Randall-Sundrum Model and Cosmology

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A dissertation submitted for the partial fulfilment of
BS-MS dual degree in Science

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Dedicated to my mother for making me who I am and my father for supporting me all the way......

Certificate of Examination

This is to certify that the dissertation titled "Randall Sundrum Model and Cosmology" submitted by Ms. Diksha Jain (Reg. No. MS11022) for the partial fulfillment of BS-MS dual degree programme of the Indian Institute of Science Education and Research, has been examined by the thesis committee duly appointed by the Institute. The committee finds the work done by the candidate satisfactory and recommends that the report be accepted.

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Dated : April 21, 2016

Declaration

The work presented in this dissertation has been carried out by me under the guidance of Dr. H.K. Jassal at the Indian Institute of Science Education and Research Mohali.

This work has not been submitted in part or in full for a degree, a diploma, or a fellowship to any other university or institute. Whenever contributions of others are involved, every effort is made to indicate this clearly, with due acknowledgment of collaborative research and discussions. This thesis is a bonafide record of original work done by me and all sources listed within have been detailed in the bibliography.

Signed:

Date:

"For me, it is far better to grasp the Universe as it really is than to persist in delusion, however satisfying and reassuring."

Carl Sagan

INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH MOHALI

Abstract

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Randall-Sundrum Model and Cosmology

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We, study cosmology of the higher dimensional scenarios, mainly that of the Randall Sundrum (RS) model. We have examined the effective cosmology on the visible brane of RS model by considering the FRW spacetime, with a warped extra dimension. We found that no viable cosmological solution can be obtained in this ansatz, if the radius of extra dimension is taken to be dynamic. In this case, the system becomes over constrained. We also analyze the effects of adding a radion stabilizing potential on the effective $4D$ cosmology and we found that there is a possibility of generating viable cosmological solutions if this potential is appropriately fine tuned. We also comment on the possibility that this potential might remove the extra constraint that we get when radion is not stabilized.

Acknowledgements

I would like to express my deepest appreciation to all those who provided me the possibility to complete this project. A special gratitude I give to my MS thesis guide Dr. H.K. Jassal, for suggesting me to work on this interesting problem. The door to her office was always open whenever I ran into a trouble spot or had a question about my research. She steered me in the right direction whenever she thought I needed it.

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List of Symbols

c	speed of Light
G	$4D$ Gravitational Constant
κ^2	$5D$ Gravitational Constant
b	Radius of extra dimension
m_0	Warp factor
Λ	$5D$ Cosmological Constant
S	The action functional
H	Hubble Parameter
ρ	Matter density
Ω_M	Matter density parameter
Indices	**values they can take**
A, B, C...	0,1,2,3,4
μ, ν...	0,1,2,3
i, j, k...	1,2,3

Chapter 1

Introduction

The last two centuries were marked with the development of two of the most beautiful and well-tested theories in physics namely, Electromagnetism and General Relativity. These theories explained two of the four fundamental forces of nature: Electromagnetic force and Gravitational force. Maxwell, in 19th century, made a successful attempt to unify electricity and magnetism. Inspired by his work, many attempts were made to unify the four fundamental forces, one involving postulating presence of extra dimensions. In this project, I worked on a theory containing five dimensions which was successful in unifying two of the four fundamental forces of nature : gravity and electromagnetism.

1.1 Why Extra Dimensions

It is natural to ask that if the universe is four dimensional (three spatial and one time) then why should we consider extra dimensions? Lisa Randall, in her book *Warped Passages* [Ran] motivates the consideration of extra dimensions by arguing that, as of now there is no Physical theory that puts a bound on the number of dimensions. Therefore, physicists we not be reluctant to tinker with the dimensionality of the universe. To quote, "Dismissing the possibility of extra dimensions before even considering their existence might be very premature."

But if there exist other dimensions, why haven't we observed them yet? One can argue that these dimensions might be too large or too small and hence beyond our experimental reach. For example, a two dimensional folded plane will appear to us as a one dimensional line, if its radius is small as shown in figure 1.1 [Ran]. Similarly, the world we live in might have a small extra dimension, that we are not

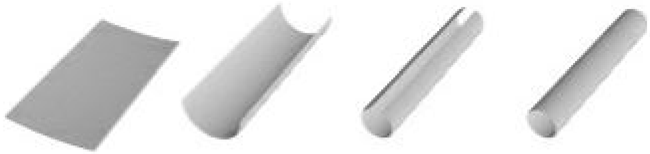

FIGURE 1.1: A 2D plane looking one dimensional

able to observe.

Other possibility is that the extra dimension might be too large. In such a case we can think of us living on a four-dimensional hyper-plane in this large five dimensional bulk. In this case, any other 4D hyper-plane will be too far from us and hence we effectively see a 4D world.

1.2 Kaluza-Klein Theory

In 1919, just after the development of general relativity, Kaluza [Kal21] attempted to unify gravity and electromagnetism in a theory containing five dimensional space-time. He was successful in his approach and was able to get both four dimensional general relativity and electromagnetism from the five dimensional Einstein equations. After Kaluza's attempt in the unification, many physicists attempted increase the number of dimensions in order to unify all the four forces in a single theory. This approach was used to construct the eleven-dimensional supergravity theories in the 1980s. It was also used in the development of the theory of ten-dimensional superstrings which is the current favorite contenders for a possible "Theory of everything". Kalzua's theory is now considered as one of the precursors of String theory.

1.3 Randall-Sundrum Model

In this project, I worked on Randall-Sundrum model, which was proposed by Lisa Randall and Raman Sundrum [LR99] in 1999. In this model, a circular extra dimension is considered with two 4D hyperplanes sitting on the fixed points of this extra dimension. These hyperplanes are called Branes and the two branes are named : the TeV brane (where all standard model particles live) and the Planck brane (the hidden brane). It is different from Kaluza's model in the sense that the extra dimension considered here is extremely warped. This model became a huge success because it was able to provide an explanation to "The Hierarchy Problem"[Her] of the Standard Model of particle physics. Due to the warping of extra dimension, the vacuum expectation value of Higgs decay from Planck brane to TeV brane, as a result all the mass parameters are exponentially suppressed, making the Weak, Strong and Electromagnetic forces stronger than gravitational force. The detailed explanation of this phenomenon is provided in Chapter 3.

The radius (length) of this extra dimension, needs to be stabilized. In 1999 itself, Goldberg and Wise [GW99] came up with a mechanism to stabilize the radius of extra dimension. They considered a massive scalar field ϕ (called it the radion field) in five dimensional bulk with a potential $V(\phi)$.In this case the radius of extra dimension is determined by the equation of motion of the radion field.

In this project, We have examined the cosmology of RS model i.e. the FRW spacetime with a warped extra dimension was considered. We found that we don't get any viable cosmology, if the radius of extra dimension is taken to be dynamic. This happens because the bulk cosmological constant and brane tensions give rise to a term which generates unconventional cosmology. We also analyze the effects of adding a radion stabilizing potential on the effective $4D$ cosmology.

The thesis is organized as follows. In Chapter 2, the basics and the features of Kaluza-Klein theory are discussed. We also explain Kaluza's original mechanism with a compactified extra dimension. In Chapter 3, we discuss the Randall-Sundrum model and explicitly shown that this warped metric satisfies Einstein equations. Further, we introduce the Hierarchy Problem of Standard Model and explicitly demonstrate how the warped metric helps in providing an explanation for this Hierarchy. The brane stabilization mechanism given by Goldberg and Wise [GW99] is also introduced. Chapter 4 is devoted to the introduction of our approach of incorporating 4D cosmology into Randall-Sundrum Model. In this Chapter, We derive the equation of motion for an effective four dimensional action. Chapter 5 is devoted to the numerical analysis of the equations derived in last chapter. We show that a viable cosmology is not achieved in this case. We also analyze the system after adding a radion stabilizing potential. In Chapter 6 we summarize our results and present concluding remarks. Additional details and explicit calculations are presented in Appendix A

Chapter 2

Kaluza-Klein Theory

Inspired by Maxwell's unification of the theory of electricity and magnetism, Nordström in [Nor14] 1914 and Kaluza[Kal21] independently in 1921 were the first to try to unify gravity and electromagnetism by considering a five dimensional space-time.

2.1 Features of Kaluza's Theory

In this subsection, we review the three main features of Kaluza's unified theory [OW97]:

2.1.1 Nature as pure geometry

One of the key assumptions of this theory is that nature is considered as pure geometry i.e. no five dimensional Energy-Momentum tensor is added. This was inspired by Einstein's vision of considering matter as a manifestation of geometry [Ein56; Whe68; Sal80]. The idea is to get all the four dimensional matters just by adding extra spatial dimensions. The Einstein equations are given by :

$$G_{AB} = 0 \tag{2.1}$$

or,equivalently :

$$\hat{R}_{AB} = 0 \tag{2.2}$$

where $G_{AB} \equiv \hat{R}_{AB} - \hat{R}g_{AB}/2$ is the five dimensional Einstein tensor and $\hat{R}_{AB}$ and $R = \hat{g}_{AB}\hat{R}^{AB}$ are the Ricci tensor and Ricci scalar respectively.Note that the indices A, B run form 0 to 4.

