Neutrino Mass in B-L Model and Left-Right Symmetric Model

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Certificate of Examination

This is to certify that the dissertation titled "Neutrino mass in B-L model, Left-Right model" submitted by **Mr. Love Grover** (Registration Number: **MS11011**) for the partial fulfillment of **BS-MS dual degree program** of the Institute, has been examined by the thesis committee duly appointed by the Institute. The committee finds the work done by the candidate satisfactory and recommends that the report be accepted.

Prof J S Bagla	Prof C S Aulakh	Dr Manimala Mitra (supervisor)

April 22, 2016

Declaration

The work presented in this dissertation has been carried out by me under the guidance of Dr Manimala Mitra at the Indian Institute of Science Education and Research Mohali.

This work has not been submitted in part or in full for a degree, a diploma, or a fellowship to any other university or institute. Whenever contributions of others are involved, every effort is made to indicate this clearly, with due acknowledgement of collaborative research and discussions. This thesis is a bonafide record of original work done by me and all sources listed within have been detailed in the bibliography.

Mohali, April 22, 2016

Love Grover

In my capacity as the supervisor of the candidate's project work, I certify that the above statements by the candidate are true to the best of my knowledge.

Mohali, April 22, 2016

Dr Manimala Mitra

Abstract

Neutrino mass has been observed in some experiments. A Beyond Standard Model theory is required to explain the masses of Neutrinos. There are many possibilities out of which the present material is mainly focused on B-L Model and Left-Right Symmetric Model. Both models are introduced along with comparisons with Standard Model. An introduction to Neutrinoless Double Beta Decay is presented. Also the Effective Neutrino Mass in the standard mechanism is calculated explicitly.

The Neutrinoless double beta decay can be mediated by many possible ways [Cha+12]

1 with W_Rs only with light neutrino exchange. (only e_L as final particle)

2 with W_Rs only with heavy neutrino exchange. (only e_R as final particle)

3 with W_Ls only with light neutrino exchange. (only e_L as final particle)

4 with W_Ls only with heavy neutrino exchange. (only e_R as final particle)

5 with W_R, W_L and with light neutrino exchange. (e_L and e_R as final particle)

6 with W_R, W_L and with heavy neutrino exchange. (e_L and e_R as final particle)

Only two of the above have a significant contribution, which are shown in figure 5.1 and 5.9. We analyzed only these parts in different hierarchies. We calculated

dependence of Dirac and Majorana phases on effective mass by plotting effective mass (due to above mentioned both Feynman diagrams) vs lightest mass in the normal hierarchy and inverted hierarchy.

Acknowledgement

I would like to express my gratitude to my supervisor Dr Manimala Mitra for the useful comments, remarks and engagement through the learning process of this master thesis. I would also like to thank my parents for their endless love and support.

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Notations and Conventions

The references in present material is shown for figure as “fig. 2.1", for equation as “2.9" and for some section as “§3"

Einstein summation convention is implied where index is repeated except where defined otherwise.

The following notions are implied except where defined otherwise.

$\delta_{\alpha\beta}$	Kronecker-Delta function
δ_{x_0} or $\delta(x - x_0)$	Dirac-Delta function
A	Atomic Mass Number
Z	Atomic Number
N	Neutron Number
$\alpha, \beta, \gamma, \ldots$	index for mass Eigenstate
i,j,k,...	index for flavor Eigenstate (i.e., e, μ, τ)
$\mathcal{T}$	Time Order Operator
c_{ij}	$\cos(\theta_{ij})$
s_{ij}	$\sin(\theta_{ij})$
ch_{ij}	$\cosh(\theta_{ij})$
sh_{ij}	$\sinh(\theta_{ij})$
P	Parity Operator
C	Charge Conjugation Operator
T	Time Reversal Operator
$\Re e$	Real Part extraction operator
$\Im m$	Imaginary Part extraction operator

Contents

1

Introduction

The Standard Model is the most successful attempt of humankind to understand the fundamental laws of nature. In standard model the particles obtain masses by the Higgs mechanism i.e., Electroweak Symmetry Breaking by adding a scalar Higgs Field. After the EWSB the $SU(2)_L \times U(1)_Y$ symmetry is broken down to $U(1)_{EM}$. The four generators break down to three gauge boson ($W^{\pm}$and Z^0) and the fourth is Standard model Higgs state h. Fermion masses are generated by Yukawa interaction with Higgs field.

Quantum gravity is not incorporated in Standard Model. To solve this we need to work at Plank-scale and for now we can leave this problem. [GC+12] In Standard Model we set up the neutrinos to be different from other fermions {charged leptons (e, μ, τ) & quarks (u, s, t, d, c, b)}. We set up the system such that the neutrinos remain massless. But after some predictions of neutrino mass theoretically, we have observations of neutrino oscillations, as discussed below, suggesting massive neutrinos.

We need some extension to the standard model and there are numerous possibilities by the extension of Standard Model, and we will follow more than one option and later stick to one of them to focus our work.

1.1 Motivation and Problem Statement

To extend the standard model, we can use two different approaches - top-down approach and the bottom up approach. The top-down approach involves selecting a gauge group which has the an embedding of the Standard model's symmetry group. After selecting the gauge group, the study of its phenomenological consequences comprise the top-down approach.[Pru11] This can get us to the Standard model as effective low energy scale theory. In the bottom-up approach one can add some minimal extension to the Standard model. We know some extensions of model in bottom-up approach and here we discuss the B-L model. Then we also consider the Left-Right symmetric model. These models lead us to the possibility of Neutrinoless double beta decay.

We need to understand the phenomenological consequences regarding the Neutrinoless double beta decay in left-right symmetric model. The dependence of Decay rate of the 4 possible Feynman diagrams on Nuclear Mass Matrix, Phase Space Factor & effective Neutrino mass is analyzed. The analysis of effective mass with different hierarchy.

2

Neutrino Oscillations

The similar case of Kaons

The states K^0 and $\hat{K}^0$ have well defined strangeness, but they are not the physical states. There are two physical states (mass eigen states) K_L and K_S are a superposition of K^0 and $\hat{K}^0$ and vice-versa. If K^0 is produced at some event and we know it is a superposition of K_L and K_S (and same for $\hat{K}^0$). The evolution of physical states (K_L and K_S) are different where K_S decays quickly. Starting from K^0 or $\hat{K}^0$, after short time leads to the state K_L i.e., equal mixture of K^0 and $\hat{K}^0$. [RP91] There is also some same oscillation character exists in Majorana Particle.

2.1 General Case of Neutrino Oscillations

Considering the ν_α is stable state. Here we are also assuming the propagating beam have different stable states with the *same 3-momentum*. Since the masses are different, the energies associated with each mass eigenstate should be different.[RP91]

$$|\nu_l\rangle = \sum_\alpha U_{l\alpha} |\nu_\alpha\rangle \tag{2.1}$$

whereas the time evolution is given by following:

$$|\nu_l(t)\rangle = \sum_\alpha e^{iE_\alpha t} U_{l\alpha} |\nu_\alpha\rangle \tag{2.2}$$

we start by calculating the value of $\langle \nu_{l'}|\nu_l(t)\rangle$, which is the amplitude of conversing from $l \to l'$ in t time.

$$
\begin{aligned}
\langle \nu_{l'}|\nu_l(t)\rangle &= \sum_{\alpha,\beta} \left\langle \nu_\beta \right| U^\dagger_{\beta l'} e^{iE_\alpha t} U_{l\alpha} \left|\nu_\alpha\right\rangle \\
&= \sum_{\alpha,\beta} e^{iE_\alpha t} U^\dagger_{\beta l'} U_{l\alpha} \left\langle \nu_\beta \middle| \nu_\alpha \right\rangle \\
&= \sum_{\alpha,\beta} e^{iE_\alpha t} U^\dagger_{\beta l'} U_{l\alpha} \delta_{\alpha\beta}
\end{aligned}
$$

$$
\langle \nu_{l'}|\nu_l(t)\rangle = \sum_{\alpha} e^{iE_\alpha t} U^*_{l'\alpha} U_{l\alpha} \tag{2.3}
$$

The probability of finding in l' state on measurement after 't' time evolution, starting from state l it is denoted by $P(l \to l', t)$

$$
\begin{aligned}
P(l \to l', t) &= |\langle \nu_{l'}|\nu_l(t)\rangle|^2 \\
&= \sum_{\alpha,\beta} \left| U^*_{l'\beta} U_{l\beta} U^*_{l'\alpha} U_{l\alpha} \right| \cos\left[(E_\alpha - E_\beta)t - \arg(U^*_{l'\beta} U_{l\beta} U^*_{l'\alpha} U_{l\alpha})\right]
\end{aligned} \tag{2.4}
$$

The following **Approximations** and **Definitions** is applied in further work unless explicitly said otherwise.

Approximations

The neutrinos in practical situations have high velocities. Time spend, t, can be approximated by beam traveled distance, x. Now P will become the function of x instead of t which is stipulated as per our practical knowledge. To get the oscillation form we need to approximate $E^2 = p^2 + m^2$ in relativistic limit as [Zub12]

$$
E_\alpha \approx \|\boldsymbol{p}\| + \frac{m_\alpha^2}{2\|\boldsymbol{p}\|} \tag{2.5}
$$

We can put Neutrino energy, $E \approx \|\boldsymbol{p}\|$ in final expression for relativistic neutrinos as

$$
E_\alpha \approx E + \frac{m_\alpha^2}{2E} \tag{2.6}
$$

Definitions

Mass square difference, $\Delta m^2_{\alpha\beta}$

$$\Delta m^2_{\alpha\beta} := m^2_\alpha - m^2_\beta \tag{2.7}$$

Oscillation lengths, $X_{\alpha\beta}$

$$X_{\alpha\beta} := \frac{4\pi E}{\Delta m^2_{\alpha\beta}} \tag{2.8}$$

Further calculations

Probability $P(l \to l', x)$ with approximations become

$$P(l \to l', x) = \sum_{\alpha,\beta} \left| U^*_{l'\beta} U_{l\beta} U^*_{l'\alpha} U_{l\alpha} \right| \cos \left[\left(\frac{2\pi}{X_{\alpha\beta}} \right) x - \arg(U^*_{l'\beta} U_{l\beta} U^*_{l'\alpha} U_{l\alpha}) \right] \tag{2.9}$$

We can write it in other form as

$$P(l \to l', x) = \sum_\alpha |U_{l\alpha} U^*_{l'\alpha}|^2 + 2\,\Re\mathrm{e} \sum_{\beta>\alpha} U_{l'\beta} U^*_{l\beta} U^*_{l'\alpha} U_{l\alpha}\, \mathrm{e}^{-i\frac{\Delta m^2_{\alpha\beta}}{2E}x} \tag{2.10}$$

Using B.4 and 2.10

$$\begin{aligned} P(l \to l', x) = \sum_\alpha |U_{l\alpha} U^*_{l'\alpha}|^2 + 2\,\Re\mathrm{e} \sum_{\beta>\alpha} U_{l'\beta} U^*_{l\beta} U^*_{l'\alpha} U_{l\alpha} \\ +2 \sum_{\beta>\alpha} \Re\mathrm{e} \left(U_{l'\beta} U^*_{l\beta} U^*_{l'\alpha} U_{l\alpha} \right) \left[\Re\mathrm{e} \left(\mathrm{e}^{-i\frac{\Delta m^2_{\alpha\beta}}{2E}x} \right) - 1 \right] \\ -2 \sum_{\beta>\alpha} \Im\mathrm{m} \left(U_{l'\beta} U^*_{l\beta} U^*_{l'\alpha} U_{l\alpha} \right) \Im\mathrm{m} \left(\mathrm{e}^{-i\frac{\Delta m^2_{\alpha\beta}}{2E}x} \right) \end{aligned} \tag{2.11}$$

Since U is a unitary matrix, using B.3, we can write

$$\begin{aligned} P(l \to l', x) = \delta_{ll'} - 4 \sum_{\beta>\alpha} \Re\mathrm{e} \left(U_{l'\beta} U^*_{l\beta} U^*_{l'\alpha} U_{l\alpha} \right) \sin^2 \left(\frac{\Delta m^2_{\beta\alpha}}{4E} x \right) \\ +4 \sum_{\beta>\alpha} \Im\mathrm{m} \left(U_{l'\beta} U^*_{l\beta} U^*_{l'\alpha} U_{l\alpha} \right) \sin \left(\frac{\Delta m^2_{\beta\alpha}}{4E} x \right) \cos \left(\frac{\Delta m^2_{\beta\alpha}}{4E} x \right) \end{aligned} \tag{2.12}$$

In CP-Conserving case (since U is real and real analogue of unitary matrix is orthogonal matrix) it becomes

$$P(l \to l', x) = \delta_{ll'} - 4 \sum_{\beta > \alpha} U_{l'\beta} U_{l\beta} U_{l'\alpha} U_{l\alpha} \sin^2 \left(\frac{\Delta m^2_{12}}{4E} x \right) \tag{2.13}$$

A two flavor case

For two flavor case (CP-conserving) the U matrix can be written in following form

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{2.14}$$

Where the definition of U is in 2.1. Without CP-violation phases, in two flavor case, 2.9 takes the form (No effect is on the Neutrino oscillation results due to Majorana phases and can be seen from [Giu10])

$$P(1 \to 2, x) = \sum_{\alpha,\beta} U_{2\beta} U_{1\beta} U_{2\alpha} U_{1\alpha} \cos\left(\frac{2\pi x}{X_{\alpha\beta}} \right) \tag{2.15}$$

Complex conjugate doesn't change anything because U is real

$$\begin{aligned} P(1 \to 2, x) = U_{21}U_{11}U_{21}U_{11} + U_{21}U_{11}U_{22}U_{12} \cos\left(\frac{2\pi x}{X_{12}} \right) \\ + U_{22}U_{12}U_{21}U_{11} \cos\left(\frac{2\pi x}{X_{21}} \right) + U_{22}U_{12}U_{22}U_{12} \end{aligned}$$

from 2.14

$$\begin{aligned} P(1 \to 2, x) = (-\sin\theta)(\cos\theta)(-\sin\theta)(\cos\theta) + (-\sin\theta)(\cos\theta)(\cos\theta)(\sin\theta) \cos\left(\frac{2\pi x}{X_{12}} \right) \\ + (\cos\theta)(\sin\theta)(-\sin\theta)(\cos\theta) \cos\left(\frac{2\pi x}{X_{21}} \right) + (\cos\theta)(\sin\theta)(\cos\theta)(\sin\theta) \end{aligned}$$

$$P(1 \to 2, x) = 2\sin^2\theta \cos^2\theta \left(1 - \cos\left(\frac{2\pi x}{X_{21}} \right) \right)$$

Finally, we got the approximated form of conversing probability in two flavor case in oscillating form

$$P(1 \to 2, x) = \sin^2 2\theta \sin^2 \left(\frac{\pi x}{X_{21}} \right) \tag{2.16}$$

$$P(1 \to 2, x) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m_{12}^2}{4E} x \right) \tag{2.17}$$

Apparently, the survival probability is

$$P(1 \to 1, x) = 1 - \sin^2 2\theta \sin^2 \left(\frac{\pi x}{X_{21}} \right) \tag{2.18}$$

$$P(1 \to 1, x) = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m_{12}^2}{4E} x \right) \tag{2.19}$$

Clearly from 2.17 and 2.19, we can see that the converting and surviving probabilities are dependent on mass square difference and mixing angle. Probabilities oscillates as the particle moves some distance. This was the case of two flavor oscillations, to get a real world picture where 3 flavors are mixed we have done some calculations in following part.

