CP Violation in Quark and Leptonic Sectors

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A dissertation submitted for the partial fulfilment of

BS-MS dual degree in Science

IISER Mohali April 22, 2016

Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other University. This dissertation is the result of my own work and includes nothing which is the outcome of work done in collaboration, except where specifically indicated in the text.

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In my capacity as the supervisor of the candidate's project work, I certify that the aforesaid statements by the candidate are true to the best of my knowledge.

Prof. Manmohan Gupta
(supervisor)

Dated: April 22, 2016

Certificate of Examination

This is to certify that the dissertation titled **CP Violation in Quark and Leptonic Sectors**, submitted by **Mr. Akhil Francis** (Registration Number: MS11030) for the partial fulfilment of the BS-MS dual degree programme of the Indian Institute of Science Education and Research, Mohali, has been examined by the thesis committee duly appointed by the institute. The committee finds the work done by the candidate satisfactory and recommends that the report be accepted.

Prof. Arvind

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Prof. Manmohan Gupta
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Dated: April 22, 2016

Acknowledgements

I express my gratitude to all those who helped me, in making this project a pleasant experience. I would like to extend my sincere thanks to all of them.

I am highly indebted to Prof. Manmohan Gupta for his guidance and constant supervision. I am really obliged to him for giving me this opportunity.

I sincerely thank Dr. Ketan Patel and Dr. Manimala Mitra for the fruitful discussions which helped me to understand the subject better. I would also like to thank Prof. Sudeshna Sinha and Dr. K.P. Yogendran for the mentorship.

I would like to express my gratitude towards my parents and friends for their support and encouragement which helped me in completion of this project.

I acknowledge IISER Mohali for providing best facilities and environment for carrying out the project. I also thank Department of Science and Technology, Govt. Of India for supporting me with INSPIRE fellowship during the past five years.

Abstract

Parity is a symmetry of physical laws except for weak interactions. Combined operations of Charge Conjugation (C) and Parity (P) serves as an approximate symmetry of weak interactions while it is a good symmetry for other physical interactions. Despite being an approximate symmetry which is broken only in certain weak processes it is important to understand the phenomenon. Even this can answer the baryogenesis problem in cosmology. CP violation has been observed in quark sector but not yet in leptonic sector. This projects aims to estimate Jarlskog invariant and correspondingly cp violating phase in leptonic sector using the idea of unitarity triangles taking analogy from the quark sector.

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Chapter 1

Introduction

The composite operation of charge conjugation (C) and parity inversion (P) is not a symmetry of the nature i.e, laws of physics are not the same when particles are replaced with anti particles and spatial coordinates inverted. This is known as CP Violation.

Parity was thought to be an obvious symmetry of physical laws until 1956. Parity conservation has already been experimentally verified in electromagnetic and strong interactions. T.D. Lee and C.N. Yang proposed experiments to check parity conservation in weak interactions. One of their suggested experimental idea on beta decay process was carried out by C.S Wu in 1956 proved that parity is violated in weak interactions. A year after the experiment was performed Lee and Yang received nobel prize. Parity conservation was then replaced by CP conservation. 'CP mirror' was then thought to be an appropriate mirror which not only inverts spatial coordinates(i.e, changing left handed particles to right handed and vice versa) but also replacing particles with its anti particle(i.e, changing the sign of intrinsic quantum numbers like charge). But in 1964 James Cronin and Val Fitch proved CP violation experimentally in nuetral kaon decays. Later CP violation effects were also observed in other experiments like B meson decays etc...

1.1 CP Violation in Neutral Kaon System

There are four strange pseudoscalar mesons K^+, K^-, K^0, and $\overline{K}^0$, which are all eigenstates of strong Hamiltonian interaction. Their quark contents are $K^+ = (\bar{s}u)$, $K^- = (\overline{u}s)$, $K^0 = (\bar{s}d)$, $\bar{K}^0 = (s\overline{d})$. K^+ and K^- are particle - antiparticle pair, so does K^0 and $\overline{K}^0$. Neutral kaons can be produced from the following strong interaction processes in definite strangeness.

$$K^- + p \longrightarrow \overline{K}^0 + n$$

$$K^+ + n \longrightarrow K^0 + p$$

$$\pi^- + p \longrightarrow \Lambda^0 + K^0$$

Strangeness S of $K^0 = 1$and $\bar{K}^0 = -1$. Also $K^0$is identified as the $I_3 = -\frac{1}{2}$ partner of K^+, and $\overline{K}^0$as the $I_3 = \frac{1}{2}$ partner of K^-.Since $K^0$and $\overline{K}^0$are particle anti particle pairs CPT theorem infers that both of them have same mass and life times.

The decay time study of neutral kaons infer that they are not eigenstate of free hamiltonian which would have had exponential decay. The data set infers that neutral kaons are superposition of states with two distinct life times K_s^0short lived one and K_L^0long lived one. K_s^0corresponds to the θ^0 decay channel (two pion decay channel) and $K_L^0$to the τ^0 decay channel (three pion decay channel).[DF94]

$$\theta^0 \longrightarrow \pi^0 + \pi^0$$

$$\theta^0 \longrightarrow \pi^+ + \pi^-$$

$$\tau^0 \longrightarrow \pi^0 + \pi^0 + \pi^0$$

$$\tau^0 \longrightarrow \pi^+ + \pi^- + \pi^0$$

The decay modes and life times observed for$K_s^0$and $K_L^0$are consistent with each other. Since both $K^0$and $\overline{K}^0$ have same decay channels they can mix with each other through higher order processes in weak interaction. That is eventhough $K^0$and $\overline{K}^0$ are states with distinct strangeness they can transform their states to each other through weak interaction as weak interaction does not preserve strangeness.

$$K^0 \xrightarrow{H_{wk}} \pi^0 + \pi^0 \xrightarrow{H_{wk}} \overline{K}^0$$

and similarly for other decay channels.

The CP operations on neutral kaons is given as.

$$CP \mid K^0 >= -C \mid K^0 >= - \mid \overline{K}^0 >$$

$$CP \mid \overline{K}^0 >= -C \mid \overline{K}^0 >= - \mid K^0 >$$

This allows the construction of CP eigenstates K_1^0 and K_2^0 with eigen values 1 and -1 respectively as

$$K_1^0 = \frac{1}{\sqrt{2}}(\mid K^0 > - \mid \overline{K}^0 >)$$

$$K_2^0 = \frac{1}{\sqrt{2}}(\mid K^0 > + \mid \overline{K}^0 >)$$

If CP is conserved then K_1^0 and K_2^0 can be identified as eigenstates of weak interaction i.e, as $K_s^0$and K_L^0 respectively. Because the total orbital angular momentum of two π^0 is zero infers that this state is a CP eigenstate of value +1. Also the orbital angular momentum of three pion states being -1 infers that this an eigenstate of CP with eigenvalue -1.

i.e, If CP is conserved K_2^0 could not decay into two pion states and K_1^0 could not decay to three pion states. But an experiment performed by James Christenson, James Cronin, Val Fitch and Rene‘ Turley in 1963 showed that K_L^0 infact deayed in two pion channel. This shows that K_1^0 and K_2^0 are not same as $K_s^0$and K_L^0 . The branching rates of K_L^0 into $\pi^0\pi^0$ and $\pi^+\pi^-$are of the order of 0.1% of all K_L^0 decays.