2.1.2 Minimal Extension of General Relativity

Kaluza's theory is just minimal extension of Einstein's 4D general relativity. Therefore all the quantities like five dimensional Ricci tensor, Christoffel symbols are defined similarly to their four dimensional counterparts.

$$R_{AB} = \partial_C \Gamma^C_{AB} - \partial_B \Gamma^C_{AC} + \Gamma^C_{AB}\Gamma^D_{CD} - \Gamma^C_{AD}\Gamma^D_{BC} \tag{2.3}$$

$$\Gamma^C_{AB} = \frac{1}{2} g^{CD}(\partial_A g_{DB} + \partial_B g_{DA} - \partial_D g_{AB}) \tag{2.4}$$

But now the indices A,B,C,D run from 0 to 4 instead of 0 to 3.

Since there is no five dimensional energy momentum tensor, everything now depends on the choice of metric. The metric can be parametrized as follows [OW97]:

$$g_{AB} = \begin{pmatrix} g_{\alpha\beta} + \kappa^2\phi^2 A_\alpha A_\beta & \kappa^2\phi^2 A_\alpha \\ \kappa^2\phi^2 A_\beta & \phi^2 \end{pmatrix} \tag{2.5}$$

where the electromagnetic potential A_α is scaled by a constant κ which can be used to get the correct multiplicative factors in the action.

2.1.3 The Cylinder Condition

Since the physics that have been observed in experiments till now is not seen to be dependent on the extra dimension, Kaluza assumed the derivatives of all the physical quantities with respect to extra dimension to be zero. This strict condition was criticized a lot. Later on a compactification mechanism was proposed which made the extra dimensions unobserved at the energy scales of current experiments. This approach has been very successful, and currently it is the dominant approach in higher-dimensional unification. Some of the review articles on this approach are available in [BL87],[Duf94] and [App84].

Using the metric (2.5) and the equations (2.4), (2.3), one can find the five dimensional Einstein field equations [Les82], [TAF87]. The $\alpha\beta-$, $\alpha 4-$ and $44-$ components of the field equations become:

$$G_{\alpha\beta} = \frac{\kappa^2\phi^2}{2}T^{EM}_{\alpha\beta} - \frac{1}{\phi}[\nabla_\alpha(\partial_\beta\phi) - g_{\alpha\beta}\Box\phi] \tag{2.6a}$$

$$\nabla_\alpha F_{\alpha\beta} = -3\frac{\partial^\alpha\phi}{\phi}F\alpha\beta \tag{2.6b}$$

$$\Box\phi = \frac{\kappa^2\phi^3}{4}F_{\alpha\beta}F^{\alpha\beta} \tag{2.6c}$$

where $G_{\alpha\beta} = R_{\alpha\beta} - Rg_{\alpha\beta}/2$ is the $4D$ Einstein tensor and $T^{EM}_{\alpha\beta} \equiv g_{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}/4 - F^\gamma_\alpha F_\beta\gamma$ is the electromagnetic energy-momentum tensor and $F_{\alpha\beta} \equiv \partial_\alpha A_\beta - \partial_\beta A_\alpha$ is the usual electromagnetic tensor.

2.2 Field equations in special cases

2.2.1 The case $\phi = constant$

If the scalar field ϕ in above equations (2.6) is kept constant, the first two equations just give us back the Einstein and Maxwell equations.

$$G_{\alpha\beta} = 8\pi G\phi^2 T^{EM}_{\alpha\beta} \tag{2.7a}$$

$$\nabla_\alpha F_{\alpha\beta} = 0 \tag{2.7b}$$

where the parameter κ is defined in terms of 4D Gravitational constant G.

$$\kappa^2 = 16\pi G \tag{2.8}$$

These two equations show that we are getting $4D$ General relativity and electromagnetism from a single $5D$ equation, hence unifying two of the three fundamental forces of nature. This result was first obtained by Kaluza and Klein who took $\phi = 1$. The third equation (2.6c) becomes $F_{\alpha\beta}F^{\alpha\beta} = 0$, which is true only for electromagnetic waves.

The above equations can also be derived by varying the action functional. Using the metric (2.5) and the definitions (2.3-2.4), one can write down the 5D action for this case. But now using the cylinder condition not only means to drop derivatives with respect to y, but we also need to pull the factor $\int dy$ out of the action integral. Using this, one finds that the action contains three components [App84]

$$S = -\int d^4x\sqrt{-g}\phi\left(\frac{R}{16\pi G} + \frac{1}{4}\phi^2 F_{\alpha\beta}F^{\alpha\beta} + \frac{2}{3\kappa^2}\frac{\partial^\alpha\phi\partial_\alpha\phi}{\phi^2}\right) \tag{2.9}$$

where G is defined in terms of $5D$ counterpart $\hat{G}$:

$$G \equiv \hat{G}/\int dy$$

Now if one takes $\phi = constant$ in this case, then the first two terms of this action are just the Einstein's action for gravity and Maxwell's action for electromagnetic radiation respectively. The third term in the action (2.9), is the action for a massless Klein-Gordon scalar field.

The fact that the source-less equation (2.1) leads to the equations (2.6) with source terms was the biggest achievement of Kaluza-Klein theory. The $4D$ matter is shown purely as a manifestation of $5D$ geometry.

2.2.2 The case $A_\alpha = constant$

If ϕ is not kept constant in Kaluza-Klein theory, it contains Brans-Dicke type scalar field theory besides General relativity and Electromagnetism. This can be seen by setting $A_\alpha = 0$. This constraint limits us to "graviton -scalar" sector of Kaluza-Klein theory. In this case the metric (2.5) becomes block diagonal:

$$g_{AB} = \begin{pmatrix} g_{\alpha\beta} & 0 \\ 0 & \phi^2 \end{pmatrix} \tag{2.10}$$

With this metric and the field equation (2.1), the effective 4D action becomes:

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R\phi \tag{2.11}$$

This is the special case of Brans-Dicke action with $\omega = 0$.[BD87]

$$S = -\int d^4x\sqrt{-g}\bigg(\frac{R\phi}{16\pi G} + \omega\frac{\partial^\alpha\phi\partial_\alpha\phi}{\phi^2}\bigg) + S_M \tag{2.12}$$

where ω is the Brans-Dicke constant and S_M is the action for the matter fields. These matter fields may be coupled to the metric or to the scalar field.

By experimental observations [Wil81], the value of ω is constrained to be greater than 500, which implies that this model is not viable, in the present era. This constraint can be satisfied by adding a nonzero potential $V(\phi)$ to the above action [DLB89]. In that case, the Brans-Dicke parameter ω should be allowed to vary as a function of ϕ. In the model that we analyzed in this project, we are getting a similar type of action in which ω is a function of ϕ.

2.3 Compactification Mechanism

Kaluza assumed the "Cylinder Condition" (no physical quantity is dependent on extra dimension) in his calculations, without giving any explanation behind this assumption. Klein came up with a mechanism to explain the physically silent nature of extra dimension [Kle26].

He assumed that the extra coordinate to be a lengthlike one and assigned two key properties to it : (1) A circular topology (S^1) and (2) small length. Due to the property (1), all the quantity $f(x_0, x_1, x_2, x_3, y)$ becomes periodic in y i.e.$f(x, y) = f(x, y + 2\pi r)$ where $x = (x_0, x_1, x_2, x_3)$ and r is the radius of the fifth dimension. Therefore we can now, Fourier-expand all the fields:

$$g_{\alpha\beta}(x,y) = \sum_{n=-\infty}^{n=+\infty} g^{(n)}_{\alpha\beta}(x)e^{iny/r} \qquad A_\alpha(x,y) = \sum_{n=-\infty}^{n=+\infty} A^{(n)}_\alpha(x)e^{iny/r}$$

$$\phi(x,y) = \sum_{n=-\infty}^{n=+\infty} \phi^{(n)}(x)e^{iny/r}$$

where the superscript $^{(n)}$ refers to the nth Fourier mode. Now from the quantum mechanics, we know that these modes carry a momentum of the order $|n|/r$ in y-direction. Now property (2) says that the radius of extra dimension is very small.

Now, if radius r of extra dimension is very small, then the y-momenta of even the $n = 1$ modes will become very large and hence making it unaccessible by the experiments. Therefore, the only visible modes are the $n = 0$ modes, which are y-independent, as required in Kaluza's theory.

Till now the experiments of the kind [KS91] constrain r to be less than $10^{-18}m$ in size. Theorists often set r equal to the Planck length $l_{pl} \backsim 10^{-35}m$, which is the natural value (obtained by dimensional analysis). This value is small enough to make all the $n \neq 0$ Fourier modes unaccessible.

In general, the five-dimensional metric (2.5) is the one containing all the Fourier modes. One then makes a "Kaluza - klein ansatz" in which all the massive $(n \neq 0)$ fourier modes are discarded. In the five-dimensional case, "Kaluza-Klein ansatz" amounts to simply dropping y dependence in $g_{\alpha\beta}, A_\alpha$ and ϕ. This gives us the effective "low energy" theory containing the graviton $g^{(0)}_{\alpha\beta}$, the photon $A^{(0)}_\alpha$ and the scalar $\phi^{(0)}$. In higher dimensions, the relationship between Kaluza-Klein ansatz and the full metric is a bit complicated [Duf86].

