–432, 2011.

[147] Paul R. Anderson and Emil Mottola. Quantum vacuum instability of “eternal” de Sitter space. *Phys. Rev.*, D89:104039, 2014.

[148] Paul R. Anderson, Emil Mottola, and Dillon H. Sanders. Decay of the de Sitter Vacuum. *Phys. Rev.*, D97(6):065016, 2018.

[149] T. Padmanabhan and T. P. Singh. RESPONSE OF ACCELERATED DETECTORS IN COHERENT STATES AND THE SEMICLASSICAL LIMIT. *Phys. Rev.*, D38:2457–2463, 1988.

[150] L. H. Ford and Thomas A. Roman. Minkowski vacuum stress tensor fluctuations. *Phys. Rev.*, D72:105010, 2005.

[151] T. Padmanabhan, T. R. Seshadri, and T. P. Singh. Making Inflation Work: Damping of Density Perturbations Due to Planck Energy Cutoff. *Phys. Rev.*, D39:2100, 1989.

[152] Kaixi Feng, Yi-Fu Cai, and Yun-Song Piao. IR Divergence in Inflationary Tensor Perturbations from Fermion Loops. *Phys. Rev. D*, 86:103515, 2012.

[153] Kaixi Feng and Yun-Song Piao. Gravitational waves induced by spinor fields. *Phys. Rev. D*, 92:023535, 2015.

[154] Marco Peloso and Lorenzo Sorbo. Preheating of massive fermions after inflation: Analytical results. *JHEP*, 05:016, 2000.

[155] G. F. Giudice, M. Peloso, A. Riotto, and I. Tkachev. Production of massive fermions at preheating and leptogenesis. *JHEP*, 08:014, 1999.

[156] Daniel G. Figueroa and Tuukka Meriniemi. Stochastic Background of Gravitational Waves from Fermions – Theory and Applications. *JHEP*, 10:101, 2013.

[157] Vinod B. Johri. Phantom cosmologies. *Phys. Rev. D*, 70:041303, 2004.

[158] R. R. Caldwell. A Phantom menace? *Phys. Lett. B*, 545:23–29, 2002.

[159] Parampreet Singh, M. Sami, and Naresh Dadhich. Cosmological dynamics of phantom field. *Phys. Rev. D*, 68:023522, 2003.

[160] T. Prokopec and Richard P. Woodard. Production of massless fermions during inflation. *JHEP*, 10:059, 2003.

[161] Bjorn Garbrecht and Tomislav Prokopec. Fermion mass generation in de Sitter space. *Phys. Rev. D*, 73:064036, 2006.

[162] L. D. Duffy and R. P. Woodard. Yukawa scalar self-mass on a conformally flat background. *Phys. Rev. D*, 72:024023, 2005.

[163] Ilya L. Shapiro. Covariant derivative of fermions and all that. 11 2016.

[164] Ankit Dhanuka. Fluctuations in the stress energy tensor of spinor fields evolving in general FRW spacetimes. *Phys. Rev. D*, 106(2):023518, 2022.

[165] *NIST Digital Library of Mathematical Functions*. http://dlmf.nist.gov/, Release 1.1.4 of 2022-01-15. F. W. J. Olver, A. B. Olde Daalhuis, D. W. Lozier, B. I. Schneider, R. F. Boisvert, C. W. Clark, B. R. Miller, B. V. Saunders, H. S. Cohl, and M. A. McClain, eds.

[166] Álvaro M. Alhambra, Achim Kempf, and Eduardo Martín-Martínez. Casimir forces on atoms in optical cavities. *Phys. Rev. A*, 89(3):033835, 2014.

[167] Eduardo Martin-Martinez, Miguel Montero, and Marco del Rey. Wavepacket detection with the Unruh-DeWitt model. *Phys. Rev. D*, 87(6):064038, 2013.

[168] Hari K and Dawood Kothawala. Effect of tidal curvature on dynamics of accelerated probes. *Phys. Rev. D*, 104(6):064032, 2021.

[169] Bjorn Garbrecht and Tomislav Prokopec. Unruh response functions for scalar fields in de Sitter space. *Class. Quant. Grav.*, 21:4993–5004, 2004.

[170] Bjorn Garbrecht and Tomislav Prokopec. Energy density in expanding universes as seen by Unruh's detector. *Phys. Rev. D*, 70:083529, 2004.

[171] Kushal Chakraborty and Bibhas Ranjan Majhi. Detector response along null geodesics in black hole spacetimes and in a Friedmann-Lemaitre-Robertson-Walker Universe. *Phys. Rev. D*, 100(4):045004, 2019.

[172] Masahiro Hotta, Achim Kempf, Eduardo Martín-Martínez, Takeshi Tomitsuka, and Koji Yamaguchi. Duality in the dynamics of Unruh-DeWitt detectors in conformally related spacetimes. *Phys. Rev. D*, 101(8):085017, 2020.