A three flavor case

The U matrix in three flavor case is

$$U = \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{13} & s_{13}e^{i\delta} \\ s_{12}c_{23} + c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & -s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \mathrm{diag}(1, e^{i\alpha}, e^{i\beta}) \tag{2.20}$$

Where δ is CP violation phase and α & β are Majorana phases

Whereas, the CP-conserving case with three flavor have U matrix that will take the form [Giu10][Zub12]

$$U = \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{13} & s_{13} \\ s_{12}c_{23}+c_{12}s_{23}s_{13} & c_{12}c_{23}-s_{12}s_{23}s_{13} & -s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13} & c_{12}s_{23}+s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \tag{2.21}$$

Some more *Approximations* and their analysis in three flavor

Case 1

Approximation

Consider the case of one mass square difference to be very small compare to other. Since we don't know the sign of one mass difference, there are three possible mass hierarchy with known values and approximation out of which two are shown in fig. 2.1

$$\Delta m_{12}^2 = \Delta m_{sol.}^2 \ll \Delta m_{13}^2 \approx \Delta m_{23}^2 = m_{atm.}^2 \tag{2.22}$$

The third can be shown as

$$\Delta m_{12}^2 \approx \Delta m_{13}^2 \approx \Delta m_{23}^2 \tag{2.23}$$

In fig. 2.1 the m_1, m_2 and m_3 are masses of mass Eigen-State of Neutrinos.

The 2.22, 2.12 and 5.15 lead us to the following oscillation results of three flavor system

$$P(\nu_e \to \nu_\mu, x) = \sin^2(2\theta_{13})\sin^2(\theta_{23})\sin^2\left(\frac{\Delta m_{atm}^2 x}{4E}\right) \tag{2.24}$$

We can see from 2.13

$$P(\nu_e \to \nu_\tau, x) = \sin^2(2\theta_{13})\cos^2(\theta_{23})\sin^2\left(\frac{\Delta m_{atm}^2 x}{4E}\right) \tag{2.25}$$

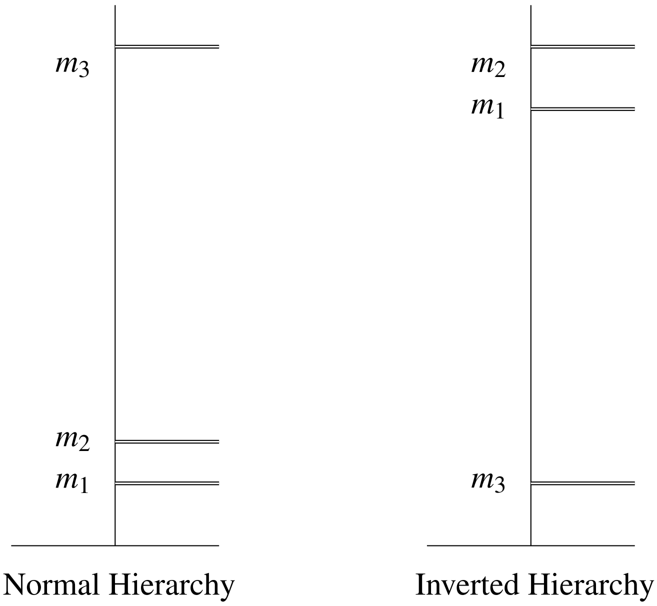

Fig. 2.1.: Possible mass hierarchy with known mass differences

[GF96]

$$P(\nu_e \rightarrow \nu_e, x) = 1 - \sin^2(2\theta_{13}) \sin^2 \left(\frac{\Delta m^2_{atm} x}{4E} \right) \tag{2.26}$$

3

Mass

Majorana mass is differently defined from Dirac mass. The Majorana and Dirac mass terms can appear in Lagrangian as follows:

Majorana mass term

$$-\frac{1}{2}m(\nu_L)^T C^{-1} \nu_L \tag{3.1}$$

whereas,

Dirac mass term

$$-\frac{1}{2}m\bar{\Psi}\Psi \tag{3.2}$$

And in two component spinor with $\Psi = \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix}$,

Dirac mass in two component spinor

$$-\frac{1}{2}m\{(\Psi_R)^\dagger \Psi_L + (\Psi_L)^\dagger \Psi_R\} \tag{3.3}$$

In the following section it can be found that the there is no difference in Particle and Anti-Particle in case of Majorana Particle which in case of Dirac particle is different.

3.1 Majorana vs Dirac Mass

Case of Dirac Particle

Consider a left handed massive spin half particle moving in positive-x direction. By definition the x-component of the spin is $+\frac{1}{2}$. Particle is massive i.e., there can be a frame in which particle moving in negative-x direction. Now particle have spin in opposite direction to the velocity i.e., it is right-handed in this frame.

There are 2 right-handed particle but clearly one of them has opposite charge and since charge is independent of frame therefore only one of them is possible, which is shown in following example.

Initial frame	e_L moving in positive-x direction and with spin in same direction.
Changed frame	particle moving in opposite direction and spin and charge must be the same as in initial frame.

The particle in new frame

It's either $\hat{e}_R$ or e_R. But $\hat{e}_R$ has the opposite charge of e_L. This means particle in new frame is e_R.

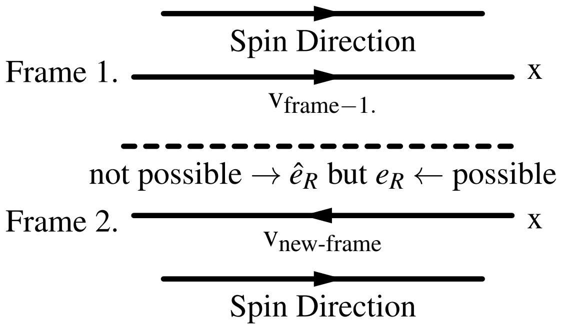

Fig. 3.1.: Observing particle from different frames

Weyl (Massless) particle

Clearly for massless particle, which moves by speed of light, can have only one handedness since you can never jump to a frame that can reverse the direction of moving particle. The handedness cannot be changed. To define above feature there is a defining feature Helicity. (which can't be changed of a particle by moving to some other frame)

Positive Helicity	Spin same in direction of velocity.
Negitive Helicity	Spin opposite in direction of velocity.

New Possibility (Majorana Mass)

Consider a massive particle with no Right-handed particle moving along positive-x direction, New boosted frame make the direction of particle opposite, and no Right-handed conjugate particle in first place are in our consideration. There was $\hat{v}_R$ which was not allowed in case of charged particle (in above discussion). Here if the particle is chargeless and even if there is mass, the $\hat{v}_R$ can become the possible solution.

The Neutrinos are chargeless, massive, with no Right-handed neutrino observed. It is possible that there is a Majorana mass term in Lagrangian for Neutrinos and Neutrinos are Majorana Particle.

3.2 Propagator of Majorana mass

The propagator for Dirac particle is same as the Majorana particle except the fact that the propagator is defined as two point 'Green's Function' and can be written in other ways too, because of the fact that it is its own antiparticle. Consider the following [RP91]

$$\langle 0| \mathcal{T}\left(\psi_A(x)\bar{\psi}_B(y)\right)|0\rangle = \int \frac{\mathrm{d}^4 \boldsymbol{p}}{(2\pi)^4} \, \mathrm{e}^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})} \left[iS_F(p)\right]_{AB} \tag{3.4}$$

where,

$$S_F(p) = \frac{\not{p}+m}{p^2-m^2+i\varepsilon}$$

We can see that a two-point function in following that if it is Dirac type then leads to zero

$$\langle 0|\,\mathcal{T}\left(\psi_A(x)\psi_B(y)\right)|0\rangle$$

But not in case of Majorana particle because Majorana field operator can create and annihilate a particle so matrix element of type $\langle 0|\,\psi_A\psi_B\,|0\rangle$ are non-zero, while matrix element of this type are zero in Dirac case. We know that

$$\bar{\psi} = \lambda\bar{\bar{\psi}} = \lambda\psi^T C^{-1} \tag{3.5}$$

We can write the following using 3.4 & 3.5

$$\begin{aligned}\langle 0|\,\mathcal{T}(\psi_A(x)\psi_B(y))\,|0\rangle &= \lambda^* C_{DB}\,\langle 0|\,\mathcal{T}(\psi_A(x)\bar{\psi}_D(y))\,|0\rangle \\ &= \lambda^*\int\frac{\mathrm{d}^4\boldsymbol{p}}{(2\pi)^4}\,\mathrm{e}^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}\left[iS_F(p)\mathrm{C}\right]_{AB}\end{aligned} \tag{3.6}$$

This implies

$$S_{\psi\psi}(p) = \lambda^* S_F(p)C \tag{3.7}$$

3.3 Seesaw Mechanism

3.3.1 Introduction

We can start from a Weinberg d=5 operator having lepton number violation by 2 for Neutrinoless double beta decay [Ble+10]

$$\frac{c_{\alpha\beta}}{\Lambda}\left(\overline{L^c}_\alpha\tilde{\phi}^*\right)\left(\tilde{\phi}^\dagger L_\beta\right)+h.c. \tag{3.8}$$

Where, ϕ is the SM Higgs field with $\tilde{\phi} = i\tau_2\phi^*$. Λ is the scale of new physics that gives rise to the operator. This is the only d=5 operator can be made from SM particle content respecting both Lorentz and gauge invariance [Ble+10]. Clearly, after the Higgs mechanism this lead us to the Majorana mass term for neutrinos ($c_{ll'}\left(\overline{\nu_{lL}^c}\nu_{l'L}\right)$) suppressed by scale Λ. There are three possible extension. All these extra degree of freedom required to introduce the Majorana nature of neutrinos and can also contribute to the "Neutrinoless double beta decay".

3.3.2 Type-I

We here introduce the Right handed neutrino, N_l as fermion gauge singlet and some of the terms comes out of extension are [RP91]

$$\mathcal{L}_{ext} = -\frac{1}{2}\overline{N_l}(M_N)_{ll'}N_{l'}^c - (Y_N)_{lm}\overline{N_l}\tilde{\phi}^\dagger L_m + h.c. \tag{3.9}$$

This lead us to a mass matrix

$$M_\nu = \begin{pmatrix} 0 & \frac{Y_n v}{\sqrt{2}} \\ \frac{Y_n^T v}{\sqrt{2}} & M_N \end{pmatrix} \tag{3.10}$$

Diagonalization lead us to the light neutrino mass and heavy sterile neutrino mass. [GC+12]

3.3.3 Type-II

In Type-II seesaw model we introduce a scalar $SU(2)$ triplet with hypercharge 2. And using $Q = T_3 + \frac{Y}{2}$ [RP91][Ble+10]

$$\Delta = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta^+ & \delta^{++} \\ \delta^0 & -\frac{1}{\sqrt{2}}\delta^+ \end{pmatrix} \tag{3.11}$$

In this case the Yukawa terms are

$$\mathcal{L}_{ext} = -(Y_\Delta)_{ll'}\overline{L^c}_l i\tau_2 \Delta L_{l'} + h.c. \tag{3.12}$$

Scalar Triplet Δ is coupled to the Higgs field with μ coupling constant. After Electroweak Symmetry breaking, $v_\Delta = \frac{\mu v^2}{2M_\Delta^2}$. Triplet VEV induces a Majorana neutrino mass m_ν^Δ

$$m_\nu^\Delta = 2Y_\Delta v_\Delta = Y_\Delta \frac{muv^2}{M_\Delta^2} \tag{3.13}$$

3.3.4 Type-III

We introduce a fermionic triplet in making of III type seesaw mechanism with zero hypercharge [Ble+10]

$$\Sigma = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ \\ \Sigma^- & \frac{\Sigma^0}{\sqrt{2}} \end{pmatrix} \tag{3.14}$$

Some of the Lagrangian terms on extending with triplets are as follows

$$\mathcal{L}_{ext} = -\frac{1}{2}(M_\Sigma)_{ll'}\mathrm{Tr}(\overline{\Sigma_l}\Sigma^c_{l'}) - (Y_\Sigma)_{lm}\tilde{\phi}^\dagger\overline{\Sigma_l} i\tau_2 L_m + h.c. \tag{3.15}$$

This setting is analogous to type-I with the neutral component of the triplet playing the role of the right handed neutrino. The situation then reduces to the one for the type-II seesaw instead the old thing replaces with

$$m_\nu^\Delta \longrightarrow m\nu^\Sigma = \frac{v^2}{2}Y_\Sigma^T M_\Sigma^{-1} Y_\Sigma \tag{3.16}$$

3.3.5 Mixed seesaw mechanism

We can get heavy and light eigenstate from adding type-II or type-III with type-I sterile neutrinos in light regime.

$$M_\nu = \begin{pmatrix} m^{\Delta,\Sigma} & Y_N \frac{v}{\sqrt{2}} \\ Y_N \frac{v}{\sqrt{2}} & M_N \end{pmatrix} \tag{3.17}$$

4

Beyond Standard Model Theories

4.1 Introduction to B-L model

In order to use bottom-up approach we start our analysis with B-L model which extend the minimal standard model by exploiting the accidental symmetries of Standard model viz. conservation of Lepton number (L) and Baryon number (B) as well as the difference of B and L. We can use the $B-L$ symmetry in gauge form as $U(1)_{B-L}$ with addition of right-handed neutrinos an one complex scalar field. [Pru11] We can add this symmetry minimally to form an extended gauge group as

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \tag{4.1}$$

We will specify the new charges and the changed charges, otherwise the charges are same as in Minimal Standard model.