2.4 Extension to Higher Dimensions

In order to incorporate strong and weak nuclear interactions in Kaluza-Klein formalism, one needs to recognize that electromagnetism was incorporated into general relativity by adding $U(1)$ local gauge invariance to the theory. This was done by imposing local coordinate invariance with respect to the extra dimension $(y = x^4)$.

Assuming the extra dimension to be circular and small, we know that the theory is now invariant under the coordinate transformation:

$$y \rightarrow y^{'} = y + f(x) \tag{2.14}$$

where $x = (x^1, x^2, x^3, x^4)$. Now the metric transforms as :

$$\hat{g}_{AB} \rightarrow \hat{g}^{'}_{AB} = \frac{\partial x^C}{\partial x'^A}\frac{\partial x^D}{\partial x'^B}\hat{g}_{CD} \tag{2.15}$$

Under the transformation (2.14), the only change in metric (2.5) is given by :

$$A_\alpha \to A^{'}_\alpha = A_\alpha + \partial_\alpha f(x) \tag{2.16}$$

which is just $U(1)$ local guage transformation. Hence by imposing cylinder condition (i.e. invariance along extra dimension), one is basically imposing $U(1)$ guage invariance. It is thus not surprising that we were able to incorporate electromagnetism and general relativity in a single five dimensional theory.

The same approach can be extended to include strong and weak nuclear forces, one just needs to incorporate the corresponding symmetry groups and hence have to include higher dimensions. The corresponding "Kaluza-Klein ansatz" can be written as :

$$\hat{g}^{(0)}_{AB} = \begin{pmatrix} g_{\alpha\beta} + \tilde{g}_{\mu\nu} K^\mu_i A^i_\alpha K^\nu_j A^j_\beta & \tilde{g}_{\mu\nu} K^\mu_i A^i_\alpha \\ \tilde{g}_{\mu\nu} K^\nu_i A^i_\beta & \tilde{g}_{\mu\nu} \end{pmatrix} \tag{2.17}$$

where $\tilde{g}_{\mu\nu}$ is defined to be the metric of the d-dimensional space. Indices $\mu, \nu, ..$ run from 1 to d, $\alpha, \beta, ...$ run from 0 to 3 and $A, B, ...$ run from 0 to $(3 + d)$. Here K^ν_i are the set of linearly independent Killing vectors. Similar to eq. (2.14), the theory can be assumed to be invariant under the transformations:

$$y^\mu \to y^{'\mu} = y^\mu + \sum_{i=1}^{n} p^i(x) K^\mu_i \tag{2.18}$$

where $p^i(x)$ is the set of n infinitesimal parameters. The effect of this transformation on the metric (2.17) is :

$$A^i_\alpha \to A^{i'}_\alpha = A^i_\alpha + \partial_\alpha p^i(x) \tag{2.19}$$

which is a local gauge transformation. Thus higher- dimensional general relativity could in principle contain any gauge theory.

Chapter 3

Randall-Sundrum Model

The Randall-Sundrum model was proposed in 1999 mainly address the Higgs Hierarchy Problem in the Standard Model of the particle physics. This model has been widely explored in order to study the physics of extra dimensions. There are two popular models : the Randall Sundrum I ($RS1$) model and the Randall Sundrum II ($RS2$) model. In the $RS1$ model, a small extra dimension is considered but in $RS2$ model a large extra dimension is considered. These models are explained below in detail.

3.1 The Randall Sundrum I Scenario

In the $RS1$ model a small and finite extra dimension is considered. This model was developed to solve The Hierarchy Problem (3.1.3) of the Standard Model [LR99] of particle physics.

3.1.1 The Model

In the $RS1$ model, the existence of one lengthlike extra dimension is assumed. This dimension is compactified on a circle whose upper and lower half are identified [Gab06]. This means that we work on $S^1/\mathbb{Z}_2$ orbifold where $\mathbb{Z}_2$ is the group $\{-1, 1\}$ and S^1 is the one-dimensional sphere as shown in the fig.(3.1)

This construction gives us two fixed points, $y = 0$ and $y = \pi R \equiv L$. One then considers a four-dimensional world, standing on each of these fixed points. These worlds with $3+1$ dimensions enclosing the $5D$ bulk are called $3-$branes. Two three branes seprated by a distance L in $5D$ bulk are shown in the fig.(3.2)

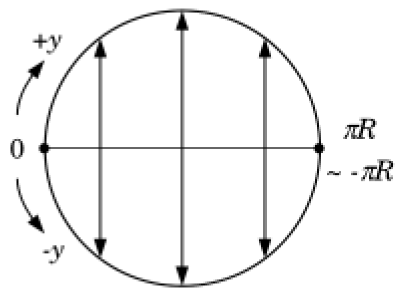

FIGURE 3.1: $S^1/\mathbb{Z}_2$ orbifold

Taking the $5D$ cosmological constant Λ into account, the fundamental action can be written as follows. Note that unlike the effective 4D cosmological constant, the $5D$ Λ does not need to be small.

$$S = S_H + S_M = \int d^4x \int_{-L}^{L} dy \sqrt{-g}(M^3 R - \Lambda) \tag{3.1}$$

where S_H is the Einstein-Hilbert action and S_M is the matter part. Here R is the $5D$ Ricci scalar, M is the fundamental $5D$ mass scale and g is the determinant of $5D$ metric.

3.1.2 The Metric

Now we need to find a suitable metric, satisfying $5D$ Einstein equation, for this setup. Since this $5D$ theory should give us back a flat and static universe in its $4D$ limit, the metric must preserve Poincare invariance. This leads to the following Ansatz: [Gab06]

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{3.2}$$

where $\eta_{\mu\nu} = diag(-1,1,1,1)$ is the $4D$ Minkowski metric. The factor $e^{-2A(y)}$ is called the **warp factor**. Since it is dependent on extra dimension, the metric now is non-factorisable as in usual Kaluza-Klein scenarios. The dependence of $A(y)$ on y can

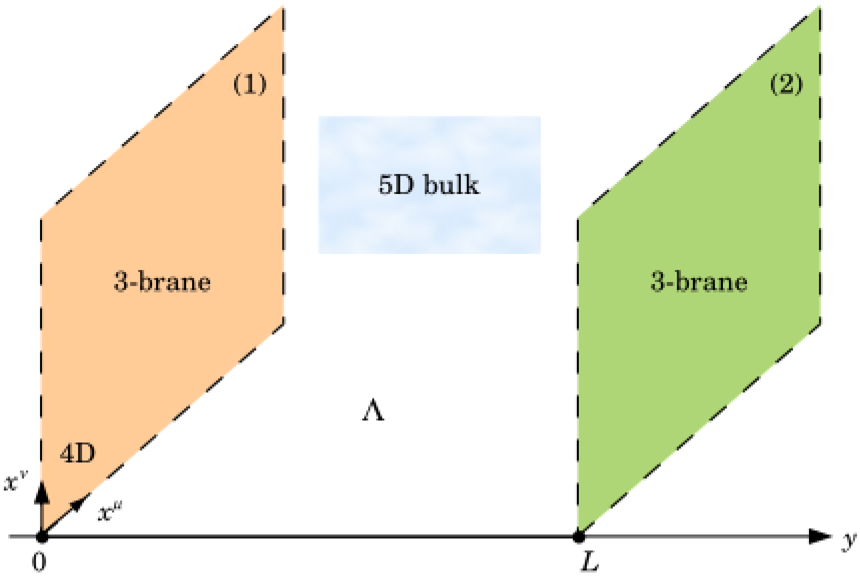

FIGURE 3.2: Randall Sundrum Setup

be found using $5D$ Einstein equations.

$$G_{MN} = R_{MN} - \frac{1}{2} g_{MN} R = \kappa^2 T_{MN} \tag{3.3}$$

where $\kappa^2 \equiv \frac{1}{2M^3}$ is the $5D$ Newton's constant and the energy momentum tensor is defined as :

$$T_{MN} = \frac{-2}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{MN}} \tag{3.4}$$

The Einstein tensor for above ansatz (3.2) can be found easily. The 55 component of Einstein equations give :

$$G_{55} = 6A'^2 = \frac{-\Lambda}{2M^3} \tag{3.5}$$

This implies that A will be real if the $5D$ cosmological constant is negative. This means that the space between the Planck brane and the TeV brane is anti-de Sitter i.e AdS_5. Also from equation(3.5), we see that A'^2 is equal to a constant, lets call it k^2:

$$A'^2 = \frac{-\Lambda}{12M^3} \equiv k^2 \tag{3.6}$$

Integrating over y, we get :

$$A(y) = \pm ky$$

Since we want our solution to respect orbifold's symmetry under the transformation $y \to -y$ we choose

$$A(y) = k|y| \tag{3.7}$$

Finally the metric for Randall-Sundrum model is parametrized by:

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{3.8}$$

with $-L \leq y \leq L$
The $\mu\nu$ components of Einstein's field equations is given by :

$$G_{\mu\nu} = (6A'^2 - 3A'')g_{\mu\nu}$$

From equation (3.7) we find that

$$A' = sgn(y)k$$

Now the term $sgn(y)$ can be written as a combination of Heaviside functions [Sgn]:

$$sgn(y) = \theta(y) - \theta(-y)$$

so now,

$$A'' = 2k\delta(y)$$

This delta function arose from the kink of A at the origin $y = 0$ as shown in fig. (3.3) A similar delta function will appear at $y = L$. So $A'\prime$ can be written as:

$$A'' = 2k(\delta(y) - \delta(y - L))$$

Plugging this in the $\mu\nu$ component of Einstein equation, we get

$$G_{\mu\nu} = 6k^2 g_{\mu\nu} - 6k(\delta(y) - \delta(y - L))g_{\mu\nu} \tag{3.9}$$

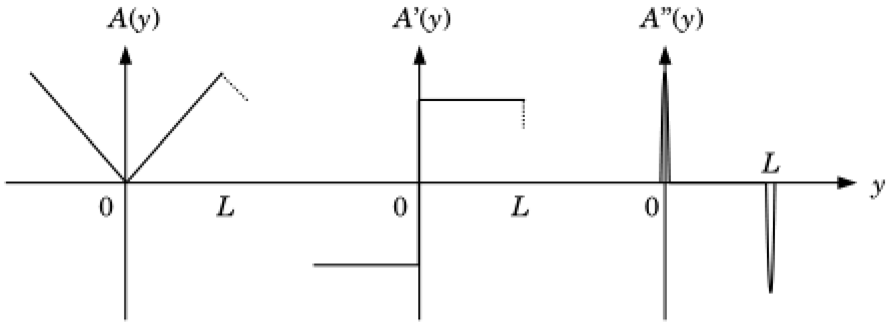

FIGURE 3.3: The function $A(y)$ and its first and second derivatives.

Now the $\mu\nu$ component of energy-momentum tensor can be found using equation (3.4) :

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\mu\nu}} = -\Lambda g_{\mu\nu}$$

Now

$$\kappa^2 T_{\mu\nu} = \frac{1}{2M^3}(-\Lambda g_{\mu\nu}) = 6k^2 g_{\mu\nu}$$

So the first component of equation (3.9) is identified by $\kappa^2 T_{\mu\nu}$. The second term can be obtained by taking the energy densities of the branes themselves into account. These terms are called the brane tensions.

$$S_1 = -\int d^4x dy \sqrt{-g}\lambda_1 \delta(y) \quad (3.10a)$$

$$S_2 = -\int d^4x dy \sqrt{-g}\lambda_2 \delta(y - L) \quad (3.10b)$$

where g stands for the determinants of the metrics induced on the Planck brane and the TeV brane. The Einstein equations impose

$$\lambda_1 = -\lambda_2 = 12kM^3 \quad (3.11)$$

3.1.3 The Hierarchy Problem

Three out of four fundamental forces viz. strong, weak and electromagnetic are well explained by the Standard Model, but it still have some unattractive features.

One of such features is the gauge Hierarchy Problem, which refers to the large discrepancy between the aspects of weak force and gravity (weak force is 10^{32} times greater than gravitational force) [Her]. This discrepancy is a problem because if we calculate the quantum corrections to Fermi constant (constant for weak force) using the Standard model, it appears to be very large and closer to Newton's constant (gravitational force constant), unless the quantum corrections are delicately canceled the bare value of Fermi constant. This fine tuning constitutes "The Hierarchy Problem". A number of solutions have been proposed to solve this problem, namely Supersymmetry [Mar11], Conformal Standard Model [MN06] etc. One such proposed solution is the Randall-Sundrum Model [LR99].

Solution to Hierarchy Problem

The proposed solution to above problem using RS model is as follows:
Consider the Higgs scalar field on the visible brane in RS model, the action for it can be written as :

$$S_{Higgs} = \int d^4x \sqrt{g_2} \quad [g_2^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda(H^\dagger H - v^2)^2]$$

where g_2 is the determinant of effective metric on visible brane, V is the Higgs vacuum expectation value.

$$S_{Higgs} = \int d^4x e^{-4KL}[e^{2KL}\eta^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda(H^\dagger H - v^2)^2]$$

This action can be re-written in canonical form by redefining H as $H = e^{KL}\tilde{H}$. Then the action becomes

$$S_{Higgs} = \int d^4x[\eta^{\mu\nu} D_\mu \tilde{H}^\dagger D_\nu \tilde{H} - \lambda(\tilde{H}^\dagger \tilde{H} - (e^{-KL}v)^2)^2] \tag{3.12}$$

This is the action of a normal Higgs scalar but its vacuum expectation value (VeV) is exponentially suppressed.

$$v_{eff} = e^{-KL}v \tag{3.13}$$

Now all the mass parameters in Standard model are set by Higgs VeV, hence all the mass parameters suffer an exponential suppression on the visible brane. Now the

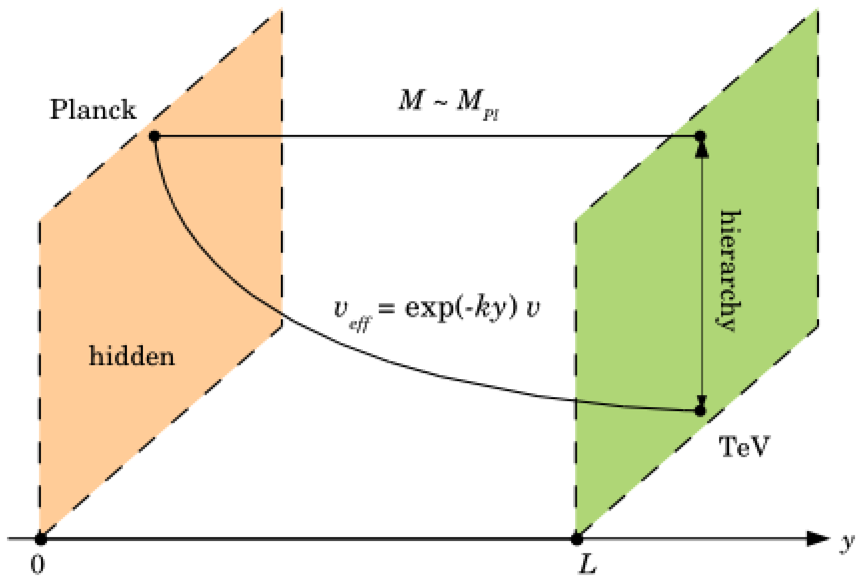

FIGURE 3.4: Generation of exponential Hierarchy

fermi constant (ignoring the muon mass against the mass of the W boson) is given by [Fec]

$$G_F = (\hbar c)^3 \frac{\sqrt{2}}{8} \frac{g^2}{m_w^2} \tag{3.14}$$

where m_w is the mass of W boson (mediator of weak force) and g is the coupling constant. Now we have seen that m_w will undergo exponential suppression (because of Higgs VeV), this implies that G_F will increase exponentially on visible brane.

Let us now examine what happens to gravitational force when it reaches to visible brane from the planck brane. For this we need to calculate the effective $4D$ action and find the $4D$ planck mass. This is done by perturbing the action (3.1) around the background metric (3.2) [Gab06].

$$S \ni M^3 \int d^4x \int_{-L}^{L} dy e^{-2k|y|} \sqrt{-g^{(0)}} R^{4D}(h^{(0)}_{\mu\nu})$$

$$= M^3 \frac{1 - e^{-2KL}}{k} \int d^4x \sqrt{-g^{(0)}} R^{4D}(h^{(0)}_{\mu\nu})$$

Therefore, the effective $4D$ planck mass is given by

$$M_{pl}^2 = \frac{1 - e^{-2KL}}{k} M^3 \tag{3.15}$$

We see that it is very weakly dependent on the size of extra dimension. Therefore, we conclude that the Newton's constant given by

$$G = \frac{\hbar c}{m_{pl}^2}$$

doesn't change much from planck to TeV brane.
From the last two results we conclude that the weak scale is exponentially suppressed while the gravity scale not changed when an extra dimension is added. (see fig. (3.4)).
On the TeV brane mass of W boson and planck mass are related by $M_W \simeq 10^{-16} M_{pl}$, therefore, the appropriate size of extra dimension which can generate this Hierarchy is given by

$$kL \simeq ln10^{16} \simeq 35 \tag{3.16}$$

3.1.4 Radius Stabilization

Till now we have considered radius of extra dimension was treated as a free parameter in our theory and its value was put in by hand in order to solve the Hierarchy problem. However, this degree of freedom implies that there exists a massless scalar field, corresponding to the fluctuations of the radius, let us call it that field: the radion. This massless radion field would result in a long range fifth force and hence will violate the equivalence principle [PS]. Therefore, in order to preserve the viability of the RS model, the radion must to be stabilized.