Now the eigenstates of weak interaction can be represented as follows,

$$\begin{aligned}
\mid K_s^0 > &= \frac{1}{\sqrt{2(1+\mid\varepsilon\mid^2)}}[(1+\varepsilon)\mid K^0 > -(1-\varepsilon)\mid \overline{K}^0 >] \\
&= \frac{1}{\sqrt{1+\mid\varepsilon\mid^2}}[\mid K_1^0 > +\varepsilon\mid K_2^0 >] \\
\mid K_L^0 > &= \frac{1}{\sqrt{2(1+\mid\varepsilon\mid^2)}}[(1+\varepsilon)\mid K^0 > +(1-\varepsilon)\mid \overline{K}^0 >] \\
&= \frac{1}{\sqrt{1+\mid\varepsilon\mid^2}}[\mid K_2^0 > +\varepsilon\mid K_1^0 >]
\end{aligned}$$

where ε is a very small complex parameter representing the deviation of $K_s^0$and K_L^0 from the CP eigenstates and hence the CP violation effect. These states are not even orthogonal which is expected as they have common decay channels.

$$< K_L^0 \mid K_S^0 > = \frac{2Re(\varepsilon)}{1+\mid\varepsilon\mid^2} = < K_S^0 \mid K_L^0 >$$

Other commonly used parameter to represent the branching ratios are η_{+-} and η_{00}

$$\eta_{+-} = \frac{K_L^0 \longrightarrow \pi^+ + \pi^-}{K_S^0 \longrightarrow \pi^+ + \pi^-}$$

$$\eta_{00} = \frac{K_L^0 \longrightarrow \pi^0 + \pi^0}{K_S^0 \longrightarrow \pi^0 + \pi^0}$$

From the definitions we can conclude $\eta_{+-} = \eta_{00} = \varepsilon$ where the experimental values are $\mid \eta_{+-} \mid = (2.29 \pm 0.02) \times 10^{-3}$ and $\mid \eta_{00} \mid = (2.27 \pm 0.02) \times 10^{-3}$.

These values indicate CP violation is effect is very small. CP Violation effects in other weak processes observed so far also very small. For practical purposes CP can be approximated to be a good symmetry. Unlike parity violation which was maximal this approximate symmetry of CP is interesting.

1.2 Baryogenesis

CP violation eventhough is a very small symmetry violation is not just related to rare weak processes but also is important in answering the baryogenesis problem of cosmology. Baryogenesis refers to physical process that produced asymmetry in the quantities of baryonic and anti baryonic matter in the early universe in big bang theory.Till now experimental evidences suggests that our universe mostly consists of matter not antimatter. Antiparticles are found in very tiny amounts in either natural or artificial high energy processes in nature. But they do not appear to have enough quantities to form macroscopic object.

Quantity of antimatter in the universe can be infered from cosmic rays because of the annihilation radiation produced by matter anti matter interaction, rays of order greater than 100MeV are deducted to be not of solar origin. The anti particles identified from cosmic rays are positrons and anti positrons. The positron fraction found to be order 0.1 is not a significant amount, as interaction of photons and particles with the interstellar gas or dense gas around stars produce positrons by following reactions[Soz08]

$$pp \longrightarrow \pi X \qquad \pi \longrightarrow \mu^{\pm}\nu_{\mu} \qquad \mu^{\pm} \longrightarrow e^{\pm}\nu_{e}\nu_{\mu}, \qquad \gamma \longrightarrow e^{+}e^{-}$$

There are other sources of positrons like decays of nuclei produced in novae etc... Many other analysis also shows that antimatter quantity is much less than matter and indeed if exists is well seperated from matter otherwise it would have been identified in cosmic rays. But no successful mechanism was found to explain this seperation.

This may be interpretted as either universe began with a bit more matter content than anti matter or they started symmetrically but then some phenomena preferred matter over anti matter. The second argument is preferred over the first by the analysis of inflation theory. In 1967 Andrei Sakharov proposed baryon assymmetry may be understood if the three requirements i.e, Baryon number Violation, C and CP symmetry violation, and departure from thermodynamic equillibrium to define an arrow of time, is satisfied. Till now the standard model parameters for CP violation does not match quantitatively with the cosmological prediction.In this context study of CP violation becomes more crucial.

Chapter 2

Formalism

2.1 Discrete Symmetry

In Quantum Field Theory continuous symmetry gives conserved current according to noethers theorem. Discrete symmetry does not give any conserved current but it puts restrictions on possible lagrangian terms. The total lagrangian in Standard Model including the interaction terms is written in terms of free fields. Thus transformation of free fields yield the transformation of lagrangian terms. Transformation of free fields under the discrete operations of parity inversion, charge conjugation and time reversal is obtained under the assumption that the transformed field also satisfies the same equations of motion.

2.1.1 Parity Inversion (P)

Parity inversion operator (P) defines how the fields and hence the other quantities transform under the coordinate transformation from $(x) = (\mathbf{x}, t)$ to $(x') = (-\mathbf{x}, t)$. Parity is physically associated with the question whether the mirror image process is also a reality. The operator should change the direction of momenta of particles without changing the spin.Under the assumption that the free lagrangian is invariant under the transformation $(PL(x)P^{-1} = L(x'))$ and fields have a linear constant transformation, transformation of the fields can be derived.

The field transformations are tabularised below[LP01]

$$scalar\,field \qquad \phi(x) \longrightarrow \phi(x')$$

$$psuedoscalar\,field \quad \phi_p(x) \longrightarrow \phi_p(x')$$

$$Dirac\,field \qquad \psi(x) \longrightarrow \gamma_0 \psi(x')$$

$$\overline{\psi}(x) \longrightarrow \overline{\psi}(x')\gamma_0$$

$$vector\,field \qquad V_\mu(x) \longrightarrow V^\mu(x')$$

$$axial\,vector\,field \qquad A_\mu(x) \longrightarrow -A^\mu(x')$$

Using the transformation of dirac field the transformation of the dirac bilinears can be derived which will be useful in further analysis.

$$scalar \quad \overline{\psi}_1\psi_2 \longrightarrow \overline{\psi}_1\psi_2$$

$$psuedo\,scalar \quad \overline{\psi}_1\gamma_5\psi_2 \longrightarrow -\overline{\psi}_1\gamma_5\psi_2$$

$$vector \qquad \overline{\psi}_1\gamma_\mu\psi_2 \longrightarrow \overline{\psi}_1\gamma^\mu\psi_2$$

$$axial\,vector \qquad \overline{\psi}_1\gamma_\mu\gamma_5\psi_2 \longrightarrow -\overline{\psi}_1\gamma^\mu\gamma_5\psi_2$$

$$tensor \qquad \overline{\psi}_1\sigma_{\mu\nu}\psi_2 \longrightarrow \overline{\psi}_1\sigma^{\mu\nu}\psi_2$$

For the free field the transformation is associated with a complex phase. It is the interaction term which fixes the relative parities of different fields given that parity is a symmetry of the system.For example consider the interaction term $L_{int} = \phi^3(x) + \phi^4$ here ϕcannot transform like a psuedo scalar, so ϕ has positive parity.Similarly the free photon field may transform like vector or axial vector. It is the source term which fixes the transformation to be like a vector.

2.1.2 Charge Conjugation (C)

The operator for charge conjugation(C) is derived under the assumption that free lagrangian is invariant under this transformation. Charge conjugation refers to changing a particle into its anti particle, i.e, it changes the sign of internal charges like electric charge, baryon number etc. The field transformations of fields under the operator is given below,

$$scalar\,field \qquad \phi(x) \longrightarrow \phi^\dagger(x)$$

$$Dirac\,field \qquad \psi(x) \longrightarrow C\overline{\psi}^T(x)$$

$$\overline{\psi}(x) \longrightarrow -\psi^T(x)C^{-1}$$

$$Vector\,field \qquad V_\mu(x) \longrightarrow -V_\mu^\dagger(x)$$

$$Axial\,vector\,field \quad A_\mu(x) \longrightarrow A_\mu^\dagger(x)$$

Here ^{T} refers to the transpose operation and C is a 4×4 unitary matrix satisfying the condition, $C^{-1}\gamma_\mu C = -\gamma_\mu^T$. C is different in different representation. For example in dirac representation $C = i\gamma^2\gamma^0$while in majorana representation of gamma matrices, $C = i\gamma^0$. Like in the case of parity transformation, Charge conjugated field in the above table may also have arbitrary phase factor.