[173] Md. Sabir Ali, Sourav Bhattacharya, and Kinjalk Lochan. Unruh-DeWitt detector responses for complex scalar fields in de Sitter spacetime. *JHEP*, 03:220, 2021.

[174] Aindriú Conroy. Unruh-DeWitt detectors in cosmological spacetimes. *Phys. Rev. D*, 105(12):123513, 2022.

[175] D. Jaffino Stargen and Kinjalk Lochan. Cavity Optimization for Unruh Effect at Small Accelerations. *Phys. Rev. Lett.*, 129(11):111303, 2022.

[176] Ankit Dhanuka and Kinjalk Lochan. Unruh dewitt probe of late time revival of quantum correlations in friedmann spacetimes. *arXiv:2210.11186 (Acceptd in Physical Review D)*, 2022.

[177] Masafumi Fukuma, Yuho Sakatani, and Sotaro Sugishita. Master equation for the Unruh-DeWitt detector and the universal relaxation time in de Sitter space. *Phys. Rev. D*, 89(6):064024, 2014.

[178] Zehua Tian and Jiliang Jing. Geometric phase of two-level atoms and thermal nature of de Sitter spacetime. *JHEP*, 04:109, 2013.

[179] Zehua Tian and Jiliang Jing. Dynamics and quantum entanglement of two-level atoms in de Sitter spacetime. *Annals Phys.*, 350:1, 2014.

[180] Alexander Vilenkin and L. H. Ford. Gravitational Effects upon Cosmological Phase Transitions. *Phys. Rev.*, D26:1231, 1982.

[181] A. D. Dolgov, M. B. Einhorn, and Valentin I. Zakharov. On Infrared effects in de Sitter background. *Phys. Rev.*, D52:717–722, 1995.

[182] Donald Marolf and Ian A. Morrison. The IR stability of de Sitter: Loop corrections to scalar propagators. *Phys. Rev.*, D82:105032, 2010.

[183] Erickson Tjoa and Robert B. Mann. Harvesting correlations in Schwarzschild and collapsing shell spacetimes. *JHEP*, 08:155, 2020.

[184] L. Parker. ONE ELECTRON ATOM AS A PROBE OF SPACE-TIME CURVATURE. *Phys. Rev. D*, 22:1922–1934, 1980.

[185] Eric Poisson. *A Relativist's Toolkit: The Mathematics of Black-Hole Mechanics*. Cambridge University Press, 12 2009.

[186] Liang Dai, Enrico Pajer, and Fabian Schmidt. Conformal Fermi Coordinates. *JCAP*, 11:043, 2015.

[187] Kinjalk Lochan and T. Padmanabhan. Inertial nonvacuum states viewed from the Rindler frame. *Phys. Rev.*, D91(4):044002, 2015.

[188] Kinjalk Lochan. Unequal time commutators in Friedmann universes: deterministic evolution of massless fields. *Gen. Rel. Grav.*, 54(9):100, 2022.

[189] Sougato Bose, Anupam Mazumdar, Gavin W. Morley, Hendrik Ulbricht, Marko Toroš, Mauro Paternostro, Andrew Geraci, Peter Barker, M. S. Kim, and Gerard Milburn. Spin Entanglement Witness for Quantum Gravity. *Phys. Rev. Lett.*, 119(24):240401, 2017.

[190] Chiara Marletto and Vlatko Vedral. Gravitationally-induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity. *Phys. Rev. Lett.*, 119(24):240402, 2017.

[191] Ryan J. Marshman, Anupam Mazumdar, and Sougato Bose. Locality and entanglement in table-top testing of the quantum nature of linearized gravity. *Phys. Rev. A*, 101(5):052110, 2020.

[192] Alessio Belenchia, Robert M. Wald, Flaminia Giacomini, Esteban Castro-Ruiz, Časlav Brukner, and Markus Aspelmeyer. Quantum Superposition of Massive Objects and the Quantization of Gravity. *Phys. Rev. D*, 98(12):126009, 2018.

[193] Rémi Faure, T. Rick Perche, and Bruno de S. L. Torres. Particle detectors as witnesses for quantum gravity. *Phys. Rev. D*, 101(12):125018, 2020.

[194] T. Rick Perche, Boris Ragula, and Eduardo Martín-Martínez. Harvesting entanglement from the gravitational vacuum. 10 2022.

[195] Cliff P. Burgess, R. Holman, and D. Hoover. Decoherence of inflationary primordial fluctuations. *Phys. Rev. D*, 77:063534, 2008.

[196] C. P. Burgess, R. Holman, G. Tasinato, and M. Williams. EFT Beyond the Horizon: Stochastic Inflation and How Primordial Quantum Fluctuations Go Classical. *JHEP*, 03:090, 2015.

[197] Jinn-Ouk Gong and Min-Seok Seo. Quantum non-linear evolution of inflationary tensor perturbations. *JHEP*, 05:021, 2019.