How it is minimal?

a. Minimal in gauge sector by only adding one $U(1)$ symmetry to minimal Standard Model. This will give rise to a new gauge boson.

b. We just have to add only one sector to the Standard Model Fermionic sector which is right handed neutrino sector.

c. We only need to add only one more complex scalar Higgs field.

This will lead us to

a. an anomaly-free and gauge invariant model.

b. light mass neutrinos generated by seesaw mechanism and already discussed in last Chapter.

We need to introduce two new things as follows

a. Right-handed neutrino, N (or ν_R) for each generation. (singlet under Standard Model gauge group)

b. Complex scalar field, χ. (singlet under whole gauge group except $U(1)_{B-L}$)

The symmetry breaking is as follows in this model

$$\begin{gathered} SU(2)_L \times U(1)_Y \times U(1)_{B-L} \\ \downarrow \\ U(1)_Q \end{gathered} \tag{4.2}$$

Q : Electromagnetic Charge
Y : Hypercharge

We can write the Lagrangian as sum of different parts as follows (We are not considering the gluons and colour multiplets of quarks)

$$\mathcal{L}_{B-L} = \mathcal{L}_{S_{B-L}} + \mathcal{L}_{YM_{B-L}} + \mathcal{L}_{f_{B-L}} + \mathcal{L}_{Y_{B-L}} \tag{4.3}$$

$$\mathcal{L}_{SM} = \mathcal{L}_{S_{SM}} + \mathcal{L}_{YM_{SM}} + \mathcal{L}_{f_{SM}} + \mathcal{L}_{Y_{SM}} \tag{4.4}$$

S	:	Scalar
YM	:	Yang-Mills
f	:	Fermionic
Y	:	Yukawa
Subsubscript B-L (Subscript B-L)	:	Part of Lagrangian of B-L model (Lagrangian of B-L model)
Subsubscript SM (Subscript SM)	:	Part of Lagrangian of Minimal Standard Model (Lagrangian of Minimal Standard model)

4.1.1 Yang-Mills Part

$$\mathcal{L}_{YM_{B-L}} = \mathcal{L}_{YM_{SM}} - \frac{1}{4}F'^{\mu\nu}F'_{\mu\nu} \tag{4.5}$$

$F'^{\mu\nu}$: $\partial_\mu B'_\nu - \partial_\nu B'_\mu$
B'_ν : Gauge Field associated with group $U(1)_{B-L}$

4.1.2 Scalar Part.

We already mentioned that a new scalar complex Higgs Field needed to give mass to our new contestant Z'. **B-L Charge**

Y^H_{B-L} : 0
Y^χ_{B-L} : +2

The most general gauge invariant scalar part of Lagrangian

$$\mathcal{L}_{S_{B-L}} = (D^\mu H)^\dagger D_\mu H + (D^\mu \chi)^\dagger D_\mu \chi - \text{Potential}(H,\chi)_{B-L} \tag{4.6}$$

$$D_\mu : \partial_\mu + ig_S \mathcal{T}^\alpha G_\mu{}^\alpha + igT^a W_\mu{}^a + ig_1 Y B_\mu + i(\tilde{g}Y + g_1' Y_{B-L}) B_\mu'$$

We assume no mixing in two $U(1)$ gauge parts (viz. $U(1)_Y$ & $U(1)_{B-L}$) and now D_μ can be written as

$$D_\mu : \partial_\mu + ig_S \mathcal{T}^\alpha G_\mu{}^\alpha + igT^a W_\mu{}^a + ig_1 Y B_\mu + ig_1' Y_{B-L} B_\mu'$$

Where,

$$\begin{aligned}
\text{Potential}(H,\chi) &= m^2 H^\dagger H + \mu^2 |\chi|^2 + \begin{pmatrix} H^\dagger H & |\chi|^2 \end{pmatrix} \begin{pmatrix} \lambda_1 & \frac{\lambda_3}{2} \\ \frac{\lambda_3}{2} & \lambda_2 \end{pmatrix} \begin{pmatrix} H^\dagger H \\ |\chi|^2 \end{pmatrix} \\
&= m^2 H^\dagger H + \mu^2 |\chi|^2 + \lambda_1 (H^\dagger H)^2 + \lambda_2 |\chi|^4 + \lambda_3 H^\dagger H |\chi|^2 \\
&= \text{Potential}(H)_{SM} + \mu^2 |\chi|^2 + \lambda_2 |\chi|^4 + \lambda_3 H^\dagger H |\chi|^2
\end{aligned} \tag{4.7}$$

and whole part can be rewritten as,

$$\mathcal{L}_{S_{B-L}} = \mathcal{L}_{S_{SM}} + (D^\mu \chi)^\dagger D_\mu \chi - \mu^2 |\chi|^2 - \lambda_2 |\chi|^4 - \lambda_3 H^\dagger H |\chi|^2 \tag{4.8}$$

4.1.3 Fermionic Part

Clearly, the **B-L Charges** from the definition of Baryons and Leptons are as follows

$$Y_{B-L}^{\text{quarks}} : \tfrac{1}{3}; \qquad Y_{B-L}^{\text{leptons}} : -1,$$

$$\begin{aligned}
\mathcal{L}_{f_{B-L}} = \sum_{i=1}^{3} \big(& i\overline{q_{iL}} \gamma_\mu D^\mu q_{iL} + i\overline{u_{iR}} \gamma_\mu D^\mu u_{iR} + i\overline{d_{iR}} \gamma_\mu D^\mu d_{iR} \\
& + i\overline{l_{iL}} \gamma_\mu D^\mu l_{iL} + i\overline{e_{iR}} \gamma_\mu D^\mu e_{iR} + i\overline{\nu_{iR}} \gamma_\mu D^\mu \nu_{iR} \big)
\end{aligned} \tag{4.9}$$

4.1.4 Yukawa part

$$
\begin{aligned}
\mathcal{L}_{Y_{B-L}} = -y^{d}_{ij}\overline{q_{Li}}d_{Rj}H - y^{u}_{ij}\overline{q_{Li}}u_{Rj}\widetilde{H} - y^{e}_{ij}\overline{l_{Li}}e_{Rj}H \\
-y^{\nu}_{ij}\overline{l_{Li}}\nu_{Rj}\widetilde{H} - y^{M}_{ij}\overline{(\nu_R)^c_i}\nu_{Rj}\chi + h.c.
\end{aligned}
\tag{4.10}
$$

$\widetilde{H} := i\tau_2 H^*$

$$
\mathcal{L}_{Y_{B-L}} = \mathcal{L}_{Y_{SM}} - y^{\nu}_{ij}\overline{l_{Li}}\nu_{Rj}\widetilde{H} - y^{M}_{ij}\overline{(\nu_R)^c_i}\nu_{Rj}\chi + h.c. \tag{4.11}
$$

Clearly, χ needed to have $+2$ $(B-L)$ charge in order to have gauge invariant Lagrangian.

Spontaneous Symmetry Breaking of $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ The Vacuum Expectation Values (VEVs) can be assigned to the two fields as follows

$$
\langle H \rangle \equiv \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \qquad \langle \chi \rangle \equiv \frac{x}{\sqrt{2}} \tag{4.12}
$$

v & x : real and non-negative

4.2 Left-Right Symmetric Model

4.2.1 Introduction

We had, in Standard Model, the gauge group that led to a difference in the character of Left-handed particle and Right-handed particle. We will attempt to go beyond

Standard model by the means of finding the general gauge group framework. The reasons to extend this model is to restore the Parity at high energies that is broken explicitly in Standard Model. This model explain the Neutrino mass existence and predict the Majorana characters of Neutrinos. [Gri93]

4.2.2 The Two Ways to Allow P

First Method

We can keep the gauge group $SU(2) \times U(1)$ and then we can extend by adding a multiplet with opposite chirality that transform same as our old fields. for example, we can define $\psi_{\chi'}$ a multiplet of opposite chirality to ψ_χ. Here we can set,

$$P : \psi_\chi \longrightarrow \gamma^0 \psi_{\chi'} \quad \& \quad \psi_{\chi'} \longrightarrow \gamma^0 \psi_\chi \tag{4.13}$$

Here, Bosonic sector remains same but as we see it needs to extend the Fermionic sector. [Rod11]

If we set $(\psi_{\chi'})^c$ as the two component partner (mirror) to ψ_χ then it will lead us to A.6 & A.7

Second Method

Instead of using the above mirror method we can add another doublet made out of standard model $SU(2)$ singlet. The analysis of this type of extension is done in below section.

4.2.3 Motivation

Motivation for choosing the 4.14 gauge group is

α. This is a very simple and elegant extension of Standard Model gauge Group.

β. The spontaneous symmetry breaking led to the Standard model. The Right-handed part will be broken down and the Lagrangian is again asymmetric between Left & Right handed Multiplets.

γ. The extra introduced fields will lead to Majorana Masses of Neutrino and generate naturally light mass.

4.2.4 Gauge Group

We extend the gauge group minimally in this model as

$$G = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \tag{4.14}$$

New charge system

$$(l,m,n) :: (SU(2)_L \text{ charge}, SU(2)_R \text{ charge}, U(1)_{B-L} \text{ charge}) \tag{4.15}$$

We have to add some fields and some new fields charge as follows

4.2.5 New Fields (and Charge discussion)

Minimal Standard model fields and some new fields with new charge system a The N_l is the l generation right handed neutrino introduces to extend field for left-right symmetric model.

Tab. 4.1.: Lepton Doublets, L and its Quantum Numbers in gauge group of Left-Right Symmetric model

Lepton Doublets	Quantum Numbers
$\begin{pmatrix} \nu_l \\ l_L \end{pmatrix}$	$(2,1,-1)$
$\begin{pmatrix} N_l \\ l_R \end{pmatrix}$	$(1,2,-1)$

Tab. 4.2.: Quark Doublets, Q and its Quantum Numbers in gauge group of Left-Right Symmetric model

Quark Doublets	Quantum Numbers
$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\left(2,1,\frac{1}{3}\right)$
$\begin{pmatrix} u_R \\ d_R \end{pmatrix}$	$\left(1,2,\frac{1}{3}\right)$

4.2.6 Higgs Sector

We start by making a Higgs triplet by using the form $H^T i\tau_2\vec{\tau}H$, where H is the Higgs $SU(2)$ doublet.

Tab. 4.3.: Scalar Bidoublet and its Quantum Numbers in gauge group of Left-Right Symmetric model

Scalar Bidoublet	Quantum Numbers
$\begin{pmatrix} \phi_{11}^0 & \phi_{12}^+ \\ \phi_{21}^- & \phi_{22}^0 \end{pmatrix}$	$(2,2,0)$

Tab. 4.4.: Scalar triplets and its Quantum Numbers in gauge group of Left-Right Symmetric model

Scalar Triplets	Quantum Numbers
$\begin{pmatrix} \Delta_{11,L}^+ & \Delta_{12,L}^{++} \\ \Delta_{21,L}^0 & -\Delta_{22,L}^+ \end{pmatrix}$	$(3,1,2)$
$\begin{pmatrix} \Delta_{11,R}^+ & \Delta_{12,R}^{++,} \\ \Delta_{21,R}^0 & -\Delta_{22,R}^+ \end{pmatrix}$	$(3,1,2)$

We can construct scalar triplet in 2×2 representation $\vec{\Delta_{L,R}} = \begin{pmatrix} \delta_{L,R}^1 \\ \delta_{L,R}^0 \\ \delta_{L,R}^{-1} \end{pmatrix}$ using

$$\Delta_{L,R} = \frac{1}{\sqrt{2}} \vec{\tau} \cdot \vec{\Delta}_{L,R} \tag{4.16}$$

$$\Delta_{L,R} = \frac{1}{\sqrt{2}} \begin{pmatrix} \delta_{L,R}^3 & \delta_{L,R}^1 - i\delta_{L,R}^2 \\ \delta_{L,R}^1 + i\delta_{L,R}^2 & -\delta_{L,R}^3 \end{pmatrix} \tag{4.17}$$

Now we will calculate the charge of each elements of scalar parts (already assigned in tables 4.3 & 4.4)

$$Q = T_{3L} + T_{3R} + \frac{1}{2}(B - L) \tag{4.18}$$

$$Q\Delta = \left[\frac{1}{2}\tau_3, \Delta\right] + \frac{1}{2}\Delta_{(B-L)} \tag{4.19}$$

Where, $\Delta_{(B-L)}$ is the B-L charge of Δ and we assigned it 2 for triplets, $\Delta_{L,R}$ and 0 for Bi-doublet, ϕ.

$$Q\Delta = \frac{1}{2} \cdot \begin{pmatrix} 0 \cdot \Delta_{11} & 2 \cdot \Delta_{12} \\ -2 \cdot \Delta_{21} & 0 \cdot \Delta_{22} \end{pmatrix} + \frac{1}{2} \cdot 2 \cdot \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{pmatrix} \tag{4.20}$$