The transformations of the dirac bilinears are useful in analysis which may be obtained from the transformation of fields.

$$\begin{array}{ll} scalar & \overline{\psi}_1\psi_2 \longrightarrow \overline{\psi}_2\psi_1 \\ psuedo\,scalar & \overline{\psi}_1\gamma_5\psi_2 \longrightarrow \overline{\psi}_2\gamma_5\psi_1 \\ vector & \overline{\psi}_1\gamma_\mu\psi_2 \longrightarrow -\overline{\psi}_2\gamma_\mu\psi_1 \\ axial\,vector & \overline{\psi}_1\gamma_\mu\gamma_5\psi_2 \longrightarrow \overline{\psi}_2\gamma_\mu\gamma_5\psi_1 \\ tensor & \overline{\psi}_1\sigma_{\mu\nu}\psi_2 \longrightarrow -\overline{\psi}_2\sigma_{\mu\nu}\psi_1 \end{array}$$

2.1.3 Time Reversal (T)

Time reversal operator describes how the fields transform under the coordinate transformation$(x) = (\mathbf{x},t)$ to $(x") = (\mathbf{x},-t) = -(x')$. Unlike P and C operator T cannot be a linear unitary operator. It needs to be anti unitary and anti linear.

Suppose an initial state $\mid \psi >$ time evolved to a state $\mid \psi' >$. In the time reversed scenario initial and final state gets interchanged.

$$< T\psi' \mid T\psi > = < \psi \mid \psi' >$$

implying T is anti unitary in contrast to the unitary operator where,

$$< U\psi' \mid U\psi > = < \psi' \mid \psi >$$

There is a theorem by Wigner which says that any symmetry operation that leaves probability of all physical processes invariant can be represented either by a unitary linear operator or by an anti unitary operator which is anti linear. Anti linear refers to taking complex conjugate of the numbers like shown below,

$$O(a \mid \psi_1 > +b \mid \psi_2 >) = a^* O \mid \psi_1 > +b^* \mid \psi_2 >$$

here O is the anti linear operator and a and b are complex numbers .

Time reversing should invert the direction of momenta as well as spin.

The transformation of photon field is$A_\mu(x) \longrightarrow A^\mu(x")$. Time reversal operator for the dirac field from the free lagrangian turns out to be $T = C^{-1}\gamma_5$ upto a phase factor. In the dirac representation $T = \gamma^1\gamma^3$. Using this the transformation of bilinears is derived.

$$\begin{array}{ll} scalar & \overline{\psi}_1\psi_2 \longrightarrow \overline{\psi}_1\psi_2 \\ psuedo\,scalar & \overline{\psi}_1\gamma_5\psi_2 \longrightarrow -\overline{\psi}_1\gamma_5\psi_2 \end{array}$$

$$vector \qquad \overline{\psi}_1\gamma_\mu\psi_2 \longrightarrow \overline{\psi}_1\gamma_\mu\psi_2$$

$$axial\,vector \qquad \overline{\psi}_1\gamma_\mu\gamma_5\psi_2 \longrightarrow \overline{\psi}_1\gamma_\mu\gamma_5\psi_2$$

$$tensor \qquad \psi_1\sigma_{\mu\nu}\psi_2 \longrightarrow -\psi_1\sigma_{\mu\nu}\psi_2$$

As usual the intraction term fixes the relative phase factor.

2.1.4 CPT Theorem

In quantum field theory CPT is a good symmetry under the assumptions that lagrangian is lorentz invariant, spinor fields anticommute, and the integaral spin fields are quantized by commutation relations. So it is a more genral result and holds so far experimentaly also. Because of CPT theorem CP violation implies violation of T symmetry also.

2.2 CP Violation In Standard Model

In Standered Electro Weak Model CP violation is possible in the quark sector. This model describes CP violation using the quark mixing matrix. The $SU(2)\times U(1)_Y$ lagrangian after taking the the vacuum expectation value of higgs field breaks to $U_{em}(1)$. This spontaneous symmetry breaking process creates mass matrices (and inturn masses to particles) in the Standard Model. The quark mixing matrix origintes as result of diagonalising the mass matrices which is necessary to change the fields to physical basis. This explained in detail in the following section.

Electro weak lagrangian density can be written symbolically as ,[GM09]

$$L = L_{int}(f,G) + L_{int}(f,H) + L_{int}(G,H) + L_{free}(G) + L_{free}(f) + L_{free}(H) - V(H)$$

where f refers to fermions, H to higgs doublet, G to gauge bosons.

Using the transformation properties of fields under C and P each of these terms can be shown to be CP invariant.L_{int} for hadronic part is given as

$$L_{int}(f,H) = \sum_{j,k=1}^{N}\{Y_{jk}(q,q')_{jL}H^c q_{kR} + Y'_{jk}\overline{(q,q')}_{jL}\bar{H}q'_{kR} + h.c\}$$

Here Y_{jk} and Y'_{jk} are the Yukawa coupling constants and H and H^C are the Higgs doublet and the its C conjugate .But after spontaneous symmetry breaking the the $L_{int}(f,H)$ transforms as (2.1) which generates mass for the fermions.

$$L_{int}(f,H)\xrightarrow{SSB} -\sum_{j,k=1}^{N}\{m_{jk}\bar{q}_{jL}q_{kR} + m'_{jk}\bar{q}'_{jL}q'_{kR} + h.c\}(1+\frac{\phi_0}{v}) \tag{2.1}$$

where $m_{jk} = -\frac{v}{\sqrt{2}}Y_{jk}$ and $m'_{jk} = -\frac{v}{\sqrt{2}}Y'_{jk}$ are the quark mass matrices.

Naming the first part of (2.1) as E , E can be rewritten as $\frac{1}{2}\bar{q}[(m+m^{\dagger})+(m-m^{\dagger})\gamma_5]q$. Under CP transformation E transforms as $\bar{q}[(m+m^{\dagger})^T-(m-m^{\dagger})^T\gamma_5]q$. HenceCP invariance implies mass matrices be real i.e, $m = m^*$and $m' = m'^*$.Flavour fields discussed so far are unphysical, physical fields are obtained by diagonalising the mass matrices. Any square matrice can be diagonalised using two unitary matrices. This implies

$$U_L m U_R^{\dagger} = D \equiv (m_u, m_c, m_t, ...)diagonal$$

$$U'_L m U_R^{'\dagger} = D \equiv (m_d, m_s, m_b, ...)diagonal$$

Now the lagrangian terms have to be rewritten in terms of the physical fields in which the mass matrices are diagonal.

The physical fields are

$$q_L^{phy} = U_L q_L \quad q_L^{'phy} = U'_L q'_L$$

$$q_R^{phy} = U_R q_R \quad q_R^{'phy} = U'_R q'_R$$

Now inorder to check the symmetry properties, the lagrangian needs to be rewritten in terms of the physical fields. Charged current term is then the only CP violating possible term. Charged current term upto some numerical factors and coupling constants is given as

$$X_c \equiv [W_\mu^1 - iW_\mu^2]\bar{q}_L\gamma^\mu q'_L + h.c$$

$$= [W_\mu^1 - iW_\mu^2]\bar{q}_L^{phy}\gamma^\mu U_L \bar{U}_L^{'\dagger} q_L^{'phy} + h.c$$

$$= [W_\mu^1 - iW_\mu^2]\bar{q}_L^{phy}\gamma^\mu V q_L^{'phy} + h.c$$

$$= [W_\mu^1 - iW_\mu^2]J_c^\mu + h.c$$

where V is the quark mixing matrix and J_c^μdenotes the charged current. The physical gauge bosons are $(W^{(+)} = \frac{(W_\mu^1 - iW_\mu^2)}{\sqrt{2}})$ and $(W^{(-)} = W^{(+)\dagger})$.