Goldberg and Wise [GW99] in 1999 came up with a mechanism to stabilize the radius of extra dimension. It involves adding a massive scalar field ϕ in bulk. A potential $V(\phi)$ is added in bulk and two potentials $V_1(phi)$ and $V_2(phi)$ are added on the two branes. The corresponding action is :

$$S = \int d^4xdy\sqrt{-g}[M^3R + \frac{1}{2}\partial^M\phi\partial_M\phi - V(\phi) - V_1(\phi)\delta(y) - V_2(\phi)\delta(y-L)]$$

Poincare invariance restricts the dependence of ϕ only on y coordinate. The scalar field equation is given by:

$$\phi'' - 4A'\phi' = \frac{\partial V}{\partial \phi} + \frac{\partial V_1}{\partial \phi}\delta(y) + \frac{\partial V_2}{\partial \phi}\delta(y-L) \tag{3.17}$$

The $\mu\nu$ and 55 components of Einstein equations give:

$$2A'^2 - A'' = \frac{\kappa^2}{6}\phi'^2 - \frac{\kappa^2}{3}(V + V_1\delta(y) + v_2\delta(y-L)) \tag{3.18}$$

$$A'^2 = \frac{\kappa^2}{12}\phi'^2 - \frac{\kappa^2}{6}V(\phi) \tag{3.19}$$

These three equations (3.17), (3.18) and (3.19) form a gravity-scalar system which are usually difficult to solve for an arbitrary potential. The value of radius is thus determined by the equation of motion. Therefore, the solution to Hierarchy problem provided by RS model doesn't arise at the cost of another fine-tuning.

3.2 The Randall Sundrum II Scenario

The geometry of RS2 model is similar to that of RS1 model i.e. a circular extra dimension with AdS_5 geometry. But in this case, the extra dimension is considered to be infinitely large, which is equivalent to saying that there is only one brane (the Planck brane) and no TeV brane. All the standard model particles are presumed to stay on the Planck brane. Such an infinitely large dimension would easily have escaped our attention. In this model, the gravitational fluctuations on the brane reproduces Newtonian gravity (More details can be found in [LL04]). This model is of interest for the study of the AdS/CFT conjecture.

Chapter 4

Cosmology of Randall-Sundrum Model

4.1 FRW Cosmology

4.1.1 Simplifying assumptions of Cosmology

In order understand the complete evolution of universe, we need to solve the Einstein equation for all the gravitating objects present in the universe. This system will be impossible to solve. Fortunately, the real universe appears to be much more simpler by making a reasonable simplifying assumption known as the Cosmological Principle.

Cosmological Principle

One of the most important assumption of cosmology is contained in the cosmological principle [Nar].

It states that at any given cosmic time, the universe is homogeneous and isotropic.

The surfaces shown below in fig. (4.1) satisfy the above two properties.
Let us derive the metric for a positive curvature surface i.e a 3-sphere. Now the 3-sphere satisfies

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = S^2$$

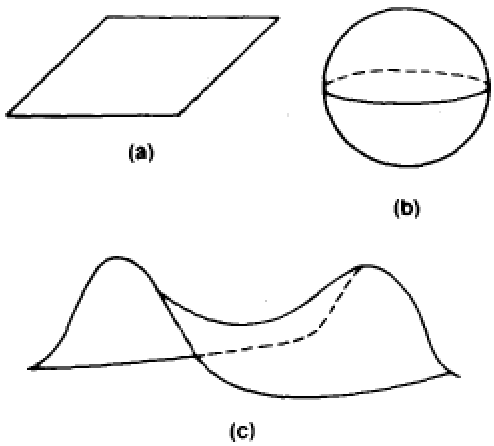

FIGURE 4.1: Examples of surfaces of (a) zero curvature (b) positive curvature, and (c) negative curvature

which is satisfied if we choose:

$$x_4 = aCos\xi \qquad x_1 = aSin\xi Cos\theta$$
$$x_2 = aSin\xi Sin\theta Cos\phi \qquad x_3 = aSin\xi Sin\theta Sin\phi$$

Therefore, any line element on this surface is given by

$$d\sigma^2 = a^2[d\xi^2 + Sin^2\xi(d\theta^2 + sin^2\theta d\phi^2)]$$

where $\xi \in [0, \pi]$, $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$. Let $r = Sin\xi$ then

$$d\sigma^2 = a^2\left[\frac{dr^2}{1 - r^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right]$$

The surface we considered (sphere) had a positive curvature, for negative curvature surface we have $1 + r^2$ in the first term. Therefore, in general, we can write.

$$d\sigma^2 = a^2\left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{4.2}$$

where k is the curvature of space. Using equation (4.2), we can write down the cosmological line element as follows:

$$ds^2 = c^2dt^2 + a^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{4.3}$$

4.1.2 FRW Equations of motion

For the line element derived above (4.3), one can write down Einstein field equations [Nar]:

$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G\rho}{3} + \frac{\lambda c^2}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\lambda c^2}{3}$$

where λ is the cosmological constant and the universe has been modeled via Energy momentum tensor of a perfect fluid. i.e. $T = diag(\rho, -p, -p, -p)$

4.2 Cosmology of Randall Sundrum Model

We here examine the cosmology of Randall Sundrum model. In Chapter 3, we have considered RS model in static background. In order to model the real expanding universe on TeV brane, we need to analyze RS model in presence of FRW metric given in equation (4.3). For simplicity, we consider flat spacetime i.e. $k = 0$. Now the action for RS1 model is given by :

$$\begin{aligned} S &= 2\int d^4x \int_0^{1/2} dy\sqrt{-G}(M^3R - \Lambda) + \int d^4x\sqrt{-g^{(+)}}(L^+ - V^+) \\ &\quad + \int d^4x\sqrt{-g^{(-)}}(L^- - V^-) \end{aligned} \tag{4.4}$$

where Λ is the 5D cosmological constant, V^+ and V^- are the brane tensions on positive and negative branes respectively and L^+, L^- are the matter actions on Planck and TeV brane respectively. Since we model the expanding universe, we choose the following metric [RT99]

$$ds^2 = e^{-2m_0 b(t)|y|}g_{\mu\nu}dx^\mu dx^\nu + b^2(t)dy^2 \tag{4.5}$$

where $b(t)$ is the scale factor corresponding to the extra dimension and the $4D$ metric $g_{\mu\nu}$ is given by equation (4.3) for spatially flat universe ($k = 0$) i. e.

$$g_{\mu\nu} = diag(1, -a^2(t), -a^2(t), -a^2(t))$$

We are looking for the solutions corresponding to $\dot{b} = 0$ i.e solutions with stabilized radius of extra dimension.

We integrate out y from the action in order to find the effective dynamics of our brane. After integration, the effective action is found to be (Details can be found in Appendix A):

$$S = \frac{3}{\kappa^2 m_0} \int d^4x\, a^3 \{ \frac{\dot{a}^2}{a^2}[1 - \Omega_b^2] + m_0 \Omega_b^2 \frac{\dot{a}}{a}\dot{b} - \frac{m_0^2}{4}\Omega_b^2 \dot{b}^2 \} + \frac{1}{\kappa^2} \int d^4x\, 2m_0 a^3 (1 - \Omega_b^4)$$

where $2M^3 = \frac{1}{\kappa^2}$ and $\Omega_b^2 = e^{-m_0 b}$.

The cross term in the above action can be removed by integration by parts. Therefore, the above action can be re-written as :

$$S = \frac{1}{2\kappa^2 m_0} \int d^4x a^3 [(1 - \Omega_b^2)R - \frac{3}{2} m_0^2 \Omega_b^2 \dot{b}^2] - \int d^4x a^3 V_r(b) \tag{4.6}$$

where

$$\Omega_b = e^{-m_0 b/2}$$
$$R = -6[\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}]$$
$$V_r(b) = \frac{1}{\kappa^2} 2m_0 (1 - \Omega_b^4)$$
$$\kappa^2 = \frac{1}{2M^3}$$

Now let us consider the presence of non-relativistic matter on TeV brane and no matter is considered on Planck brane. In this case, the effective action becomes:

$$\begin{aligned} S &= \frac{1}{2\kappa^2 m_0} \int d^4x a^3 [(1 - \Omega_b^2)R - \frac{3}{2} m_0^2 \Omega_b^2 \dot{b}^2 - V(b)] + \int d^4x \sqrt{-g^{(-)}} L^- \\ &= \frac{1}{2\kappa^2 m_0} \int d^4x a^3 [(1 - \Omega_b^2)R - \frac{3}{2} m_0^2 \Omega_b^2 \dot{b}^2 - V(b) + \Omega_b^4 S_M^{TeV}] \end{aligned}$$

where $V(b) = 4m_0^2(1-\Omega_b^4)$ and S_M^{TeV} is the matter term for TeV brane. This action can be converted into the well-known Brans-Dicke form [BD61] by defining $\phi = 1-\Omega_b^2$

$$S = \frac{1}{2\kappa^2 m_0}\int d^4xa^3[\phi R - \frac{3}{2}(\frac{\phi}{1-\phi})\frac{\dot{\phi}^2}{\phi}] - \int d^4xa^3V_r(\phi) \tag{4.8}$$

where $\kappa^2 m_0 \equiv 8\pi G$ and the Brans Dicke parameter $\omega(\phi)$ is given by:

$$w(\phi) = \frac{3}{2}(\frac{\phi}{1-\phi})$$

Equations of motion for this system are given by [Sct] :

$$3H^2 = \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 3H\frac{\dot{\phi}}{\phi} - \frac{1}{2}\frac{V(\phi)}{\phi} + \frac{\kappa^2 m_0}{\phi}(1-\phi)^2\rho \tag{4.9a}$$

$$2\dot{H} + 3H^2 = -\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{\ddot{\phi}}{\phi} - 2H\frac{\dot{\phi}}{\phi} - \frac{1}{2}\frac{V(\phi)}{\phi} \tag{4.9b}$$

$$(2\omega+3)(\ddot{\phi}+3H\dot{\phi}) = -\frac{\partial\omega}{\partial\phi}\dot{\phi}^2 - 2V(\phi) + \phi\frac{\partial V}{\partial\phi} - \kappa^2 m_0(1-\phi)^2\rho \tag{4.9c}$$

where $H = \frac{\dot{a}}{a}$ and $V(\phi) = 4m_0^2\phi(2-\phi)$. Also, note that we have taken matter dominated era i.e. we have ignored pressure density term.