We have $\Delta_{22} = -\Delta_{11}$ from 4.17. We can write charge elements as

$$Q\Delta = \begin{pmatrix} +1 \cdot \Delta_{11} & +2 \cdot \Delta_{12} \\ 0 \cdot \Delta_{21} & +1 \cdot \Delta_{22} \end{pmatrix} = \begin{pmatrix} \Delta_{11}^{+} & \Delta_{12}^{++} \\ \Delta_{21}^{0} & -\Delta_{11}^{+} \end{pmatrix} \tag{4.21}$$

Now we can go back to our $\vec{\Delta}$ and write it as

$$\vec{\Delta} = \begin{pmatrix} \delta^1 \\ \delta^2 \\ \delta^3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(\Delta^{++} + \Delta^{0}\right) \\ \frac{i}{\sqrt{2}}\left(\Delta^{++} - \Delta^{0}\right) \\ \Delta^{+} \end{pmatrix} \tag{4.22}$$

Similar analysis can be done with Bi-Doublet, ϕ scalar part too and the analysis will go as follows (with $\phi_{(B-L)}$ B-L charge of ϕ and zero in Left-Right Symmetric model)

$$Q\phi = \left[\frac{1}{2}\tau_3, \phi\right] + \frac{1}{2}\phi_{(B-L)} \tag{4.23}$$

This lead us to

$$Q\phi = \begin{pmatrix} 0\cdot\phi_{11} & +\phi_{12} \\ -\phi_{21} & 0\cdot\phi_{22} \end{pmatrix} \tag{4.24}$$

The charge distribution from above is

$$\begin{pmatrix} \phi_{11}^{0} & \phi_{12}^{+} \\ \phi_{21}^{-} & \phi_{22}^{0} \end{pmatrix} \tag{4.25}$$

Clearly, from 4.21 & 4.25 we can see the chargeless components of Higgs-sector and name them as $u_{L,R}$ of triplet and u, v of Bi-Doublet in in vacuum expectation value as discussed below

Vacuum Expectation Values, VEVs

The vacuum expectation given to Higgs Triplets as

$$\langle\Delta_{L,R}\rangle_0 = \begin{pmatrix} 0 & 0 \\ u_{L,R} & 0 \end{pmatrix} \tag{4.26}$$

and the vacuum expectation values given to Bi-Doublet as

$$\langle\phi\rangle_0 = \begin{pmatrix} v & 0 \\ 0 & w \end{pmatrix} \tag{4.27}$$

4.2.7 Symmetry Breaking

The symmetry breaking can be think in the following way

$$\begin{array}{c} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ \Big\downarrow{\scriptstyle u_L,u_R} \\ SU(2)_L \times U(1)_Y \\ \Big\downarrow{\scriptstyle v,w} \\ U(1)_Q \end{array} \tag{4.28}$$

We will assume that the Higgs is Electromagnetism charge conserving. Then the expectation values u_R, u_L & v and u used above is the values assigned to zero charged of field parts only. We get back Electroweak gauge group after symmetry breaking with vacuum expectation value of u_R and u_L.

From here, we can see that the u & v are the expectation value given at low energy scale and the $U(1)_{(B-L)}$ symmetry was already broken. This led to the reasoning of "why ϕ has 0 B-L charge". and "why $\Delta_{L,R}$ has 2 B-L charge"

The $\tilde{\phi}$ used below can be defined as $\tau_2\phi^*\tau_2$.

Transformations

$$\Psi_{L,R} \rightarrow e^{i(B-L)}U_{L,R}\Psi_{L,R} \tag{4.29}$$

$$\phi \rightarrow U_L\phi U_R^\dagger \tag{4.30}$$

$$\tag{4.31}$$

4.3 Lagrangian Parts

The possible Lagrangian parts of Left-Right symmetric model is as follows (We are not considering the gluons and colour multiplets of quarks)

$$\mathcal{L}_{\text{Left-Right}} = \mathcal{L}_{S_{\text{Left-Right}}} + \mathcal{L}_{YM_{\text{Left-Right}}} + \mathcal{L}_{f_{\text{Left-Right}}} + \mathcal{L}_{Y_{\text{Left-Right}}} \tag{4.32}$$

4.3.1 Fermionic Part

The fermionic part of Lagrangian can be written as

$$\begin{aligned}\mathcal{L}_f = \sum_{\psi=Q,L} \Big\{ &\bar{\psi}_L i\gamma^\mu \left(\partial_\mu + ig_L \frac{\vec{\tau}}{2} \cdot \vec{W}_{L\mu} + ig' \frac{B-L}{2} B_\mu\right) \psi_L + \\ &+ \bar{\psi}_R i\gamma^\mu \left(\partial_\mu + ig_R \frac{\vec{\tau}}{2} \cdot \vec{W}_{R\mu} + ig' \frac{B-L}{2} B_\mu\right) \psi_R \Big\}\end{aligned} \tag{4.33}$$

4.3.2 Yukawa part

We can write the Lagrangian that give rise to fermion mass terms [Gri93]

$$\mathcal{L}_y = g_y l_L^T C i\tau_2 \Delta_L l_L + g'_Y l_R^T C i\tau_2 \Delta_R l_R + \tilde{l}_R (y_D + y_L \tilde{\phi}) l_L \tag{4.34}$$

l_l & l_R are left & Right handed fermions doublet under $SU(2)_L$ and $SU(2)_R$.

The above after putting the vacuum expectation value will give us

$$\mathcal{L}_Y = -\bar{l}_L^{\prime}(M_l)_{ij}l_R^{\prime} + h.c., \tag{4.35}$$

Where,

$$M_l = \frac{1}{\sqrt{2}}(wy_D + vy_L) \tag{4.36}$$

M_l become hermitian and in neutrino sector the Lagrangian of this part will become

$$\mathcal{L}_{mass}^{\nu} = -\frac{1}{2}\overline{n}_l M_{ll'}^{\nu} n_{l'}^{c} + h.c. = -\frac{1}{2}\begin{pmatrix} \nu & N \end{pmatrix}\begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}\begin{pmatrix} \nu \\ N \end{pmatrix} + h.c. \tag{4.37}$$

The diagonalization will lead to the masses as

$$m_\nu = M_L - M_D M_R^{-1} M_D^T = g_y v_L - \frac{(|w|^2 + |v|^2)}{v_R} g_D g_y' g_D^T \tag{4.38}$$

Where,

$$g_D := \frac{1}{\sqrt{2}} \frac{vy_D + wy_L}{\sqrt{|w|^2 + |v|^2}} \tag{4.39}$$

The neutrino mass matrix (6×6), W can be represented as

$$W := \begin{pmatrix} V_L^n \\ V_R^n \end{pmatrix} = \begin{pmatrix} U_L & S \\ T & U_R \end{pmatrix} \tag{4.40}$$

We will get a diagonal mass matrix as

$$W^{\dagger} M_{\nu} W^{*} = diag(m_1, m_2, m_3, M_1, M_2, M_3) \tag{4.41}$$

5

Neutrinoless Double Beta Decay

Neutrinoless Double Beta Decay analysis has a lot of parts and we will first derive & analyze the Leptonic matrix elements part of the decay rate.

5.1 Introduction and Possibilities

The motivation of our work here as “search of Neutrinoless Double Beta Decay" ($0\nu\beta\beta$) is that the discovery of “Neutrino oscillation phenomena" implies the massive Neutrino existence where the $0\nu\beta\beta$ discovery is a possible contestant explaining mass of Neutrinos and this ($0\nu\beta\beta$ discovery) can also end the quest of the discovery of first kind of Majorana particle. This is not the only reason as there is an unconfirmed claim on $0\nu\beta\beta$ in ^{76}Ge which is not yet refute.[GC+12] These reasons cause a paradigm shift in Neutrino research focus toward a quest of making an $0\nu\beta\beta$ understanding and creating an experimental quest across experimental groups across the world.

There is a “**standard mechanism**" suggested that it can, in one possible parts, involves light Neutrinos which is shown in figure 5.1.

5.2 Double Beta Decay

Double Beta decay observed and widely known with explanations in known theories (Standard Model). The process is Z protons decays into $Z+2$ protons conserving A. This decay is only possible if the initial Nucleus state is less bound than final Nucleus and both Nucleus state more bound that intermediate (prohibiting the possibility of Beta-Decay).

The Nuclear Pairing force i.e., the even-even is more bound than odd-odd Z and N with same A, make it possible for some elements to eligible for the Double-Beta decay. The decay reaction (of $2\nu\beta\beta$) can be written in the following manner

$$(Z,A) \longrightarrow (Z+2,A) + 2e^- + 2\bar{\nu}_e \tag{5.1}$$

The decay rate range of Double-Beta decay is $10^{-20} - 10^{-18}$ year^{-1}

5.2.1 $0\nu\beta\beta$ and other similar contestant

The Neutrinoless mode of Double beta decay, the reaction can be written as

$$(Z,A) \longrightarrow (Z+2,A) + 2e^- \tag{5.2}$$

Clearly, this violates the lepton conservation by 2-units. According to §**??**, the decay reaction imply the Majorana character of Neutrinos. We did not get any convincing $0\nu\beta\beta$ reaction existence results experimentally. We will focus our work on $0\nu\beta\beta$

but there are three more similar possibilities violating lepton number by two as follows

$$\begin{aligned} \beta^+\beta^+0\nu: &\quad (Z,A) &&\longrightarrow (Z-2,A)+2e^+ \\ \beta^+\mathrm{EC}0\nu: &\quad (Z,A)+e^- &&\longrightarrow (Z-2,A)+e^+ \\ \mathrm{ECEC}0\nu: &\quad (Z,A)+2e^- &&\longrightarrow (Z-2,A) \end{aligned}$$

The part a) is called double positron emission, part b) called single positron emission plus single electron capture and c) is called double electron capture.

5.3 Standard Mechanism for Neutrinoless Double Beta Decay

5.3.1 Leptonic Part Calculations Analysis

We can write the Leptonic part of fig. 5.1 (Standard Mechanism) as

$$\mathcal{M}^{(l)}_{\mu\lambda} = \left[S_{\psi\psi}(q)\right]_{AB}\left[\bar{u}_e(p_1)\gamma_\lambda P_L\right]_A\left[\bar{u}_e(p_2)\gamma_\mu P_L\right]_B - (1\leftrightarrow 2) \tag{5.3}$$

where '(l)' indicates the leptonic part of Amplitude. The above can be written in matrix form as

$$\mathcal{M}^{(l)}_{\mu\lambda} = \left[\bar{u}_e(p_1)\gamma_\lambda P_L\right]\left[S_{\psi\psi}(q)\right]\left[\bar{u}_e(p_2)\gamma_\mu P_L\right]^T - (1\leftrightarrow 2) \tag{5.4}$$

3.7 and 5.4 lead us to

$$\mathcal{M}^{(l)}_{\mu\lambda} = \lambda^*\left[\bar{u}_e(p_1)\gamma_\lambda P_L\right]S_F(p)C\left[\bar{u}_e(p_2)\gamma_\mu P_L\right]^T - (1\leftrightarrow 2) \tag{5.5}$$

Expanding further results

$$\begin{aligned} \mathcal{M}^{(l)}_{\mu\lambda} &= \lambda^* \bar{u}_e(p_1)\gamma_\mu P_L \frac{\not{q} + m_\nu}{q^2 - m_\nu^2}\gamma_\lambda P_R \nu_e(p_2) - (1 \leftrightarrow 2) \\ &= \frac{\lambda^* m_\nu}{q^2 - m_\nu^2}\left[\bar{u}_e(p_1)\gamma_\mu P_L \gamma_\lambda \nu_e(p_2) - (1 \leftrightarrow 2)\right] \end{aligned} \tag{5.6}$$

Clearly, because of B.8 the Leptonic matrix element is proportional to Neutrino mass and so the amplitude of Standard Mechanism. But in the case of Neutrino mixing consider the following analysis. Consider the neutrino ejected from point A and absorbed as anti-neutrino (which is the same as its anti-particle, neutrino) at point B. Now the ejected neutrino is $|\nu_e\rangle$ which can be written as

$$|\nu_e\rangle = \sum_\alpha U_{e\alpha} |\nu_\alpha\rangle \tag{5.7}$$

We can write the matrix as (In following, the summation convention is not applied)

$$m_{ll'} = \langle \nu_l | \mathcal{M}_{ll'} | \nu_{l'} \rangle \tag{5.8}$$

The mass term of Neutrinos in Lagrangian can be written in the following form

$$-\mathcal{L}_{\text{mass}_\nu} = \frac{1}{2}\begin{pmatrix} \langle \nu_e | & \langle \nu_\mu | & \langle \nu_\tau | \end{pmatrix} \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} \\ m_{\tau e} & m_{\tau\mu} & m_{\tau\tau} \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} \tag{5.9}$$

Here, the diognalization leads to the following changes

$$\begin{aligned} -\mathcal{L}_{\text{mass}_\nu} &= \frac{1}{2}\begin{pmatrix} \langle \nu_e | & \langle \nu_\mu | & \langle \nu_\tau | \end{pmatrix} U^\dagger \,\text{Diagonal}(m_{11}, m_{22}, m_{33}) U \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} \\ &= \frac{1}{2}\begin{pmatrix} \langle \nu_1 | & \langle \nu_3 | & \langle \nu_3 | \end{pmatrix} \text{Diagonal}(m_{11}, m_{22}, m_{33}) \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix} \end{aligned} \tag{5.10}$$

In above equation the U is the famous 'PMNS Matrix' and $|\nu_\alpha\rangle$ is the mass Eigenstate and α can take the values $1,2,3$. The following is possible with above analysis

$$\begin{aligned} m_\nu^{ee} &\equiv \langle \nu_e | \mathcal{M}_{ee} | \nu_e \rangle \\ &= (\langle \nu_1 | U_{e1} + \langle \nu_2 | U_{e2} + \langle \nu_3 | U_{e3}) \, \text{Diagonal}(m_{11}, m_{22}, m_{33}) \, (|\nu_1\rangle U_{e1} + |\nu_2\rangle U_{e2} + |\nu_3\rangle U_{e3}) \end{aligned} \tag{5.11}$$