2.2.1 Quark Mixing Matrix

CP violation requires quark mixing matrix to be ’real’ . This is shown explicitly below,

$$X_c = (W_\mu^1 - iW_\mu^2)\bar{q}_j\gamma^\mu V_{jk}(1-\gamma_5)q'_k + (W_\mu^1 + iW_\mu^2)\bar{q}'_k\gamma^\mu V_{jk}^*(1-\gamma_5)q_j$$

X_c under CP transforms as,

$$X_c \xrightarrow{CP} (W^1_\mu + iW^2_\mu)\bar{q}^{'}_k \gamma^\mu V_{jk}(1-\gamma_5)q_j + (W^1_\mu - iW^2_\mu)\bar{q}_j \gamma^\mu V^*_{jk}(1-\gamma_5)q^{'}_k$$

So if $V = V^*$, $X_c$remains the same, where V^*denotes the complex conjugate of V. Absolute phase of different fields is not measurable, what matters is the relative phase, this can be expressed mathematically as ,

$$u_L \longrightarrow e^{i\phi(u)}u_L, \quad c_L \longrightarrow e^{i\phi(c)}, ...$$

$$d_L \longrightarrow e^{i\phi(d)}d_L, \quad s_L \longrightarrow e^{i\phi(s)}, ...$$

Here all the phasesϕ's are arbitrary real numbers. For N families there are 2N such phases. Under the above phase transfomations the change in V is not physically relevant. For the case of three families

$$V \longrightarrow \begin{bmatrix} e^{-i\phi(u)} & 0 & 0 \\ 0 & e^{-i\phi(c)} & 0 \\ 0 & 0 & e^{-i\phi(t)} \end{bmatrix} \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} e^{-i\phi(d)} & 0 & 0 \\ 0 & e^{-i\phi(s)} & 0 \\ 0 & 0 & e^{i\phi(b)} \end{bmatrix}$$

i.e,

$$V_{\alpha j} \longrightarrow exp\{i[\phi(j) - \phi(\alpha)]\}V_{\alpha j}$$

This is true for any number of families. Since in each element of V only relative phase appears, any one of these phases can be set arbitrarily say zero. So only $2N-1$ of phases are relevant for the transformation. Thus for CP conservation V needs to be real after removing these unmeasurable phases.

2.2.2 Parameterisation Of n Dimensional Unitary Matrices

An n dimensional unitary matrix U is the one which satisfies $UU^+ = I$. This implies U has n^2parameters. U can be parameterised using $\frac{n(n-1)}{2}$ angles and $\frac{n(n+1)}{2}$ phases. Consider the 2 dimensional case, a 4 parameter 2 dimensional unitary matrix can be written in the form $D(\delta_1, \delta_2)U(\phi, \sigma)$; where $D(\delta_1, \delta_2)$ the diagonal matrix is given as $\begin{bmatrix} exp(i\delta_1) & 0 \\ 0 & exp(i\delta_2) \end{bmatrix}$ and $U(\phi, \sigma)$ the unimodular unitary matrix is given as $\begin{bmatrix} c & -sexp(-i\sigma) \\ sexp(i\sigma) & c \end{bmatrix}$, $c = cos(\phi)$, $s = sin(\phi)$.$\phi$and $\delta_2$are the longitudinal angles and $\sigma$and $\delta_1$are the latitudinal angles i.e, $-\pi \leq \phi < \pi, -\pi \leq \delta_2 < \pi$, $\frac{-\pi}{2} \leq \sigma \leq \frac{-\pi}{2}$, $\frac{-\pi}{2} \leq \delta_1 \leq \frac{\pi}{2}$.In order to parameterise the n dimensional unitary matrix the n dimensional unimodular unitary matrix $U_{pq}(\phi, \theta)$ $p < q$ is defined in the following way :

1)All its diagonal elements are 1 except pth and qth which are $c = cos(\phi)$

2)All its non diagonal are zero except pth row qth coloumn element which is $sexp(i\sigma)$and qth row pth coloumn element which is $-sexp(-i\sigma)$.

For example, when n=3

$$U_{23}(\phi,\sigma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & -sexp(-i\sigma) \\ 0 & sexp(i\sigma) & c \end{bmatrix}$$

U can be written as [Mur62]

$$U = U^{(n-1)}U_{12}(\theta_{n-2},\sigma_{n-1})U_{13}(\theta_{n-3},\sigma_{n-2})....U_{1n-1}(\theta_1,\sigma_1)U_{1n}(\phi_1,\sigma_1)$$

where $U^{(n-1)} = \begin{bmatrix} exp(i\delta_1) & 0 \\ 0 & V \end{bmatrix}$, V is an $(n-1)$ dimensional unitary matrix.

(1) If n=3,

we know $V = D(\delta_{2,}\phi_3)U(\phi_2,\sigma_3)$

so that $U^{(2)} = D(\delta_1,\delta_2,\phi_3)U_{23}(\phi_2,\sigma_3)$

and $U = D(\delta_1,\delta_2,\phi_3)U_{23}(\phi_2,\sigma_3)U_{12}(\theta_1,\sigma_2)U_{13}(\phi_1,\sigma_1)$

Thus 3 dimensional unitary matrix has three $\phi's$ which are longitudinal angles and the rest latitudinal angles.

(2)If n=4 we add a single ϕ, two θ's,three σ's and a single δ to the parameters of the three dimensional unitary group.

ϕ's are the longitudinal angles and the rest latitudinal angles.

$$U = D(\delta_1,\delta_2,\delta_3,\phi_4)U_{34}(\phi_3,\sigma_6)U_{23}(\theta_3,\sigma_5)U_{24}(\phi_2,\sigma_4)U_{12}(\theta_2,\sigma_3)U_{13}(\theta_1,\sigma_2)U_{14}(\phi_{1,},\sigma_1)$$

and so on.

when we move from $(n-1)$ dimension to n dimension we add a single ϕ, single δ, (n-2) θ's and (n-1) σ's . Thus in all there are $\frac{n(n-1)}{2}$angles and $\frac{n(n+1)}{2}$phases. If we take all the phases to be zero we get parameterisation for orthogonal matrices.

2.2.3 Conditions For CP Violation

From the above discussions it is clear that out of the N^2parameters of N dimensional unitary matrix $\frac{N(N-1)}{2}$ are angles and $\frac{N(N+1)}{2}$ are phases. For V, $2N-1$ of phases are unmeasurable. Therefore V has $\frac{N(N+1)}{2} - (2N-1) = \frac{(N-1)(N-2)}{2}$ phases and $\frac{N(N-1)}{2}$ angles, in all $(N-1)^2$parameters.

For the case of two families V has only one rotation angle and no phases. The quark mixing matrix is of the form

$$\begin{bmatrix} cos(\theta) & sin(\theta) \\ -sin(\theta) & cos(\theta) \end{bmatrix}$$

Thus for 2 families V is automatically real. Hence CP violation is not possible in this case. For three families V can be parameterised using three angles and one phase. Hence CP violation is possible here.

From historic point of view, Cabibbo proposed the matrix for two families which explained the GIM mechanism i.e, abscence of flavour changing nuetral currents. At that time third family was not detected.A possible solution explored then was to extend the Higgs sector.But Kobayashi and Masakawa proposed another solution by introducing third family and giving a particular parameterisation for V, KM parameterisation for the matrix. This explained CP violation within in theSM framework. Hence the quark mixing matrix for the three families is called CKM matrix, named after the three scientists behind its discovery.

Within the three family case if two quarks with same charge have same mass then there is no CP violation. This is shown through the following example, Let s and b quark be degenerate. Unitary rotations in the space spanned by the s and b quark leaves the lagrangian invariant. Therefore one can define a new b and s quark such that u quark couples only to new s quark and not to new b quark. Also using unitarity we construct the V matrix to be [C.J89]

$$\begin{bmatrix} cos(\theta) & sin(\theta) & 0 \\ -sin(\theta)cos(\phi) & cos(\theta)cos(\phi) & sin(\phi) \\ sin(\theta)sin(\phi) & -cos(\theta)sin(\phi) & cos(\phi) \end{bmatrix}$$

Since this matrix is real CP violation is not possible. Similarly for other quark pairs. Therefore six necessary condition for CP violation in three family case is that

$$m_u \neq m_c, \qquad m_c \neq m_t, \qquad m_t \neq m_u$$

$$m_d \neq m_s, \qquad m_s \neq m_b, \qquad m_b \neq m_d$$

The other constraint for CP violation is on the parameters of V_{ckm}so that the matrix is not real. This is explaned in the following section with reference to the KM parameterisation.