The only unknown variables in our system are $a(t)$ and $b(t)$, therefore, two of the above equations for analysis (one of the equations is redundant), Therefore, we use equation (4.9a) which gives the evolution of scale factor $a(t)$ and the equation (4.9c) which gives the evolution of radius of extra dimension $b(t)$.

The matter density, ρ can be re-written in the form of density parameter [Sch], which is defined as the ratio of observed matter density ρ to the critical matter density $p_{critical}$ of the Friedmann universe. Therefore, Ω_M is given by:

$$\Omega_M \equiv \frac{\rho}{\rho_{critical}}$$

The relation between ρ and $\rho_{critical}$ determines the overall geometry of universe. The expression for critical density can be found from the FRW equations

$$\rho_{critical} = \frac{3H^2}{8\pi G}$$

Therefore, we get

$$\rho = \frac{3H^2}{8\pi G}\Omega_M$$

Now the evolution of ρ with time is given by :

$$\rho = \rho_0 a^{-3}$$

where ρ_0 is the matter density at present. The above expression can be derived by considering the expansion of universe to be adiabatic [Sch]. Now, the present matter density ρ_0 is given by :

$$\rho_0 = \frac{3H_0^2}{8\pi G}\Omega_{M0}$$

where H_0 and Ω_{M0} are the values of Hubble parameter and non-relativistic matter density at present epoch. Therefore,

$$\rho = \frac{3H_0^2}{8\pi G}\Omega_{M0}a^{-3} \tag{4.10}$$

Using equation (4.10) in equations (4.9a) and (4.9c) and substituting for $V(\phi)$ and $\omega(\phi)$ we get:

$$3H^2 = \frac{3}{4}\frac{\phi}{(1-\phi)}\frac{\dot{\phi}^2}{\phi^2} - 3H\frac{\dot{\phi}}{\phi} - 2m_0^2(2-\phi) + \frac{3H_0^2}{\phi}(1-\phi)^2\frac{\Omega_{M0}}{a^3} \tag{4.11a}$$

$$(\frac{3}{1-\phi})(\ddot{\phi} + 3H\dot{\phi}) = -\frac{3}{2}\frac{\dot{\phi}^2}{(1-\phi)^2} - 8m_0^2\phi - 3H_0^2(1-\phi)^2\frac{\Omega_{M0}}{a^3} \tag{4.11b}$$

where we have used $\kappa^2 m_0 = 8\pi G$

In the next Chapter, we analyze these equations and attempt to obtain numerical solutions.

Chapter 5

Numerical Analysis and Results

5.1 Analyzing Equations of Motion

In this section, we analyze the equation (4.11a) and (4.11b) by comparing the order of different terms. We can put the factor c back in the equations (4.11a) and (4.11b) by dimensional analysis. The dimensions of the quantities appearing in equation of motion are:

$$[m_0] = [L^{-1}] \qquad [H] = [T^{-1}]$$
$$[H_0] = [T^{-1}]$$

All the other quantities like ϕ, Ω_{M0} and a^3 are dimensionless. We define dimensionless variables $y = \frac{H}{H_0}$, $x = tH_0$. These equations can be re-written as:

$$y^2 = \frac{1}{4\phi}\frac{\phi'^2}{(1-\phi)} - y\frac{\phi'}{\phi} - \frac{2m_0^2c^2}{3H_0^2}(2-\phi) + \frac{(1-\phi)^2}{\phi}\frac{\Omega_{M0}}{a^3} \tag{5.2a}$$

$$\phi'' + 3y\phi' = -\frac{1}{2}\frac{\dot{\phi}}{(1-\phi)^2} - \frac{8m_0^2c^2}{3H_0^2}\phi(1-\phi) - (1-\phi)^3\frac{\Omega_{M0}}{a^3} \tag{5.2b}$$

where $'$ denote the derivative w.r.t x.

In order to solve the Hierarchy Problem, we need [3.16] $m_0 b \simeq ln10^{16} \simeq 35$. We also know that the present experiments [KS91] constrain the value of radius of extra dimension to be below $10^{-18}m$. This implies:

$$b \sim 10^{-18}m$$

i.e.

$$m_0 \sim 10^{19} m^{-1}$$

$$\phi = 1 - e^{-m_0 b} \sim 1$$

$$1 - \phi = e^{-m_0 b} \sim 10^{-16}$$

In S.I

$$H_0 \sim 10^{-18} s^{-1}$$

This implies

$$\phi' \sim \frac{\delta\phi}{\delta(tH_0)} \sim 10^{20}$$

if we consider $\delta t = 0.01$. Now let us compare the orders of different terms in equation (5.5a):

$$\frac{1}{4\phi}\frac{\phi'^2}{(1-\phi)} \sim 10^{46}$$
$$y\frac{\phi'}{\phi} \sim 10^{20}$$
$$\frac{2m_0^2c^2}{3H_0^2}(2-\phi) \sim 10^{90}$$
$$\frac{(1-\phi)^2}{\phi}\frac{\Omega_{M0}}{a^3} \sim 10^{-23}$$

So equation (5.5a) can be approximated as:

$$y^2 = -\frac{2m_0^2c^2}{3H_0^2}(2-\phi)$$

Since $\phi \leq 1$, the above equation gives an imaginary solution for y and hence an imaginary H, which is completely against intuition as well as cosmological observation.

So we conclude that a viable cosmology is not obtained, if we analyze cosmology of RS model in the absence of any radion stability mechanism. This result is in agreement with the results presented in [DL99], [GS99] and [RT99]. In [RT99], it is argued that, we get an additional constraint on our system when we try to analyze cosmology of RS model in the absence of Radion stability mechanism. This additional constraint is in terms of a relation between the matter density on visible brane and those on hidden brane (Note that in our case no matter density is added on hidden brane). This constraint is given by:

$$\rho_* = -\rho\Omega_0^2 \tag{5.3}$$

They have also shown that the matter density on Planck brane is positive i.e. $\rho_* > 0$, therefore, the constraint (5.3) implies that matter density on visible brane must be negative. Therefore, this constraint also implies that no viable cosmology is possible in this scenario.

In our case, the constraint can obtained from the G_{55} equation which can be obtained by varying the full action function given in equation (A.1). Note that the values of $5D$ cosmological constant λ and that of the brane tensions λ_1, λ_2 that we have used, are obtained for the static case i.e. $\dot{b} = 0$. When we write down Einstein equations for dynamic case i.e. $\dot{b} \neq 0$ and use the same values for $5D$ cosmological constant and brane tensions, we will get an additional constraint. We have not checked whether this constraint and the constraint presented in [RT99] are the same or not. It is expected that both these constraints will be same because their origin is quite similar.

5.2 Cosmology in the presence of Radion potential

In [RT99], it is shown that to obtain a viable cosmology, an addition radion potential has to be added alongwith the 4D cosmological metric.

Let us examine, whether the addition of a radion potential in our case, can give us a viable cosmology or not. Let us add a radion potential of the form

$$U(\phi) = K\phi(2-\phi)$$

The effective $4D$ action is given by:

$$S = \frac{1}{2\kappa^2 m_0}\int d^4xa^3[\phi R - \frac{3}{2}(\frac{\phi}{1-\phi})\frac{\dot{\phi}^2}{\phi}] - \int d^4xa^3V_r(\phi) - \int d^4xa^3U(\phi) \tag{5.4}$$

This will just add $U(\phi)$ term in equation (5.5a) and the corresponding term in equation (5.5b). So the equations of motion for this system become :

$$y^2 = \frac{1}{4\phi}\frac{\phi'^2}{(1-\phi)} - y\frac{\phi'}{\phi} - \left[\frac{2m_0^2c^2}{3H_0^2} + \frac{K}{6}\right](2-\phi) + \frac{(1-\phi)^2}{\phi}\frac{\Omega_{M0}}{a^3} \tag{5.5a}$$

$$\phi'' + 3y\phi' = -\frac{1}{2}\frac{\dot{\phi}}{(1-\phi)^2} - \left[\frac{8m_0^2c^2}{3H_0^2} + 2K\right]\phi(1-\phi) - (1-\phi)^3\frac{\Omega_{M0}}{a^3} \tag{5.5b}$$

We can now fine-tune K such that the order of potential term is similar to that of the other terms. This fine tuning will may us a viable cosmology on the visible brane.