In 5.11, m_ν^{ee} is referred as effective Neutrino Mass and the standard definition is the following (5.12). The following is result with $m_\alpha = m_{\alpha\alpha}$

$$m_\nu^{ee} = \sum_\alpha U_{e\alpha}^2 m_\alpha \tag{5.12}$$

So, the 5.6 can be written in new following form with m_ν^{ee} in it (also, considering the high value of momentum related to the average mass). Clearly, from above analysis the m_ν^{ee} is the average mass.

$$\mathcal{M}_{\mu\lambda}^{(l)} = \frac{m_\nu^{ee}}{q^2} \left[\bar{u}_e(p_1) \gamma_\mu P_L \gamma_\lambda \nu_e(p_2) - (1 \leftrightarrow 2) \right] \tag{5.13}$$

Here a phase λ^* absorbed in m_ν^{ee} and the here it is different from 5.12 and take the following form

$$m_\nu^{ee} = \sum_\alpha \lambda^* U_{e\alpha}^2 m_\alpha \tag{5.14}$$

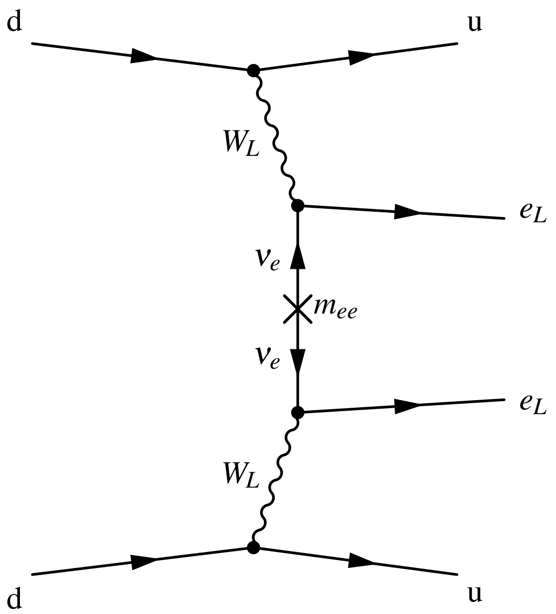

Fig. 5.1.: Feynman Diagram: Neutrinoless double beta decay in standard mechanism

5.4 Analysis of effective mass, m_ν^{ee}

The form of PMNS matrix (Neutrino mixing matrix) is already discussed in §2 is

$$U = \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{13} & s_{13}e^{i\delta} \\ s_{12}c_{23}+c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & -s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23}+s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \mathrm{diag}(1, e^{i\alpha}, e^{i\beta}) \tag{5.15}$$

The above matrix takes above form when charged leptons mass matrix is diagonal.

Neutrino oscillation data is given in Table 5.1. The C.1 contains the 1σ & 3σ ranges of mixing angles and mass square differences. C.1 is the data taken from Particle data group website.

From 5.16 we get

$$m_\nu^{ee} = \sum_\alpha \lambda^* U_{e\alpha}^2 m_\alpha \tag{5.16}$$

and putting the value of matrix give,

$$|m_\nu^{ee}| = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha_2} + m_3 s_{13}^2 e^{2i\alpha_3}| \tag{5.17}$$

Normal Hierarchy i.e., $m_1 < m_2 \ll m_3$

$$m_2 = \sqrt{m_1^2 + \Delta m_{sol}^2}, \qquad m_3 = \sqrt{m_1^2 + \Delta m_{atm}^2 + \Delta m_{sol}^2} \tag{5.18}$$

Putting back the 5.18 in equation 5.17 give

$$|m_\nu^{ee}| = \left| m_l \left(c_{12}^2 c_{13}^2 + \sqrt{m_1^2 + \Delta m_{atm}^2} s_{12}^2 c_{13}^2 e^{2i\alpha_2} + \sqrt{m_1^2 + \Delta m_{atm}^2 + \Delta m_{sol}^2} s_{13}^2 e^{2i\alpha_3} \right) \right| \tag{5.19}$$

Quasi Degenerated

$$m_1 \approx m_2 \approx m_3 \gg \sqrt{\Delta m_{atm}^2} \tag{5.20}$$

5.5 Analysis

The figure 5.2 represents the effective mass that governs $0\nu\beta\beta$ function of lightest mass in 3-neutrino picture for normal hierarchy while figure 5.5 represents the same for inverted hierarchy. The figure 5.3 & figure 5.6 represents the effective mass as a function of all neutrino mass Σ. The former is for normal hierarchy and latter is for inverted hierarchy. Similarly, we generated the effective mass as a function of m_ν^e (figure 5.4 & figure 5.7). The all above analysis is done with 3σ range of parameter given in Table 5.1. The phases, α_2 and α_3 we varied in the range $[0, 2\pi]$.

Inverted Hierarchy i.e., $m_3 \ll m_1 \approx m_2$

$$m_1 = \sqrt{m_3^2 + \Delta m_{sol}}, \qquad m_2 = \sqrt{m_3^2 + \Delta m_{atm}^2 + \Delta m_{sol}^2} \tag{5.21}$$

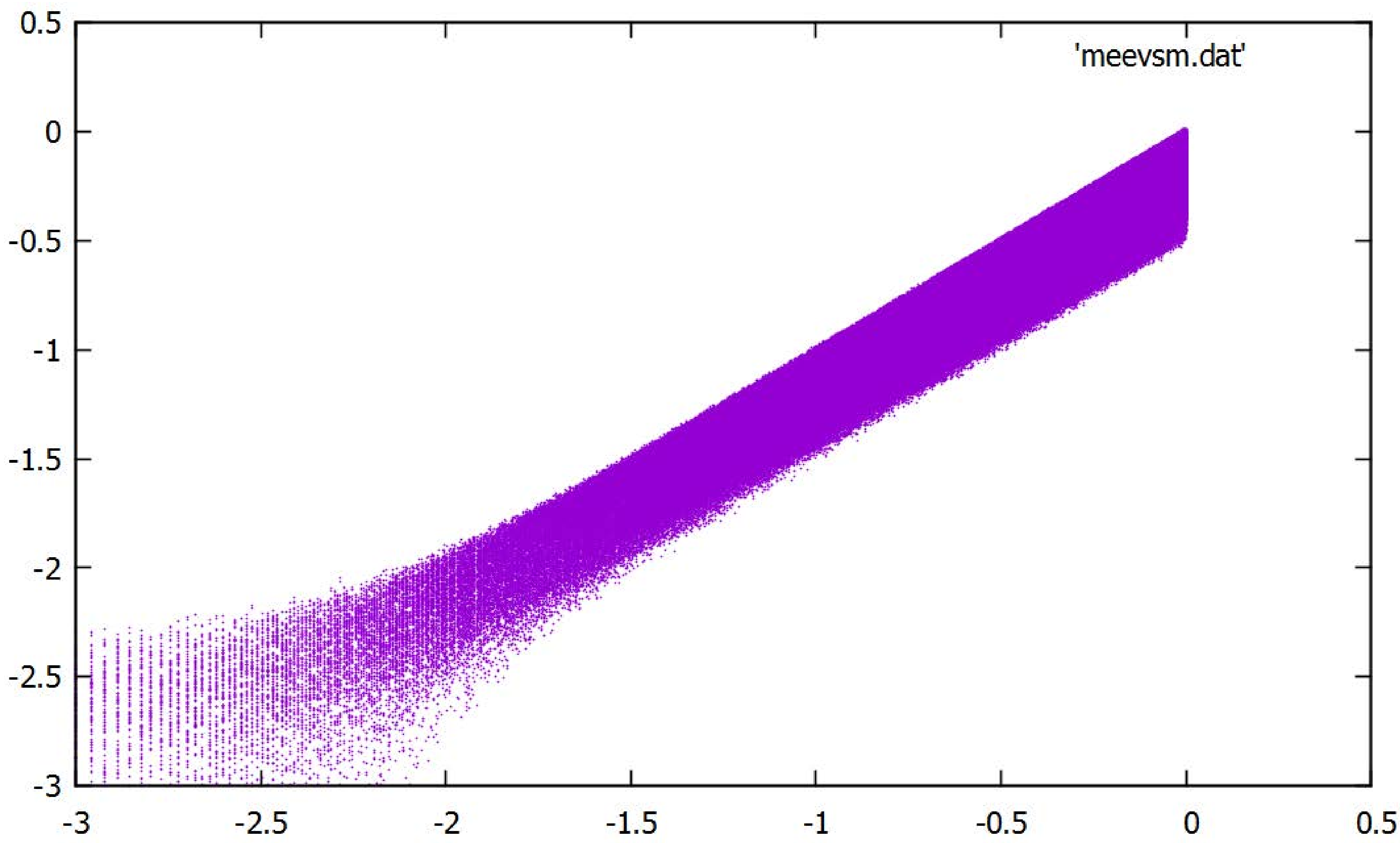

Fig. 5.2.: The graph shows the Normal Hierarchy points of possible values where m_{ee} & m_1 can lie. The graph is '$\log_{10} m_\nu^{ee}$ vs. $\log_{10} m_l$', where m_l is the lightest Neutrino.

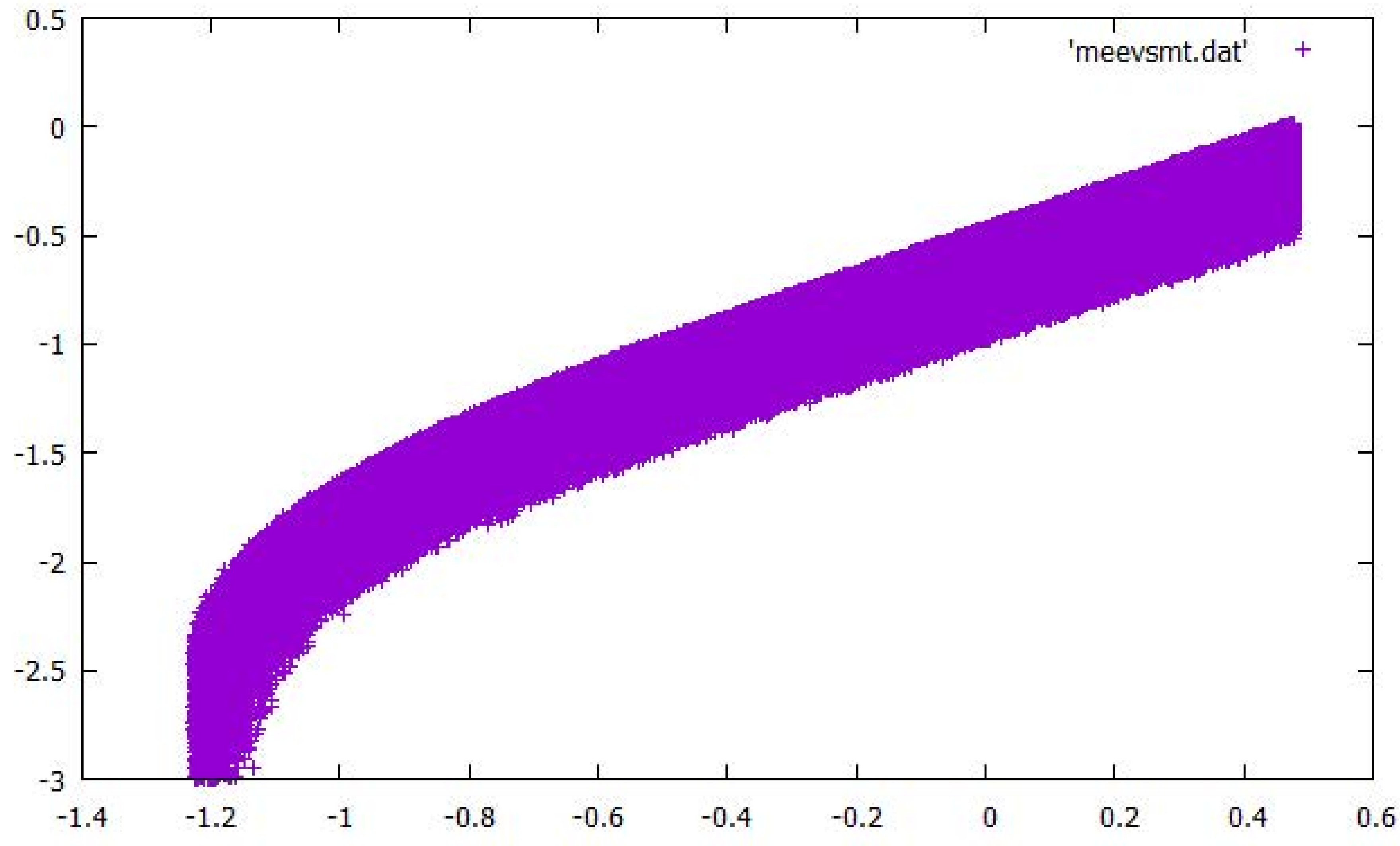

Fig. 5.3.: This shows the Normal Hierarchy points of possible values where m_{ee} & Σ can lie. The graph is '$\log_{10} m_{\nu}^{ee}$ vs. $\log_{10}\Sigma$', where Σ is the sum of Neutrino masses.