2.2.4 Parameterisation of Quark Mixing Matrix.

From the discussion we know thatCKM matrix can be parameterised using one phase and three angles. This can be done in many ways. One such way is the KM parameterisation given as below

$$V_{KM} = U_{23}(-\theta_3, \delta)U_{12}(-\theta_1, 0)U_{23}(-\theta_2, 0)$$

There are three ways of choosing the indices either 12,13 or 23. Suppose we choose the middle one to be one of the indices. Then other two indices can't be the same. Therefore there are $2 \times 3 \times 2 = 12$ ways of doing this. And there are 3 places for δ. In all there are

36 ways to parameterise V . In order to have V to complex we need to have the following constriaints on angles and phases.

$$\theta_i \neq 0, \frac{\pi}{2} \qquad \delta \neq 0, \pi \qquad i = 1,2,3.$$

Thus the requirement that V is complex implies in all 14 constraints on the masses , angles and phase. The parameterisation used by ParticleDataGroup is [AY14]

$$\begin{bmatrix} c_{12}c_{23} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

where $c_{ij} = cos(\theta_{ij})$, $s_{ij} = sin(\theta_{ij})$, the angles $\theta_{ij} = [0, \frac{\pi}{2}]$, $\delta = [0, 2\pi]$.

There is another important parameterisation known as as Wolfenstein parameterisation exploiting the eaperimental fact that $s_{13} \ll s_{23} \ll s_{12} \ll 1$which is unitary upto all orders of λ.

$$V_{CKM} = \begin{bmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} + O(\lambda^4)$$

where $\lambda = s_{12}, \quad A\lambda^2 = s_{23}, \quad A\lambda^3(\rho + i\eta) = s_{13}e^{i\delta}$.

2.2.5 Jarlskog Rephasing Invariant Parameter

For the three family case a meaning rephasing invariant known as Jarlskog invariant can be defined as follows:

$$J \sum_{k,\gamma} (\varepsilon_{ijk}\varepsilon_{\alpha\beta\gamma}) = Im(V_{i\alpha}V_{j\beta}V^*_{i\beta}V^*_{j\alpha})$$

for any $i \neq j$ and $\alpha \neq \beta$, here Im denotes the imaginary part of its argument. If either $i = j$ or $\alpha = \beta$ the quantity $V_{i\alpha}V_{j\beta}V^*_{i\beta}V^*_{j\alpha} = t_{ij\alpha\beta}$ becomes real and is a trivial rephasing invariant related to the magnitude of each elements. Rephasing invariance refers to invariance under changing the phase of quark fields(like $u_L \longrightarrow e^{i\phi(u)}u_L$ mentioned in the previous section). In other words this is invariance under different parameterisations. In standard parameterisation J can be derived as

$$J = s_{12}s_{23}s_{13}c_{12}c_{23}c^2_{13}sin(\delta) \tag{2.2}$$

It turns out that the constraints on angles and phases for CP violation described easily in terms of J i.e; if for CP invariance J should be equal to zero.In many calculations CP violating effects are proportional to J. For N family also $t_{ij\alpha\beta}$ is still a rephasing invarinat and turns out there are $\frac{(N-1)(N-2)}{2}$ independent non real $t_{ij\alpha\beta}$'s, which is exactly equal to the number of phases in the parameterisation. [MP04]

2.2.6 Unitarity Triangles of Quark Mixing Matrix

Number of independent $Im(t_{ij\alpha\beta})$'s equals one for $i \neq j$ and $\alpha \neq \beta$ can proved is a property of the unitary matrix and can be proved in the following way,

Let A be a 3×3 unitary matrix with each elements denoted as a_{ij}, i denoting the row index and j coloumn index. Defining symbols connecting first column and second coloumn elements in the following way, $a_1 = a_{11}a_{12}^*$, $a_2 = a_{21}a_{22}^*$, $a_3 = a_{31}a_{32}^*$. Similarly define b_i's connecting second and third coloumn,c_i's connecting first and third coloumn.Likewise defining symbols connecting first and second row as $A_1 = a_{11}a_{21}^*, A_2 = a_{12}a_{22}^*, A_3 = a_{13}a_{23}^*$, B_i's connecting second and third coloumn C_i's connecting first and third row. Now $AA^\dagger = 1$ gives the constraints: 1) sum of square of magnitudes elements of same row equals one, and 2) $\sum_i a_i = \sum_i b_i = \sum_i c_i = 0$. The second constraint allows to construct three triangles on complex plane one with sides a_i's , second with sides b_i's and third using c_i's. Twice the area of the formed by a_i's is $2 \mid a_1 \mid\mid a_2 \mid sin(\theta) = Im(a_1 a_2^*) = t_{1212}$, where θ is the angle between $a_1$and a_2 .Twice the area of triangle formed by a_i^*givest_{2121} .Similarly for the other two triangles. In the same way $A^\dagger A = 1$ gives the constraints 1) sum of square of magnitude of elements in each coloumn equals one, and 2) $\sum_i A_i = \sum_i B_i = \sum_i C_i = 0$. Twice the area of these triangles gives the rest of the rest of the nonreal t 's . Proving that the area of these triangles are the same proves $Im(t_{ij\alpha\beta})$ upto a sign is same which inturn equals J.

$$area(\Delta a) = Im(a_{11}a_{12}^* a_{22} a_{21}^*)$$

$$area(\triangle A) = Im(a_{11}a_{21}^* a_{12}^* a_{22})$$

This proves $area(\triangle A) = area(\triangle a)$. In a similar way on can prove that area of all six triangles are same.

Using $\sum_i A_i = 0$

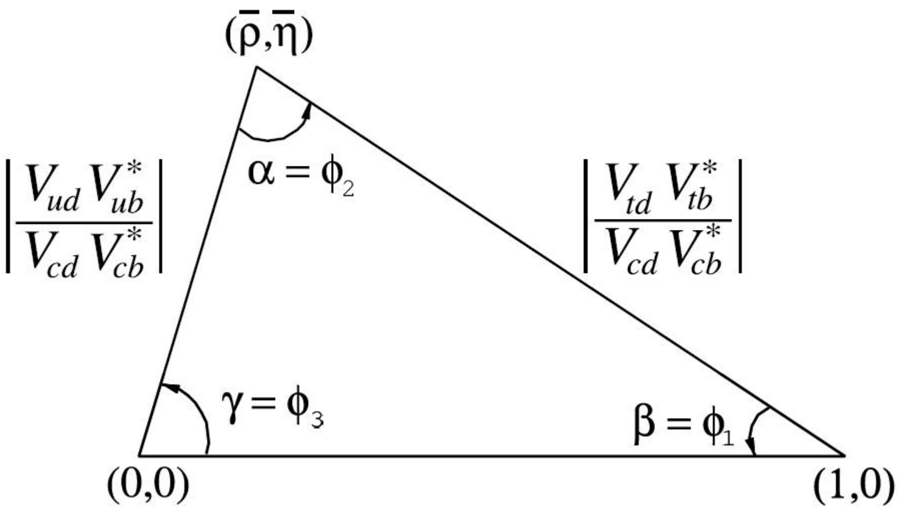
$$area(\Delta A) = Im(-(a_{21}a_{22}^* + a_{31}a_{32}^*)a_{21}^* a_{22})$$

$$= Im(-(\mid a_{12} \mid^2 \mid a_{22} \mid^2 - a_{21}^* a_{22} a_{31} a*_{32}))$$

$$= -Im(a_{21}^* a_{22} a_{31} a_{32}^*)$$

which explicitly shows that $t_{1212}$and t_{2312}corresponds to area of same triangle $\triangle A$. Similarly for other t 's.

In the context of V_{CKM} matrix these six triangles are named depending upon which rows



Source: https://inspirehep.net/record/1085541/plots

Figure 2.1: Schematic diagram of CKM 'db' triangle

or coloumns are selected.For example,

$$V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0$$

is named as uc triangle. Similarly selecting second and third row gives ct triangle, first and third row gives ut triangle. By selecting first and second coloumn i.e,

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$$

ds triangle is formed. Likewise second and third coloumn gives sb triangle, first and third coloumn gives db triangle. See figure (2.1)

2.3 CP violation in Leptonic Sector

In Standard Model neutrinos are massless therefore the mixing matrix can be shown to be diagonal and thus no mixing between families exists there. But later neutrino oscillation experiments (SNO, Super Kamiokande, reactor) proved that nuetrinos have mass in fact implies that mixing matrix (PMNS) exists in leptonic sector also. Neutrinos being chargeless can also be majorana particles, unlike in quark sector where all the quarks are dirac particles.