The constraint can also be removed in this case. By appropriately choosing the exact form of $U(\phi)$, the constraint equation obtained from G_{55} equation, can be automatically satisfied. Therefore, the system is not over constrained in this case.

Hence we conclude that a viable cosmology may be obtained from Randall Sundrum model, if we add a radion potential.

Chapter 6

Conclusions and Outlook

We have analyzed the cosmology of Randall Sundrum model by using a $4D$ FRW metric with a time-dependent warp factor. We also choose the radius of extra dimension b as a dynamic quantity in our system. We have also taken into account the bulk cosmological constant and the brane tensions, which was ignored in most of the papers analyzing similar setup.

We found that the radion potential (which gets a significant contribution from the bulk cosmological constant term and the brane tensions) generated in this cosmological setup, becomes very large and hence generates unconventional cosmology on the visible brane. This can be attributed to the fact that when radius of extra dimension is taken as a dynamic quantity, a constraint appears when $5D$ Einstein equations are written. Due to this, the system becomes over-constrained and hence cosmological solutions are absent.

We then analyzed this system by adding a radion stabilizing potential. This system is expected to give back the conventional cosmology on visible brane. This happens because this additional potential can now be fine tuned to cancel the effects of the potential generated by the cosmological setup. In this case, the additional constraint can also be avoided by choosing appropriate form of radion potential.

So we conclude that, a viable cosmology is possible in Randall Sundrum setup only in the presence of radion stabilizing potential.

Future Directions

We will explicitly calculate the exact form of radion potential that needs to be added in order to cancel the effects of potential generated via cosmological metric. Also, we need to explicitly check whether this form of potential eliminates the constraint equations, as expected or not.

In the analysis presented here, we have ignored the matter fields on Planck brane. Once we take those into account, we can then compare the results we obtain with [RT99] paper in which they have ignored the presence of bulk cosmological constant and the brane tensions.

Appendix A

4D Effective Action

$$
\begin{aligned}
S &= 2\int d^4x \int_0^{1/2} dy \sqrt{-G}(M^3R - \Lambda) + \int d^4x \sqrt{-g^{(+)}}(L^+ - V^+) \\
&\quad + \int d^4x \sqrt{-g^{(-)}}(L^- - V^-)
\end{aligned} \tag{A.1}
$$

where

$$V^+ = -V^- = 12m_0M^3$$

$$\Lambda = -12m_0^2M^3$$

Now the line element we choose is given by :

$$ds^2 = e^{-2mb(t)|y|}g_{\mu\nu}dx^\mu dx^\nu + b^2(t)dy^2 \tag{A.2}$$

$$g_{\mu\nu} = diag(1, -a^2(t), -a^2(t), -a^2(t)$$

Taking the first term from action(A.1), the action can be written as

$$
\begin{aligned}
S &= 2\int d^4x \int_0^{1/2} dy \sqrt{-G}(M^3R - \Lambda) \\
&= 2M^3 \int d^4x \int_0^{1/2} dy \sqrt{-G}(R + 12m_0^2)
\end{aligned} \tag{A.3}
$$

Now from the metric (A.2), we can found the Ricci tensor(R) to be :

$$\begin{aligned}
R &= 20m_0^2 + \{-6\frac{\dot{a}^2}{a^2} - 6\frac{\dot{a}}{a}\frac{\dot{b}}{b} - 6m_0^2y^2\dot{b}^2 - 6\frac{\ddot{a}}{a} - 2\frac{\ddot{b}}{b}\}e^{2m_0b(t)y} \\
&+ye^{2m_0b(t)y}\{18m_0\frac{\dot{a}\dot{b}}{a} + 4m_0\frac{\dot{b}^2}{b} + 6m_0\ddot{b}\} \\
R + 12m_o^2 &= 32m_0^2 + \{-6\frac{\dot{a}^2}{a^2} - 6\frac{\dot{a}}{a}\frac{\dot{b}}{b} - 6m_0^2y^2\dot{b}^2 - 6\frac{\ddot{a}}{a} - 2\frac{\ddot{b}}{b}\}e^{2m_0b(t)y} \\
&+ye^{2m_0b(t)y}\{18m_0\frac{\dot{a}\dot{b}}{a} + 4m_0\frac{\dot{b}^2}{b} + 6m_0\ddot{b}\} \\
\sqrt{-G}(R + 12m_0^2) &= 32m_0^2a^3be^{-4m_0b(t)y} + \{-6\frac{\dot{a}^2}{a^2} - 6\frac{\dot{a}}{a}\frac{\dot{b}}{b} - 6m_0^2y^2\dot{b}^2 - 6\frac{\ddot{a}}{a} - 2\frac{\ddot{b}}{b}\} \\
&a^3be^{-2m_0b(t)y} + a^3bye^{-2m_0b(t)y}\{18m_0\frac{\dot{a}\dot{b}}{a} + 4m_0\frac{\dot{b}^2}{b} + 6m_0\ddot{b}\}
\end{aligned}$$

where $G = -a^6b^2e^{-8m_0b(t)y}$ is the determinant of metric (A.2). Evaluating the y integral of (A.3), term by term

First term is given by:

$$\begin{aligned}
I &= \int_0^{1/2} 32m_0^2a^3be^{-4m_0b(t)y}dy \\
I &= 8a^3m_0[1 - e^{2m_0b(t)}] \qquad \text{(A.4)}
\end{aligned}$$

Dropping a^3 factor for some time., the second term is given by:

$$\begin{aligned}
II &= \{-6\frac{\dot{a}^2}{a^2} - 6\frac{\dot{a}}{a}\frac{\dot{b}}{b} - 6\frac{\ddot{a}}{a} - 2\frac{\ddot{b}}{b}\}b\int_0^{1/2} e^{-2m_0b(t)y}dy \\
&= \frac{1}{2m_0}\{-6\frac{\dot{a}^2}{a^2} - 6\frac{\dot{a}}{a}\frac{\dot{b}}{b} - 6\frac{\ddot{a}}{a} - 2\frac{\ddot{b}}{b}\}[1 - e^{-m_0b(t)}] \qquad \text{(A.5)}
\end{aligned}$$

Third term:

$$\begin{aligned}
III &= \int_0^{1/2} -6m_0^2\dot{b}^2 b y^2 e^{-2m_0 b(t) y} dy \\
&= -6m_0^2\dot{b}^2 b[-\frac{1}{4}\frac{e^{-m_0 b}}{m_0 b}\{\frac{1}{2}+\frac{1}{m_0 b}+\frac{1}{m_0^2 b^2}\}+\frac{1}{4m_0^3 b^3}] \\
&= \frac{3}{4}m_0 e^{-m_0 b}\dot{b}^2 + \frac{3}{2}e^{-m_0 b}\frac{\dot{b}^2}{b} + \frac{3}{2m_0}e^{-m_0 b}\frac{\dot{b}^2}{b^2} - \frac{3}{2m_0}\frac{\dot{b}^2}{b^2} \\
&= \frac{3}{4}m_0 e^{-m_0 b}\dot{b}^2 + \frac{3}{2}e^{-m_0 b}\frac{\dot{b}^2}{b} - \frac{3}{2m_0}\frac{\dot{b}^2}{b^2}[1-e^{-m_0 b}]
\end{aligned} \tag{A.6}$$

Fourth term:

$$\begin{aligned}
IV &= \int_0^{1/2} y e^{-2m_0 b(t) y} b\{18m_0\frac{\dot{a}\dot{b}}{a} + 4m_0\frac{\dot{b}^2}{b} + 6m_0\ddot{b}\}dy \\
&= b\{18m_0\frac{\dot{a}\dot{b}}{a} + 4m_0\frac{\dot{b}^2}{b} + 6m_0\ddot{b}\}\{\frac{1}{4b^2m_0^2}(1-e^{-m_0 b}) - \frac{1}{4m_0 b}e^{-m_0 b}\} \\
&= \frac{9}{2m_0}\frac{\dot{a}}{a}\frac{\dot{b}}{b}(1-e^{-m_0 b}) + \frac{1}{m_0}\frac{\dot{b}^2}{b^2}(1-e^{-m_0 b}) + \frac{3}{2m_0}\frac{\ddot{b}}{b}(1-e^{-m_0 b}) \\
&\quad -\frac{9}{2}\frac{\dot{a}}{a}\dot{b}e^{-m_0 b} - \frac{\dot{b}^2}{b}e^{-m_0 b} - \frac{3}{2}\ddot{b}e^{-m_0 b}
\end{aligned} \tag{A.7}$$