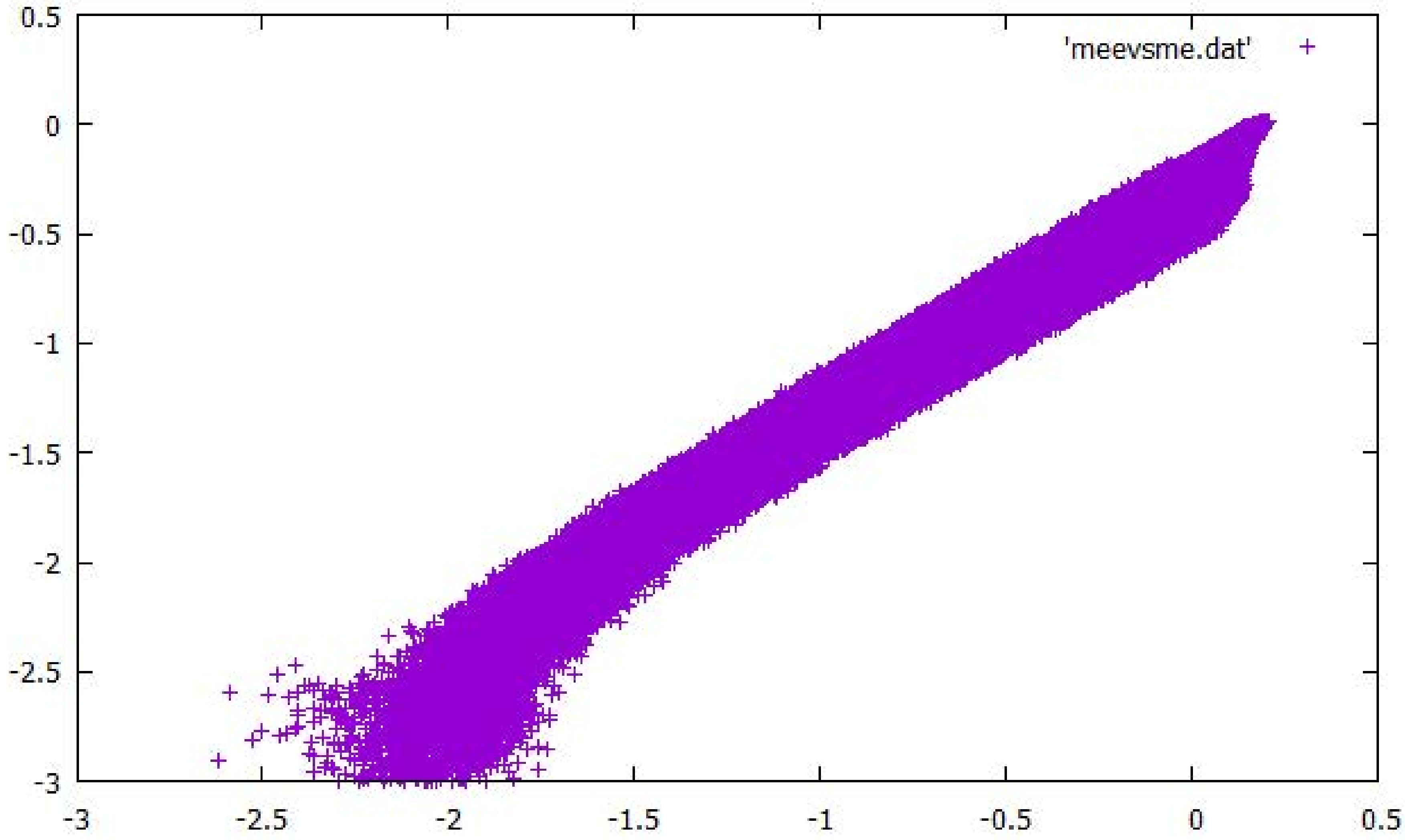

Fig. 5.4.: This shows the Normal Hierarchy points of possible values where m_{ee} & m_e can lie. The graph is '$\log_{10} m_\nu^{ee}$ vs. $\log_{10} m_e$', where $m_e = \sum_\alpha U_{e\alpha} m_\alpha$.

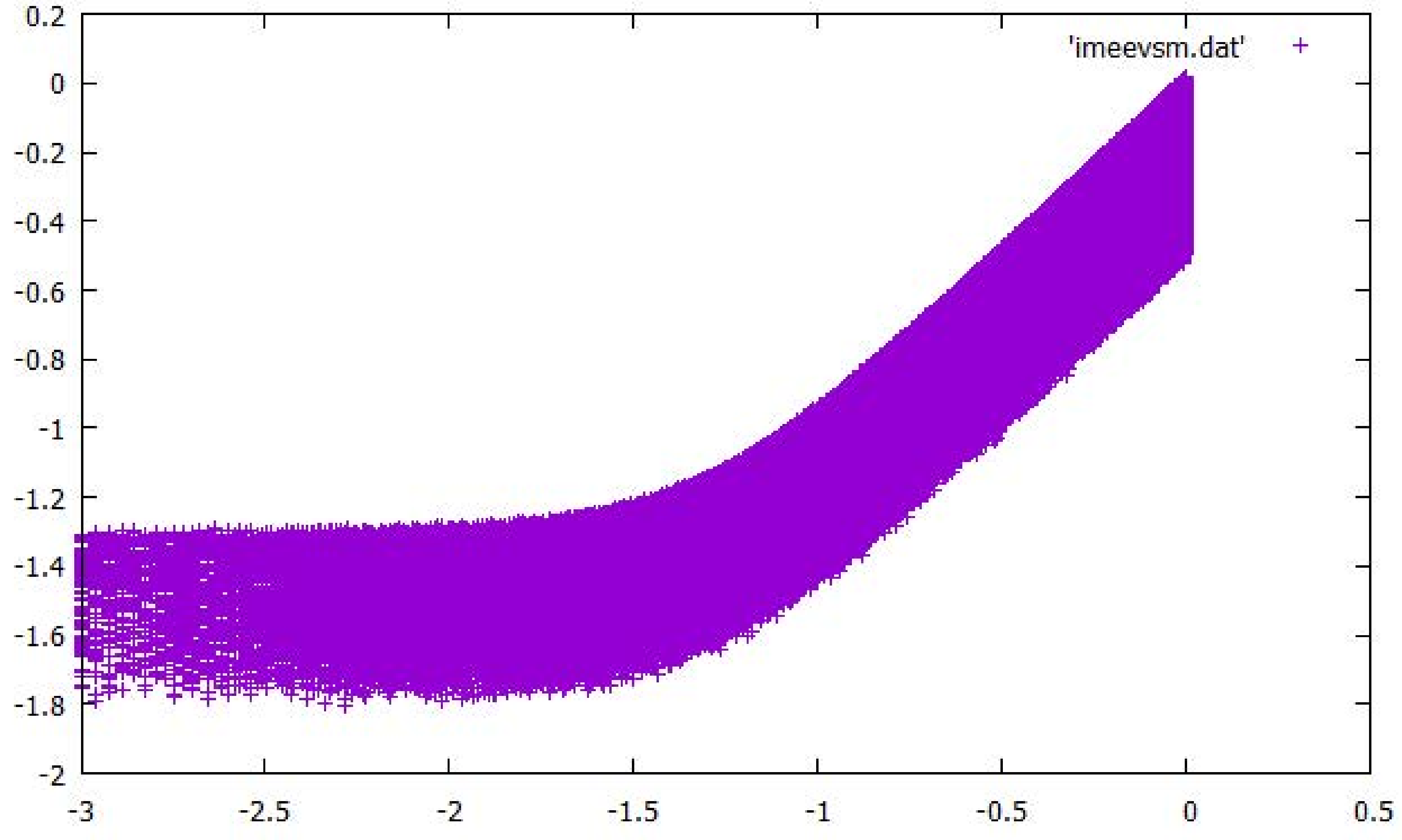

Fig. 5.5.: The graph shows the Inverted Hierarchy points of possible values where m_{ee} & m_1 can lie. The graph is '$\log_{10} m_\nu^{ee}$ vs. $\log_{10} m_l$', where m_l is the lightest Neutrino.

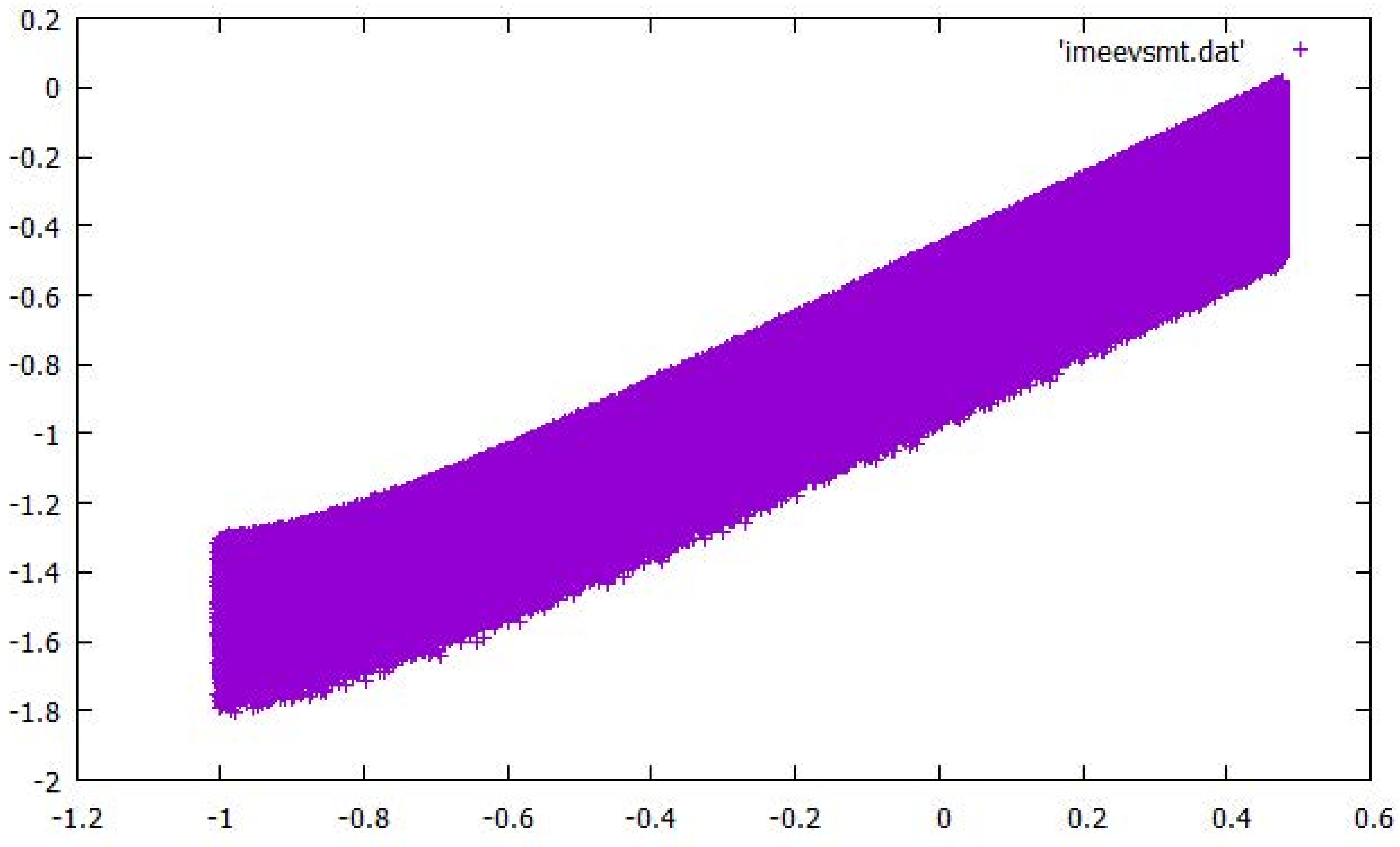

Fig. 5.6.: This shows the Inverted Hierarchy points of possible values where m_{ee} & Σ can lie. The graph is '$\log_{10} m_{\nu}^{ee}$ vs. $\log_{10} \Sigma$', where Σ is the sum of Neutrino masses.

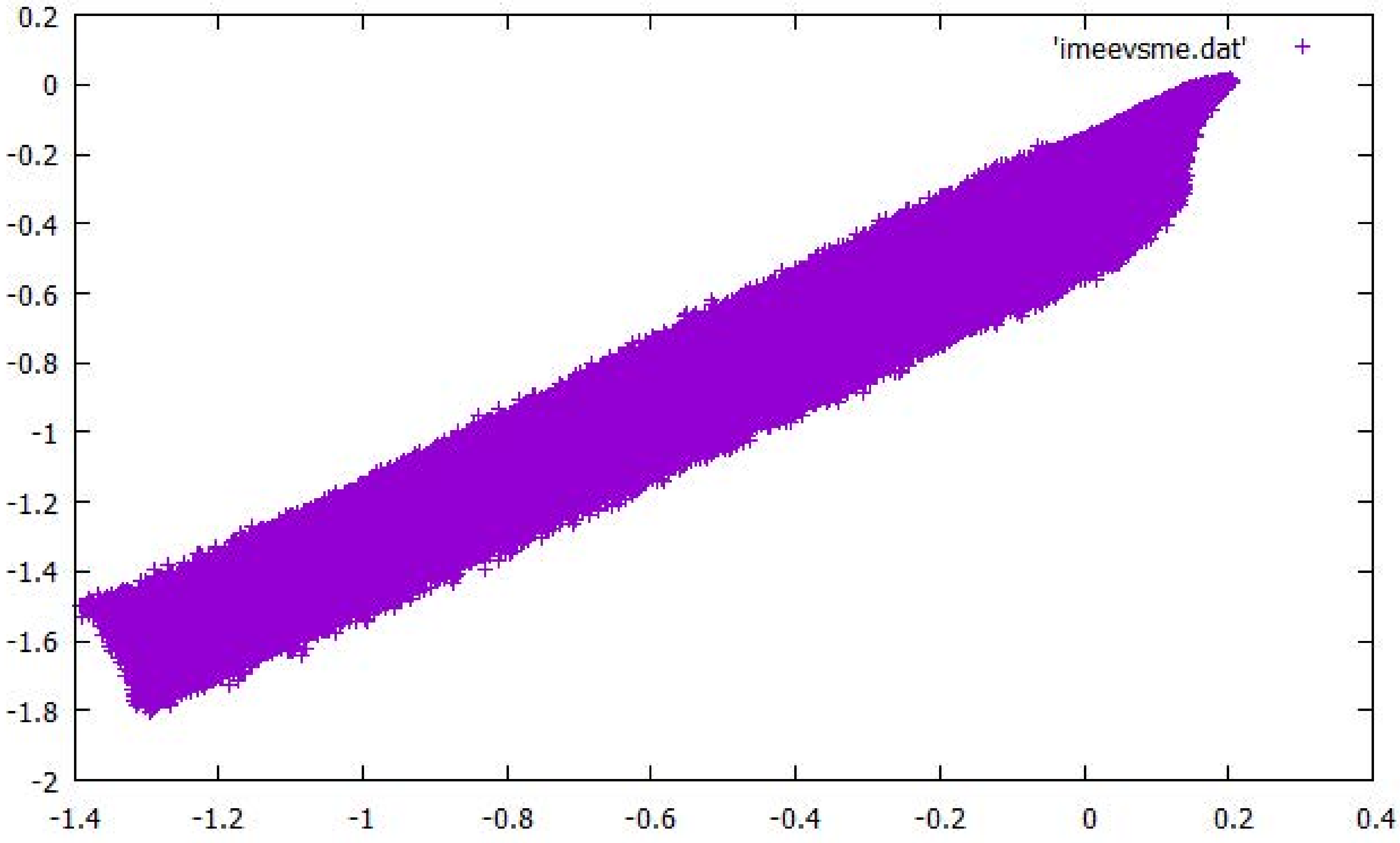

Fig. 5.7.: This shows the Inverted Hierarchy points of possible values where m_{ee} & m_e can lie. The graph is '$\log_{10} m_\nu^{ee}$ vs. $\log_{10} m_e$', where $m_e = \sum_\alpha U_{e\alpha} m_\alpha$.