After the gauge symmetry breaking like in the quark sector of SM charged lepton mass matrices is obtained through yukawa coupling with the higgs doublet, while the neutrino mass matrice is obtained from the B-L breaking mechanism. Here B-L stands for the difference in baryon number and lepton number. Lagrangian mss terms the can be written as

[EPR12]

$$L_{mass} = -\bar{l}_L m_l l_R - \frac{1}{2}\nu_L^T C m_\nu \nu_L + h.c$$

where l stands for the charged leptons and ν for the neutrinos.

Similar to the quark sector lepton mixing matrix can be obtained from the charged current interaction term

$$L_W = \frac{g}{\sqrt{2}}\bar{l}_L \gamma_\mu \nu_L W^\mu + h.c$$

The mixing matrix for three generation U can be written in the form

$$U = VP, \quad P = diag(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}})$$

where V is same as the CKM matrix of quark sector which contains the dirac CP violating phase and α_{21}, α_{31} are the two CP violating majorana phases. These extra phases arises due to the fact that only dirac fields can be rephased and not the majorana fields because they are real solutions of dirac equation. So after removing the $n(=3)$ phases by rephasing the charged lepton fields $n\frac{(n-1)}{2}$ $(=3)$ phases remains in total.

The matrix U is written symbolically as

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

Dirac and Majorana Unitary Triangles

The unitarity conditions ($UU^\dagger = I$) give rise to six triangles . The orthogonality of coloumns defines three majorana triangles while the orthogonality of the rows defines the the rest three dirac triangles. See figure (2.2). The distinction between dirac and majorana triangles is that orientation of the triangle has no meaning for dirac triangles as rephasing rotate the triangles in the complex plane while majorana triangles has a specific orientation which is invariant under rephasing. This again originates from the fact that majorana fields cant be rephased.

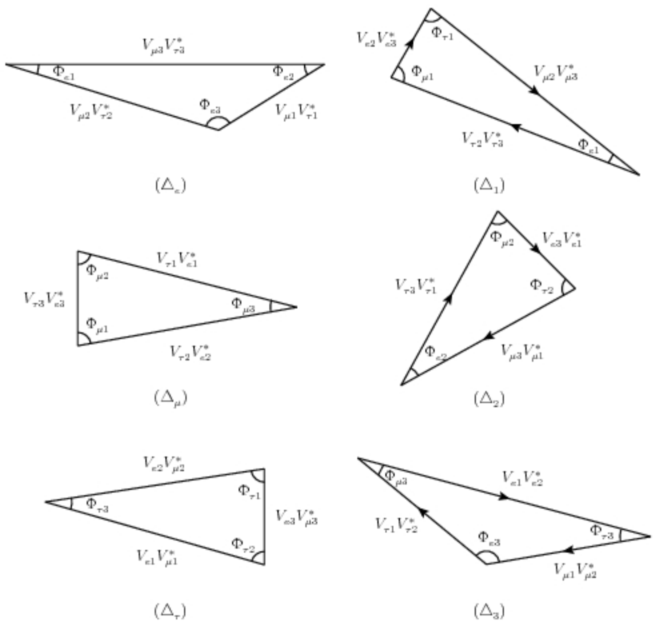


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Figure 2.2: Schematic diagram for six leptonic unitarity triangles.

Chapter 3

Calculations

In the last decade some major progress has occured in the phenomenology of CKM matrix as well as nuetrino oscillations. Detailed analysis by PDG,CKM fitter and UT fit combining the experimental data and unitarity constraints enables to fix the value of CKM matrix upto a reasonable range. In case of nuetrino oscillations there has been progress made in fixing the data from the atmospheric, solar as well as reactor and accelerator based nuetrino experiments. The Jarlskog rephasing invariant J and CP violating phase δ is calculated from the CKM and PMNS matrix.

3.1 Construction of Unitarity Triangles in the quark sector

Using the experimental data from each of the quark mixing process together with the unitarity condition for the three generation PDG has arrived at the following value of the V_{ckm} matrix [AY14]

$$\begin{bmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{bmatrix}$$

Out of the six unitarity triangles four of them(ds,sb,uc,ct) are highly skewed, i.e; length of one side is much smaller compared to the other two(atleast one order). The other two namely db and ut have sides of comparable length and provide a good opportunity to calculate J and δ. In particular db triangle is known as the reference triangle because the angles of these triangle can be constrained by various CP violating observables.

For calculating J from the triangle say db, length of each of the sides is computed by varying the matrix element values uniformly and subsequently the area is calculated using heron's formula.In order impose the unitarity constraint the fact that sum of the lengths of any two sides of a triangle should be greater than the third is used. J versus its frequency(number of J values in the particular bin) is plotted.For db and ut triangle a gaussian can be reasonably fitted using which J is estimated. See figure (3.1,3.2) [RG]

From the plot of db triangle upto one sigma(i.e, standard deviation) J is obtained as $J = (3.03 \pm 0.08) \times 10^{-5}$

From the plot of ut triangle upto one sigma J is obtained as $J = (3.03 \pm 0.07) \times 10^{-5}$

J and its error for db triangle is also estimated using the error propagation formula using differentiation. This can be obtained as follows

$$J = 2\sqrt{s(s-a)(s-b)(s-c)} \tag{3.1}$$

where a,b,c are the length of the sides($a = \mid V_{ud} \mid\mid V_{ub} \mid$, $b = \mid V_{cd} \mid\mid V_{cb} \mid$, $c = \mid V_{td} \mid\mid V_{tb} \mid$) and $s = \frac{a+b+c}{2}$. The quadrature formula for error propagation is

$$\frac{\triangle J}{J} = \sqrt{(\frac{\triangle a + \triangle b + \triangle c}{s})^2 + (\frac{-\triangle a + \triangle b + \triangle c}{s})^2 + (\frac{\triangle a - \triangle b + \triangle c}{s})^2 + (\frac{\triangle a + \triangle b - \triangle c}{s})^2} \tag{3.2}$$

Error in sides is obtained from the matrix values.

$$\frac{\triangle a}{a} = \sqrt{(\frac{\triangle \mid V_{ud} \mid}{\mid V_{ud} \mid})^2 + (\frac{\triangle \mid V_{ub} \mid}{\mid V_{ub} \mid})^2}$$

and similarly for other sides.This yield

$$J = (3.06 \pm 0.9) \times 10^{-5}$$

From the values of $\mid V_{us} \mid, \mid V_{ub} \mid, \mid V_{cb} \mid$sine of angles can obtained (3.3,3.4,3.5). Combining this with (3.1),and (2.2) δ the CP violating phase can be calculated.

$$s_{13} = \mid V_{ub} \mid \tag{3.3}$$

$$s_{12} = \frac{\mid V_{us} \mid}{c_{13}} \tag{3.4}$$

$$s_{23} = \frac{\mid V_{cb} \mid}{c_{13}} \tag{3.5}$$

While computing the values were taken uniformly distributed. In the plot of δ vs its frequency (figure 3.3) histogram was approximately fitted to get the following values.

$$\delta = (68 \pm 8)^\circ$$

The histogram plot of δ shifts to left and gets more spread when we take the three sigma values of J(3.1)