Now adding (A.5),(A.6) and (A.7) , we get

$$\begin{aligned}
II+III+IV &= \frac{1}{2m_0}\{-6\frac{\dot{a}^2}{a^2} - 6\frac{\ddot{a}}{a}\}[1-e^{-m_0 b(t)}] - \frac{6}{2m_0}\frac{\dot{a}}{a}\frac{\dot{b}}{b}[1-e^{-m_0 b(t)}] \\
&\quad -\frac{\ddot{b}}{m_0 b}[1-e^{-m_0 b(t)}] - \frac{3}{2m_0}\frac{\dot{b}^2}{b^2}[1-e^{-m_0 b}] + \frac{3}{4}m_0 e^{-m_0 b}\dot{b}^2 \\
&\quad +\frac{3}{2}e^{-m_0 b}\frac{\dot{b}^2}{b} + \frac{9}{2m_0}\frac{\dot{a}}{a}\frac{\dot{b}}{b}(1-e^{-m_0 b}) + \frac{1}{m_0}\frac{\dot{b}^2}{b^2}(1-e^{-m_0 b}) \\
&\quad +\frac{3}{2m_0}\frac{\ddot{b}}{b}(1-e^{-m_0 b}) - \frac{9}{2}\frac{\dot{a}}{a}\dot{b}e^{-m_0 b} - \frac{\dot{b}^2}{b}e^{-m_0 b} - \frac{3}{2}\ddot{b}e^{-m_0 b}
\end{aligned}$$

Multiplying by a^3 that was dropped earlier, here we also multiply by m_0(will divide by it later)

$$
\begin{aligned}
m_0(II+III+IV) &= -3a\dot{a}^2[1-e^{-m_0b(t)}]-3a^2\ddot{a}[1-e^{-m_0b(t)}]-\frac{3}{2}a^2\dot{a}\frac{\dot{b}}{b}[1-e^{-m_0b(t)}] \\
&+\frac{3}{4}m_0^2a^3e^{-m_0b}\dot{b}^2-\frac{1}{2}a^3\frac{\ddot{b}}{b}[1-e^{-m_0b(t)}]-\frac{1}{2}a^3\frac{\dot{b}^2}{b^2}[1-e^{-m_0b}] \\
&+\frac{1}{2}a^3m_0e^{-m_0b}\frac{\dot{b}^2}{b}-\frac{9}{2}m_0a^2\dot{a}\dot{b}e^{-m_0b}-\frac{3}{2}m_0a^3\ddot{b}e^{-m_0b} \\
&= 3a\dot{a}^2[1-e^{-m_0b(t)}]+3m_0a^2\dot{a}\dot{b}e^{-m_0b}-\frac{3}{4}m_0^2a^3\dot{b}^2e^{-m_0b} \\
&+\frac{d}{dt}[-3a^2\dot{a}[1-e^{-m_0b(t)}]+\frac{1}{2}a^3\frac{\dot{b}}{b}[1-e^{-m_0b(t)}]-\frac{3}{2}a^3m_0\dot{b}e^{-m_0b}]
\end{aligned}
$$

Now dropping the total derivative term in action, the effective action A.1(ignoring the matter part) is given by :

$$
\begin{aligned}
S &= 2M^3\int d^4x(I+II+III+IV)+\int d^4x\sqrt{-g^{(+)}}(-V^+)+\int d^4x\sqrt{-g^{(-)}}(-V^-) \\
&= 2M^3\int d^4x\, 8a^3m_0[1-e^{2m_0b(t)}]+\frac{2M^3}{m_0}\int d^4x\{3a\dot{a}^2[1-e^{-m_0b(t)}]+3m_0a^2\dot{a}\dot{b}e^{-m_0b} \\
&\quad -\frac{3}{4}m_0^2a^3\dot{b}^2e^{-m_0b}\}-\int d^4x\, a^3(12m_0M^3)+\int d^4x\, a^3e^{-2m_0b}(12m_0M^3) \\
&= \frac{3}{\kappa^2m_0}\int d^4x\, a^3\{\frac{\dot{a}^2}{a^2}[1-\Omega_b^2]+m_0\Omega_b^2\frac{\dot{a}}{a}\dot{b}-\frac{m_0^2}{4}\Omega_b^2\dot{b}^2\}+\frac{1}{\kappa^2}\int d^4x\, 2m_0a^3(1-\Omega_b^4)
\end{aligned}
$$

where $2M^3=\frac{1}{\kappa^2}$ and $\Omega_b^2=e^{-m_0b}$.

Bibliography

[App84] T. Appelquist. *Kaluza-Klein theories: problems and prospects*. World Scientific, Singapore, 1984.

[BD61] C. H. Brans and R. H. Dicke. "Mach's Principle and a Relativistic Theory of Gravitation". In: *Physical Review* 124 (1961).

[BD87] C. Brans and R. H. Dicke. "Kaluza-Klein theories". In: *Rep. Prog. Phys.* 50 (1987).

[BL87] D. Bailin and A. Love. "Kaluza-Klein theories". In: *Rep. Prog. Phys.* 50 (1987).

[DL99] Pierre Binetruy ; Cedric Deffayet and David Langlois. "Non-conventional cosmology from a brane-universe". In: *Nucl.Phys.B* (1999).

[DLB89] P. J. Steinhardt D. La and E. W. Bertschinger. "Prescription for successful extended inflation". In: *Phys. Lett. 231B* 231 (1989).

[Duf86] M. J. Duff. "Recent results in extra dimensions". In: *proc. 2nd Jerusalem winter school of theoretical physics* (1986).

[Duf94] M. J. Duff. "Kaluza-Klein theory in perspective". In: *SISSA preprint hep-th/9410046* (1994).

[Ein56] A. Einstein. "The meaning of General Relativity". In: *Princeton Univ. Press, Princeton* (1956).

[Fec] *Fermi's Interaction*. URL: https://en.wikipedia.org/wiki/Fermi%27s_interaction (visited on 08/07/2015).

[Gab06] Maxime Gabella. "The Randall Sundrum Model". In: *IPPC, EPFL* (2006).

[GS99] James M. Cline ; Christophe Grojean and Geraldine Servant. "Cosmological Expansion in the Presence of an Extra Dimension". In: *Phys.Rev.Lett.* 83 (1999).

[GW99] W. Goldberger and M. Wise. “Modulus stabilization with bulk fields”. In: *Physical Review Letters* 83 (1999). URL: http://journals.aps.org/prl/abstract/10.1103/PhysRevLett.83.4922.

[Her] *The Hierarchy Problem*. 2011. URL: http://profmattstrassler.com/articles-and-posts/particle-physics-basics/the-hierarchy-problem/ (visited on 08/12/2015).

[Kal21] T. Kaluza. “Zum Unitätsproblem in der Physik”. In: *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* (1921), 966–972.

[Kle26] O. Klein. “The atomicity of electricity as a quantum theory law”. In: *Nature* 118 (1926).

[KS91] V. A. Kostelecký and S. Samuel. “Experimental constraints on extra dimensions”. In: *Phys. Lett. 270B* 21 (1991).

[Les82] G. Lessner. “Unified field theory on the basis of the projective theory of relativity”. In: *Phys. Rev. D25* (1982).

[LL04] Yoonbai Kim; Chong Oh Lee and Ilbong Lee. “Brane World of Warp Geometry: An Introductory Review”. In: *J. Kor. Astron. Soc.* 37 (2004). URL: http://arxiv.org/abs/hep-th/0307023v2.

[LR99] Randall Lisa and Sundrum Raman. “Large Mass Hierarchy from a Small Extra Dimension”. In: *Physical Review Letters* 83 (1999). URL: arXiv:hep-ph/9905221.

[Mar11] Stephen P. Martin. “A Supersymmetry Primer”. In: (2011). URL: http://arxiv.org/abs/hep-ph/9709356.

[MN06] K. Meissner and H. Nicolai. “Conformal Symmetry and the Standard Model”. In: *Physics Letters B* 648 (2006). URL: https://arxiv.org/abs/hep-th/0612165.

[Nar] J.V. Narlikar. *Introduction to Cosmology*. Cambridge University PRESS.

[Nor14] G. Nordström. “Über die M/"oglichkeit, das elektromagnetische Feld und das Gravitationsfeld zu vereinigen”. In: *Phys. Zeitschr.* 15 (1914).

[OW97] J.M. Overduin and P.S. Wesson. “Kaluza Klein Gravity”. In: *Phys.Rept.* 283 (1997). URL: http://arxiv.org/abs/gr-qc/9805018v1.

[PS] Michael E. Peskin and Daniel V. Schroeder. *An introduction to Quantum Field Theory*. Addison-Wesley Publishing Company.

[Ran] Lisa Randall. *Warped Passages*. HarperCollins e-books.

[RT99] Csaba Csáki ; Michael Graesser ; Lisa Randall and John Terning. "Cosmology of Brane Models with Radion Stabilization". In: *Phys.Rev.D* 62 (1999).

[Sal80] A. Salam. "Gauge unification of fundamental forces". In: *Rev. Mod. Phys* 52 (1980).

[Sch] Peter Schneider. *Extragalactic Astronomy and Cosmology*. Springer.

[Sct] *Scalar-tensor theory*. URL: https://en.wikipedia.org/wiki/Scalar%E2%80%93tensor_theory (visited on 11/06/2015).

[Sgn] *Sign Function*. URL: https://en.wikipedia.org/wiki/Sign_function (visited on 07/15/2015).

[TAF87] A. Chodos T. Appelquist and P. G. O. Freund. "Modern Kaluza-Klein theories". In: *Addison-Wesley, Menlo Park* (1987).

[Whe68] J.A. Wheeler. "Einstein's Vision". In: *Springer-Verlag, Berlin* (1968).

[Wil81] C. M. Will. "Theory and experiment in gravitational physics". In: *Cambridge Univ.Press, Cambridge* (1981).