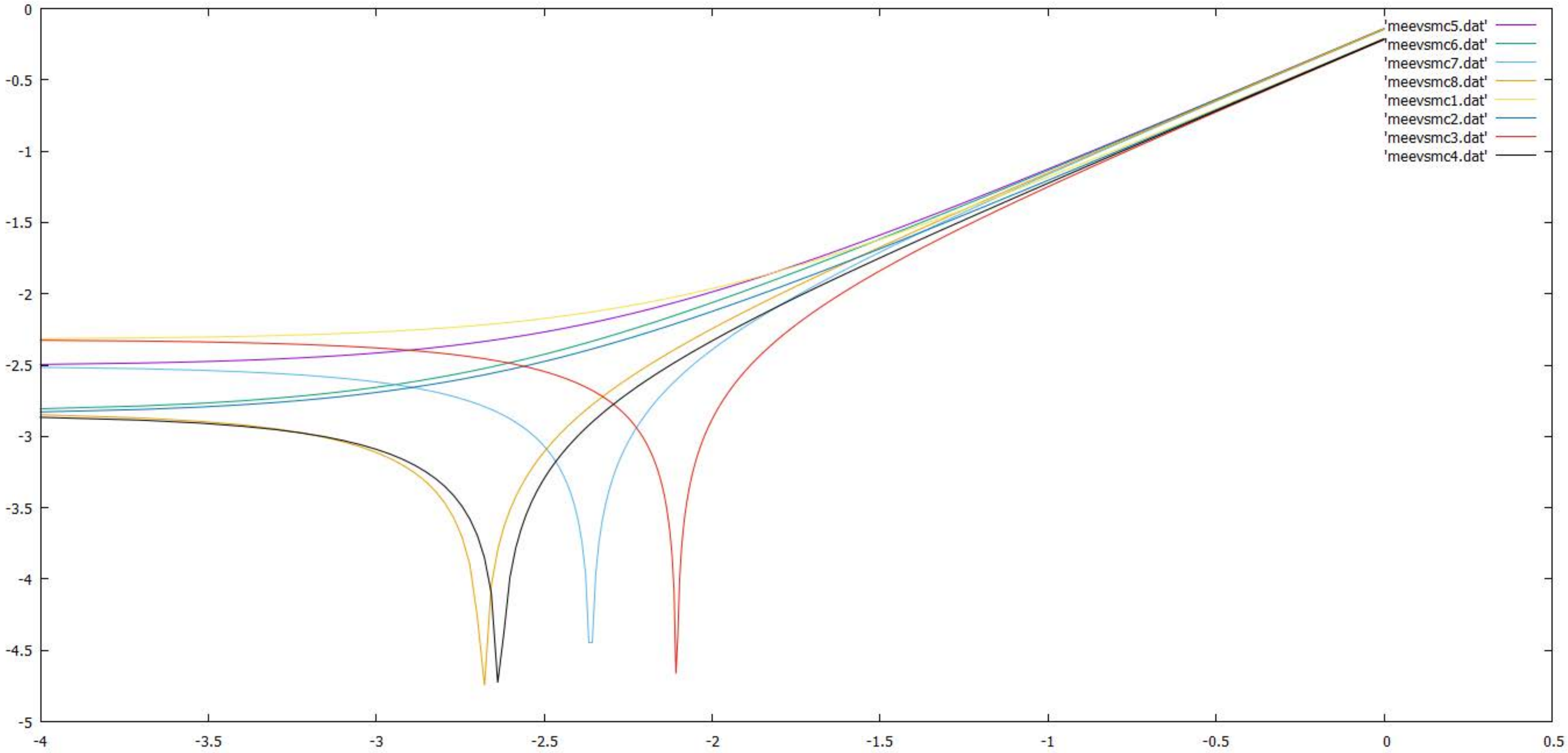

Fig. 5.8.: This shows the $\log_{10} m_\nu^{ee}$ vs. $\log_{10} m_l$ graph. (With maxima of trigonometric functions, With minima of trigonometric functions): (Yellow, Purple): (+,+), (Blue,Green): (+,-), (Red, Sky Blue): (-,-), (Black, Mustard): (-,+)

The figure 5.8 represents effective mass as a function of light mass with colour yellow & purple showing the $++$ i.e., for the value of majorana phases, α and β, 0. The colour blue & green showing the +- i.e., for the value of α and β as 0 & $\frac{\pi}{2}$ respectively. The colour red & sky blue showing the – i.e., for the value of α and β as $\frac{\pi}{2}$ & $\frac{\pi}{2}$ respectively. The colour black & mustard showing the -+ i.e., for the value of α and β as $\frac{\pi}{2}$ & 0 respectively.

There were two colors in the each of above parts. First color mentioned in above is showing the graph with 3σ added to the best fitted values (from table 5.1) of trigonometric function appeared m_ν^{ee}. The second color is 3σ subtracted from above explained trigonometric function.

The m_ν^{ee} limits depends on the relative value of m_1, Δm^2_{atm} and Δm_{sol}. We can define a mass ratio, t as

$$t = \left| \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \right| \tag{5.22}$$

We calculated some useful values in Table 5.1.

Normal Hierarchy

Strictly normal hierarchy can be shown as

$$m_1 \ll m_2 \approx \sqrt{\Delta m^2_{sol}} \ll m_3 \approx \sqrt{\Delta m^2_{atm}} \tag{5.23}$$

And we can calculate the effective mass in terms of t defined in 5.22 [Cha+12]

Tab. 5.1.: Using the definition of t from 5.22 and the table C.1 the following shows the maximum and minimum values required in further sections

	$\sqrt{t}$	$\sqrt{t}\sin^2(\theta_{12})$	$\sqrt{t}\cos^2(\theta_{12})$	$\tan^2\theta_{13}$	$\sqrt{t}\tan^2(\theta_{13})$
Maximum	0.2	0.072	0.096	0.046	0.009
Minimum	0.16	0.042	0.046	0.001	0.0001

$$|m_\nu^{ee}|_{NH} = \sqrt{\Delta m_{atm}^2}|\sqrt{t}s_{12}^2c_{13}^2e^{2i\alpha_2} + s_{13}^2e^{2i\alpha_3}| \tag{5.24}$$

To find the minimum and maximum value we can see that the 5.24 takes the form

$$\begin{aligned}
|m_\nu^{ee}|_{NH} &= \sqrt{\Delta m_{atm}^2}|\sqrt{t}s_{12}^2c_{13}^2e^{2i\alpha_2} + s_{13}^2e^{2i\alpha_3}| \\
&= \sqrt{\Delta m_{atm}^2}\sqrt{\left(\sqrt{t}s_{12}^2c_{13}^2\cos 2\alpha_1 + s_{13}^2\sin 2\alpha_2\right)^2 +} \\
&\qquad\qquad \overline{+\left(\sqrt{t}s_{12}^2c_{13}^2\sin 2\alpha_1 + s_{13}^2\cos 2\alpha_2\right)^2} \\
&= \sqrt{\Delta m_{atm}^2}\sqrt{\left(\sqrt{t}s_{12}^2c_{13}^2\right)^2 + \left(s_{13}^2\right)^2 + 2\cdot s_{13}^2\cdot\sqrt{t}s_{12}^2c_{13}\cos(2\alpha_1 + 2\alpha_2)}
\end{aligned} \tag{5.25}$$

We can see from above that the maximum value of the system will occur at $2\alpha_1 + 2\alpha_2 = 0$ and minimum at $2\alpha_1 + 2\alpha_2 = \pi$

Inverted Hierarchy

For smaller value of m_3 the hierarchy is $m_3 \ll m_2 \approx m_1 \approx \sqrt{\Delta m_{atm}^2}$. The effective mass will take form.

$$|m_\nu^{ee}|_{IH} = \sqrt{\Delta m_{atm}^2}|c_{12}^2c_{13}^2 + s_{12}^2c_{13}^2e^{2i\alpha}| \tag{5.26}$$

Clearly, the maximum value of the function is at $2\cdot\alpha = 0$

$$|m_\nu^{ee}|^{max} = c_{13}^2\sqrt{\Delta m_{atm}^2} \tag{5.27}$$

and the minimum value at $2\alpha = \pi$ with the form

$$|m_\nu^{ee}|^{min} = c_{12}^2\sqrt{\Delta m_{atm}^2} \tag{5.28}$$

5.6 Non-Standard Mechanism

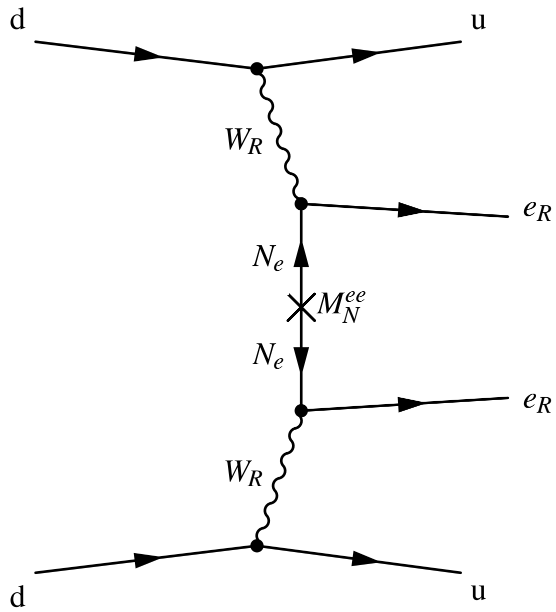

Fig. 5.9.: Feynman Diagram: Neutrinoless double beta decay with heavy neutrino exchange

The half-life of the Feynman Diagram, 5.9, is

$$\frac{\Gamma^{N}_{0\nu\beta\beta}}{\ln 2} = G\frac{|\mathcal{M}_\nu|^2}{m_e^2}|M_N^{ee}|^2 \tag{5.29}$$

Here, we can clearly compute

$$|M_N^{ee}| = \sum_i \left| \langle p \rangle^2 \frac{M_{W_L}^4}{M_{W_R}^4} \frac{(U_R)_{ei}^2}{M_i} \right| \tag{5.30}$$

We know that the mass light neutrino comes out to be the inversely proportional to the heavy neutrino mass i.e.,

$$m_i \propto \frac{1}{M_i} \tag{5.31}$$

we know from 4.2 that the mass of $M_{W_R} \sim v_R$ and we will analyze the following with $M_{W_R} \sim TeV$

We, forcefully, set M_{W_R} to be $3.5TeV$ and since the heaviest right handed neutrino to be $500GeV$ then we chose the momentum value to $(180MeV)$ the $M_{W_L} = 81GeV$ and $R_N = p^2 \left(\frac{M_W}{M_{W_R}}\right)^4 \sim 10^{10} eV^2$ (These values are taken from analysis in [Cha+12])

Normal Hierarchy

From 5.31, we can easily see that the

$$\begin{aligned} \frac{M_2}{M_1} &= \frac{m_1}{m_2} \\ \frac{M_3}{M_1} &= \frac{m_1}{m_3} \end{aligned} \tag{5.32}$$

Like, light neutrino we can write the effective mass contribution term to the decay rate, $\Gamma^N_{0\nu\beta\beta}$

$$|M_N^{ee}| = C_N \left| \frac{c_{12}^2 c_{13}^2}{M_1} + \frac{s_{12}^2 c_{13}^2}{M_2} e^{2i\alpha_2} + \frac{s_{13}^2}{M_3} e^{2i\alpha_3} \right| \tag{5.33}$$

But we can use 5.32 an get

$$|M_N^{ee}| = \frac{C_N}{M_1} \left| c_{12}^2 c_{13}^2 + \frac{m_2}{m_1} s_{12}^2 c_{13}^2 e^{2i\alpha_2} + \frac{m_3}{m_1} s_{13}^2 e^{2i\alpha_3} \right| \tag{5.34}$$

But again we can set the strictly hierarchical setting in our 5.34 $(m_1 \ll m_2 \approx \sqrt{\Delta m_{sol}^2} \ll m_3 \approx \sqrt{\Delta m_{atm}^2})$ to get the following

$$|M_N^{ee}| = \frac{C_N}{M_1} \left| c_{12}^2 c_{13}^2 + \frac{\sqrt{\Delta m_{sol}^2}}{m_1} s_{12}^2 c_{13}^2 e^{2i\alpha_2} + \frac{\sqrt{\Delta m_{atm}^2}}{m_1} s_{13}^2 e^{2i\alpha_3} \right| \tag{5.35}$$

Here, clearly, to increase the $|M_N^{ee}|$ we need to decrease m_1. This is different than the calculations of $|m_\nu^{ee}|$

As we increase m_1 we will get the ($m_1 \approx m_2 \approx \sqrt{\Delta m_{sol}^2} \ll m_3 \approx \sqrt{\Delta m_{atm}^2}$) and we can write the approximated $|M_N^{ee}|$. The form is

$$|M_N^{ee}| = \frac{C_N}{M_1}\left| c_{13}^2(c_{12}^2 + s_{12}^2 e^{2i\alpha_2}) + \frac{1}{\sqrt{t}} s_{13}^2 e^{2i\alpha_3} \right| \tag{5.36}$$