Using $\mid V_{us} \mid, \mid V_{ub} \mid, \mid V_{cb} \mid, \mid V_{cs} \mid$values δ can be determined as given below,

$$cos(\delta) = \frac{(a^2 + r^2 - \mid V_{cs} \mid^2)}{2ar} \tag{3.6}$$

where,

$$a = c_{12}c_{23}$$

$$r = s_{12}s_{23}s_{13}$$

The value of δ calculated using average values of the matrix elements is

$$\delta = 72.6^\circ$$

The quadrature error propagation formula for δ using (2.2) is

$$\frac{\triangle sin(\delta)}{sin(\delta)} = \sqrt{(\frac{\triangle J}{J})^2 + (\frac{\triangle s_{12}}{s_{12}})^2 + (\frac{\triangle s_{13}}{s_{13}})^2 + (\frac{\triangle s_{23}}{s_{23}})^2 + (\frac{\triangle c12}{c_{12}})^2 + (\frac{\triangle c_{13}^2}{c_{13}^2})^2 + (\frac{\triangle c_{23}}{c_{23}})^2} \tag{3.7}$$

$$\triangle\delta = \frac{\triangle sin(\delta)}{cos(\delta)} \tag{3.8}$$

The value of δ calculated using (2.2) and (3.7,3.8,3.3,3.4,3.5)is

$$\delta = (70 \pm 9)^\circ$$

3.2 Construction of unitarity triangles in leptonic sector

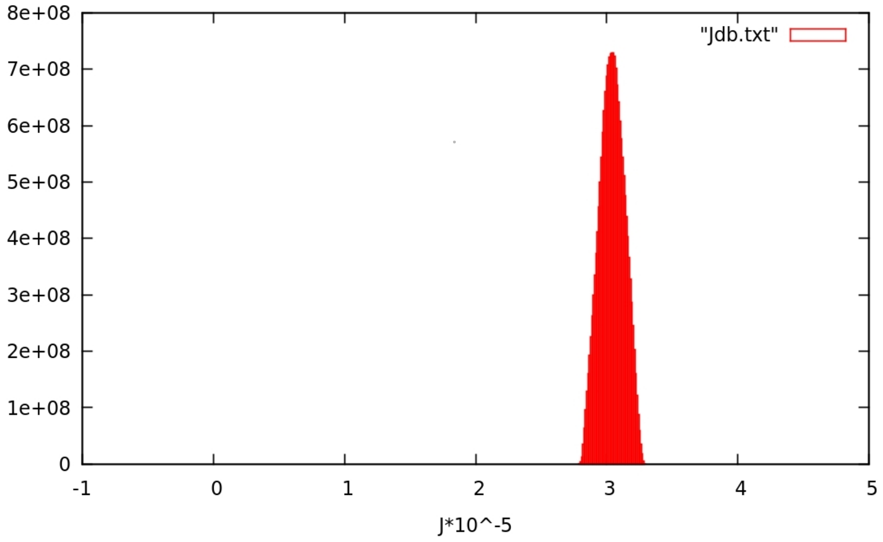
Similar to V_{ckm} PMNS matrix(U) elements have been estimated combining the experimental data and unitarity constraints as follows [Ahu]

$$U = \begin{bmatrix} 0.8190 \pm 0.0105 & 0.5516 \pm 0.0151 & 0.1581 \pm 0.0221 \\ 0.4254 \pm 0.0315 & 0.6317 \pm 0.0442 & 0.6399 \pm 0.0610 \\ 0.3620 \pm 0.0358 & 0.5376 \pm 0.0516 & 0.7520 \pm 0.0519 \end{bmatrix}$$

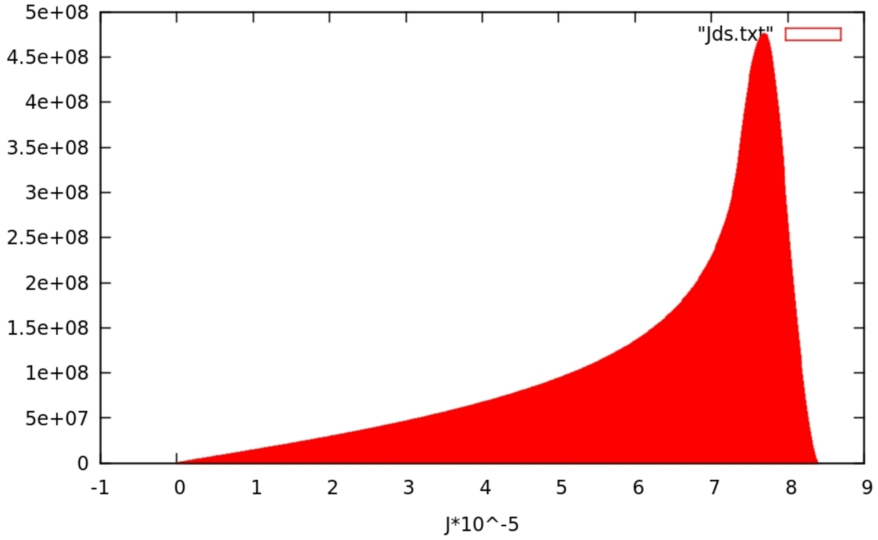
Here all the six triangles have length of sides of comparable magnitude, i.e, triangles are not much squeezed. The analysis is carried out with the $'\nu_1\nu_3'$ triangle, the one corresponding to 'db' triangle. See figure (3.4,3.5)

From the plot of J vs its frequency and fitting the gaussian,

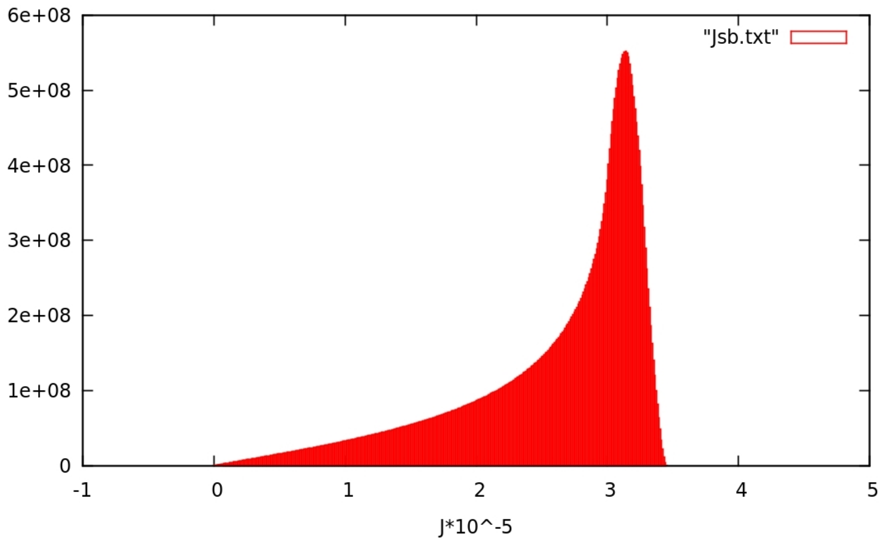
$$J = (3.3 \pm 0.4) \times 10^{-2} \tag{3.9}$$