Inverted Hierarchy

In the case of inverted hierarchy the m_3 is the lightest and M_3 will be the heaviest neutrino and we get

$$\begin{aligned} \frac{M_2}{M_3} &= \frac{m_3}{m_2} \\ \frac{M_1}{M_3} &= \frac{m_3}{m_1} \end{aligned} \tag{5.37}$$

Again the M_N^{ee} can be written in the form of

$$|M_N^{ee}| = \frac{C_N}{M_3}\left| \frac{m_1}{m_3} c_{12}^2 c_{13}^2 + \frac{m_2}{m_3} s_{12}^2 c_{13}^2 e^{2i\alpha_2} + s_{13}^2 e^{2i\alpha_3} \right| \tag{5.38}$$

But again we can set the strictly hierarchical setting in our 5.34 ($m_3 \ll m_2 \approx m_1 \approx \sqrt{\Delta m_{atm}^2}$) to get the following

$$|M_N^{ee}| = \frac{C_N}{M_3}\left| \frac{\sqrt{\Delta m_{atm}^2}}{m_3} c_{12}^2 c_{13}^2 + \frac{\sqrt{\Delta m_{atm}^2}}{m_3} s_{12}^2 c_{13}^2 e^{2i\alpha_2} + s_{13}^2 e^{2i\alpha_3} \right| \tag{5.39}$$

The Normal Hierarchy graphs of $|M_N^{ee}|$ is shown in figure 5.10

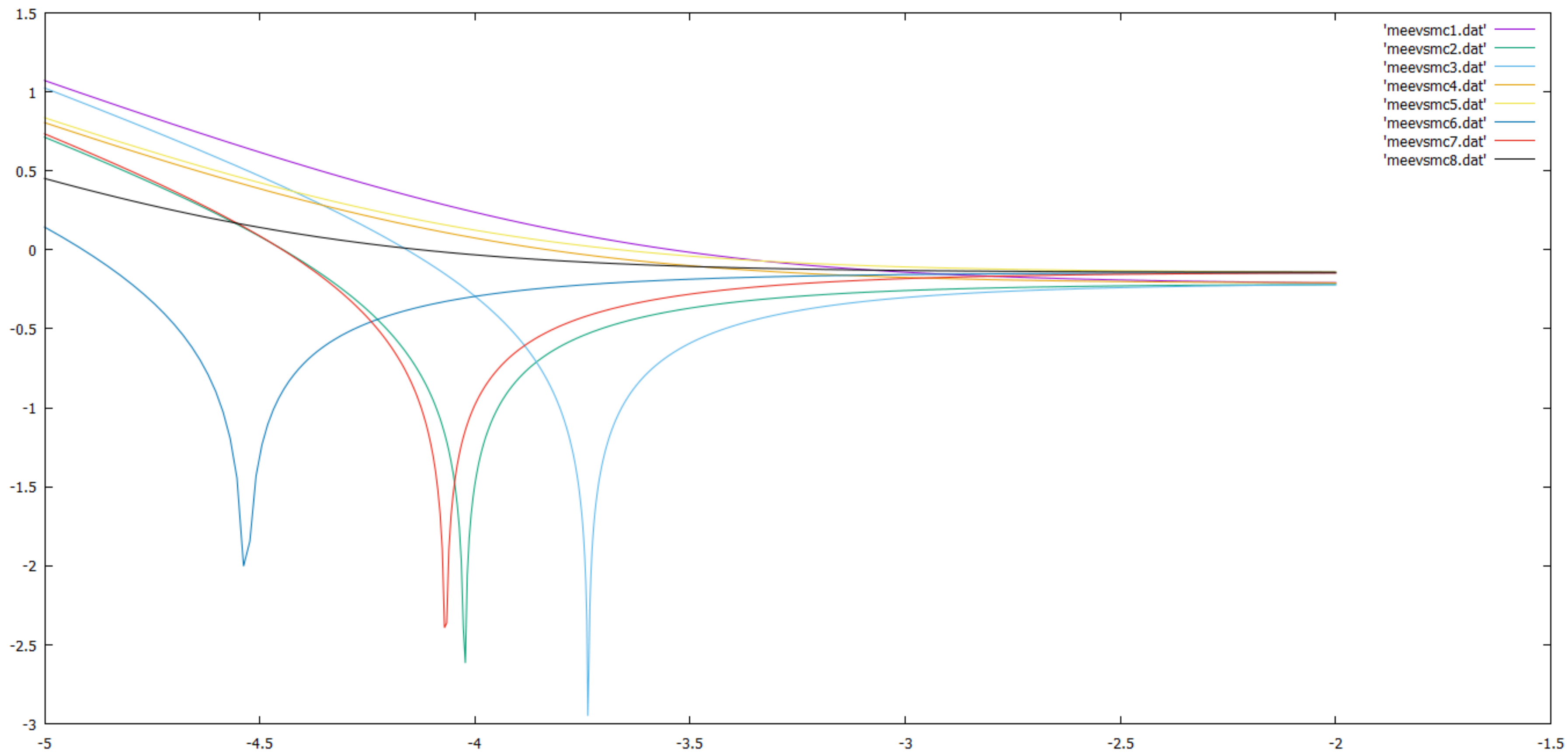

Fig. 5.10.: This shows the $\log_{10} M_\nu^{ee}$ vs. $\log_{10} m_l$ graph. (With maxima of trigonometric functions, With minima of trigonometric functions): (Yellow, Purple): (+,+), (Blue,Green): (+,-), (Red, Sky Blue): (-,-), (Black, Mustard): (-,+)

With Dirac mass dominance (type-II)

Considering the dominating $\langle \Delta_R \rangle_0$ value in compare to the elements of $\langle \phi \rangle_0$ will lead us to the $m_i \propto M_i$ and the ratios we needed in further analysis are as follows

$$\begin{aligned} \frac{M_1}{M_3} &= \frac{m_1}{m_3} \\ \frac{M_1}{M_3} &= \frac{m_1}{m_3} \end{aligned} \tag{5.40}$$

We will fix again the $M_3 = 500GeV$, we again fix the p, M_{W_L} & M_{W_R} as we did in the last analysis. We also will be using the $U_L = U_R$ Now we can from 5.33 write the equation

$$|M_N^{ee}| = \frac{C_N}{M_3} \left| \frac{m_3}{m_1} c_{12}^2 c_{13}^2 + \frac{m_3}{m_2} s_{12}^2 c_{13}^2 e^{2i\alpha_2} + s_{13}^2 e^{2i\alpha_3} \right| \tag{5.41}$$

Again in the limit of strict hierarchy i.e., $(m_1 \ll m_2 \approx \sqrt{\Delta m_{sol}^2} \ll m_3 \approx \sqrt{\Delta m_{atm}^2})$

$$|M_N^{ee}| = \frac{C_N}{M_3} \left| \frac{\sqrt{\Delta m_{atm}^2}}{m_1} c_{12}^2 c_{13}^2 + \frac{1}{\sqrt{t}} s_{12}^2 c_{13}^2 e^{2i\alpha_2} + s_{13}^2 e^{2i\alpha_3} \right| \tag{5.42}$$

From 5.41 & $m_1 \approx m_2 \approx \sqrt{\Delta m_{sol}^2} \ll m_3 \approx \sqrt{\Delta m_{atm}^2}$ will lead to

$$|M_N^{ee}| = \frac{C_N}{M_3} \left| \frac{1}{\sqrt{t}} c_{12}^2 c_{13}^2 + \frac{1}{\sqrt{t}} s_{12}^2 c_{13}^2 e^{2i\alpha_2} + s_{13}^2 e^{2i\alpha_3} \right| \tag{5.43}$$

5.7 Remarks

As discussed, Neutrino Oscillations already shows the rigid evidence of massive Neutrino existence. The B-L model and Left-Right Symmetric model explains the mass of light neutrinos . The left-right symmetric model needs ad-hoc heavy neutrino fields. The left-right symmetric model allow mass of neutrino with type-I and II seesaw mechanism. We extend the gauge field and show that the symmetry breaking leads to the standard model gauge group. The left-right symmetric model allows the Neutrinoless double beta decay process. The Neutrinoless double beta decay will evidently shows the neutrino massive character and majorana nature. That is why, from a long time people are interested in search of neutrinoless double beta decay.

The neutrinoless double beta decay is a lepton number violation process. We studied the effective mass contribution of Neutrinos in two processes.

- where W_L are the Bosons mediating the left handed electrons and light neutrino exchange occur.

- where W_R are the Bosons mediating the right handed electrons and heavy neutrino exchange occur.

While there are other diagrams too but the above analyzed diagrams contribute most (because there is low $W_L - W_R$ mixing) for effective mass.

We plotted and calculated the effective mass as a function of light mass, as a function of sum of all neutrino masses, as a function of $|m_\nu^e|$.

We analyzed the two different possibilities with *major Majorana mass term contribution* and with *major Dirac mass term contribution*.

For the above two possibilities we plotted the graphs of $|M_\nu^{ee}|$ vs. m_{light}.

Appendices

A

Discrete Symmetries

A.1 QED Analysis

Consider ψ to be electron field then

$$\begin{aligned} \text{P} &: \quad \psi(x) \longrightarrow \gamma^0 \psi(\hat{x}) \\ & \qquad\qquad\qquad \text{where, } \hat{x} = \begin{pmatrix} x^0 \\ -\vec{x} \end{pmatrix} \\ \text{CP} &: \quad \psi(x) \longrightarrow -C\psi^*(\hat{x}) \end{aligned} \tag{A.1}$$

The definition of ‘C matrix’ can be shown mathematically as

$$C^{-1}\gamma_\mu C = -\gamma_\mu^T \tag{A.2}$$

We can define the two chiral projection as

$$\psi_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)\,\psi \tag{A.3}$$

The possible two independent left-handed fields can be written in the following form

$$\psi_{L_1} \equiv \psi_L, \qquad \psi_{L_2} \equiv \mathrm{C}\,\gamma_0^T\,\psi_R^* \equiv (\psi_R)^c \tag{A.4}$$

The parity operator action on individual handedness particle

$$\mathrm{P}: \psi_L \to \gamma_0 \psi_R, \qquad \psi_R \to \gamma_0 \psi_L \tag{A.5}$$

In the ψ_{L_1} and ψ_{L_2} form the transformation will be

$$P : \begin{pmatrix} \psi_{L_1} \\ \psi_{L_2} \end{pmatrix} \longrightarrow \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathrm{C} \begin{pmatrix} \psi_{L_1} \\ \psi_{L_2} \end{pmatrix}^* \tag{A.6}$$

$$CP : \begin{pmatrix} \psi_{L_1} \\ \psi_{L_2} \end{pmatrix} \longrightarrow \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \mathrm{C} \begin{pmatrix} \psi_{L_1} \\ \psi_{L_2} \end{pmatrix}^* \tag{A.7}$$

B

Mathematical Identities used in Dissertation

B.1 Unitary Matrices

The definition of Unitary group is

$$U(N) = \{U|U \text{ is an N}\times\text{N complex matrix with } U^{\dagger}U = I\} \tag{B.1}$$

where ‘ † ’ sign is used to show the transpose of complex conjugate of the matrix. In component form the definition of transpose) we can see

$$(U_{\alpha\beta})^{\dagger} = U^{*}_{\beta\alpha} \tag{B.2}$$

From the definition of unitary matrix (B.1)

$$\sum_{\beta}(U_{\alpha\beta})^{\dagger}U_{\beta\delta} = \sum_{\beta}U^{*}_{\beta\alpha}U_{\beta\delta} = \delta_{\alpha\delta} \tag{B.3}$$

B.2 Complex Numbers

$$\Re\mathrm{e}(A\cdot B) = \Re\mathrm{e}(A)\,\Re\mathrm{e}(B) - \Im\mathrm{m}(A)\,\Im\mathrm{m}(B) \tag{B.4}$$

B.3 Parity Operator

$$P_L \gamma_\mu = \gamma_\mu P_R \tag{B.5}$$

$$P_L P_R = 0 \tag{B.6}$$

$$P_L^2 = P_L; \quad P_R^2 = P_R \tag{B.7}$$

$$P_L \left(\frac{\not{p} - m_\nu}{q^2 + m_\nu^2} \right) P_R = \frac{\not{p} P_L P_R + P_L^2 m_\nu}{q^2 - m_\nu^2} = \frac{P_L m_\nu}{q^2 - m_\nu^2} \tag{B.8}$$

C

Particle Data Tables

C.1 Neutrino Oscillation Data

Fig. C.1.: Neutrino oscillation data

Neutrino Mixing

The following values are obtained through data analyses based on the 3-neutrino mixing scheme described in the review "Neutrino Mass, Mixing, and Oscillations" by K. Nakamura and S.T. Petcov in this *Review*.

$\sin^2(2\theta_{12}) = 0.846 \pm 0.021$

$\Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5}\ \mathrm{eV}^2$

$\sin^2(2\theta_{23}) = 0.999^{+0.001}_{-0.018}$ (normal mass hierarchy)

$\sin^2(2\theta_{23}) = 1.000^{+0.000}_{-0.017}$ (inverted mass hierarchy)

$\Delta m^2_{32} = (2.44 \pm 0.06) \times 10^{-3}\ \mathrm{eV}^2$ [i] (normal mass hierarchy)

$\Delta m^2_{32} = (2.52 \pm 0.07) \times 10^{-3}\ \mathrm{eV}^2$ [i] (inverted mass hierarchy)

$\sin^2(2\theta_{13}) = (9.3 \pm 0.8) \times 10^{-2}$

Stable Neutral Heavy Lepton Mass Limits

Mass $m >$ 45.0 GeV, CL = 95% (Dirac)

Mass $m >$ 39.5 GeV, CL = 95% (Majorana)

Neutral Heavy Lepton Mass Limits

Mass $m >$ 90.3 GeV, CL = 95%
(Dirac ν_L coupling to e, μ, τ; conservative case(τ))

Mass $m >$ 80.5 GeV, CL = 95%
(Majorana ν_L coupling to e, μ, τ; conservative case(τ))

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