(a) Plot for 'db' triangle

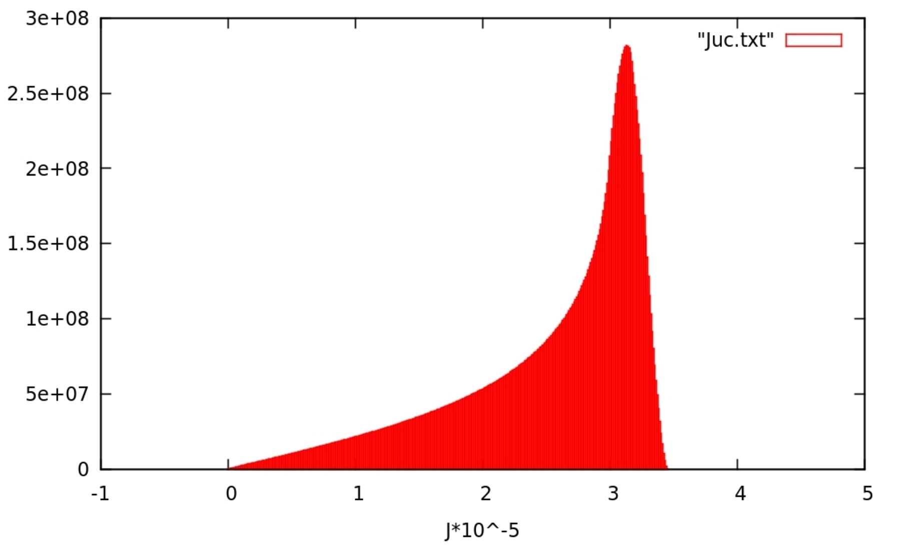


(b) Plot for 'ds' triangle

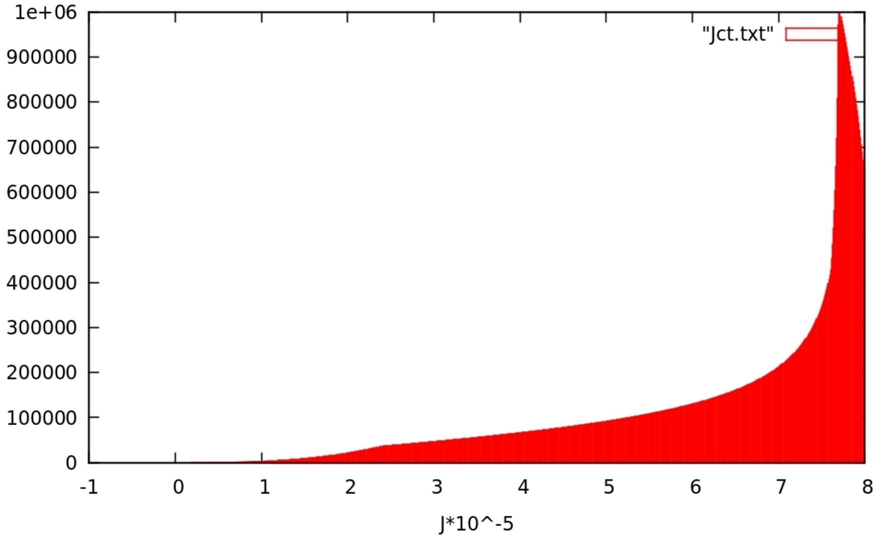


(c) Plot for 'sb' triangle

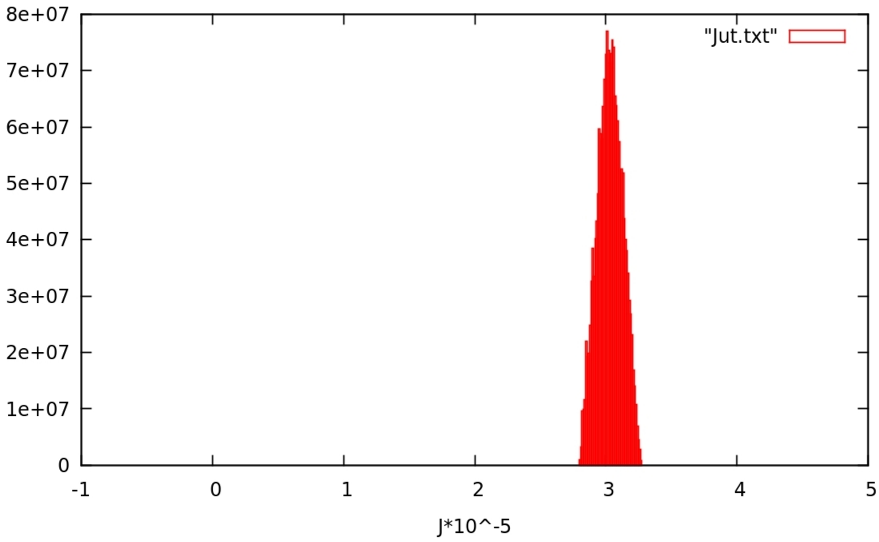
Figure 3.1: Histogram plots for Jarlskog invariant(J) in Quark Sector



(a) Plot for ’uc’ triangle

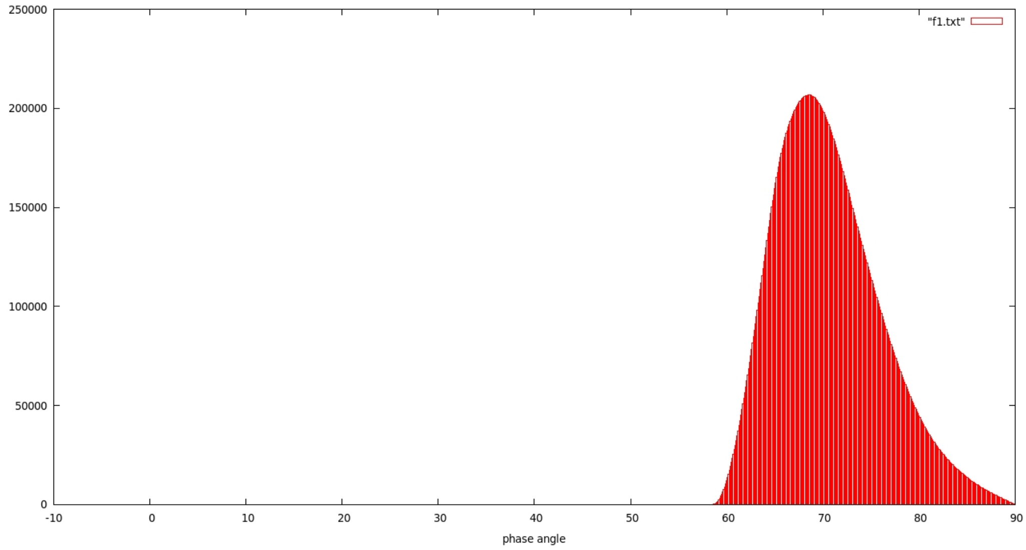


(b) Plot for ’ct’ triangle

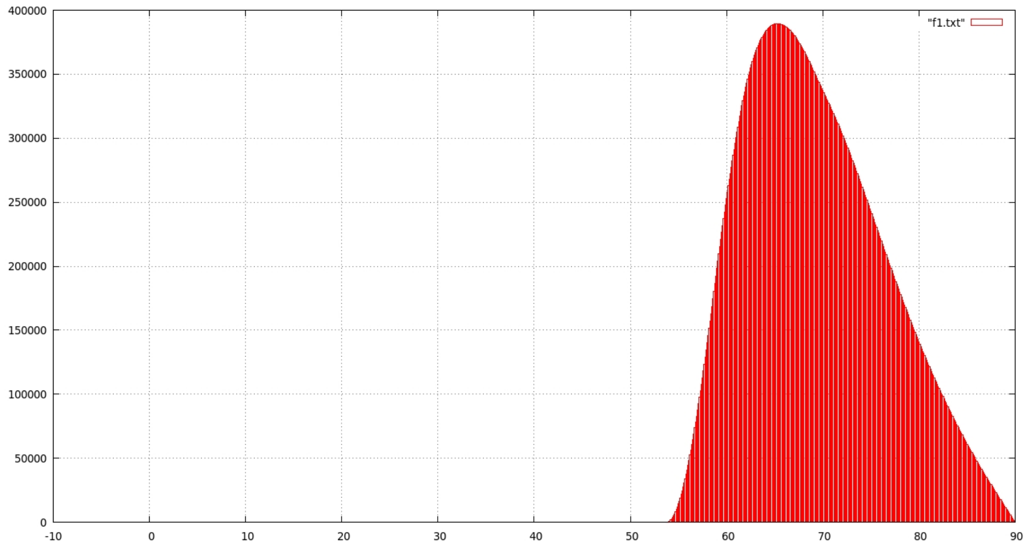


(c) Plot for ’ut’ triangle

Figure 3.2: Histogram plots for Jarlskog invariant(J) in Quark Sector



(a) Plot using one sigma error in J



(b) Plot using three sigma error in J

Figure 3.3: Histogram plots for cp violating phase(δ) in quark sector

The value of J calculated using the (2.2) and the error propagation using the quadrature formula(3.2) is

$$J = (3.4 \pm 1.8) \times 10^{-2}$$

The value of CP violating phase δ estimated from the histogram plot (figure 3.6) using (3.9,2.2,U_{e3}, U_{e2}, $U_{\mu 3}$)

$$\delta = (64 \pm 10)^{\circ}$$

Using (3.6) and the average value of$\mid U_{e2} \mid, \mid U_{e3} \mid, \mid U_{\mu 3} \mid, \mid U_{\mu 2} \mid$ δ can be determined as

$$\delta = 87.5^{\circ}$$

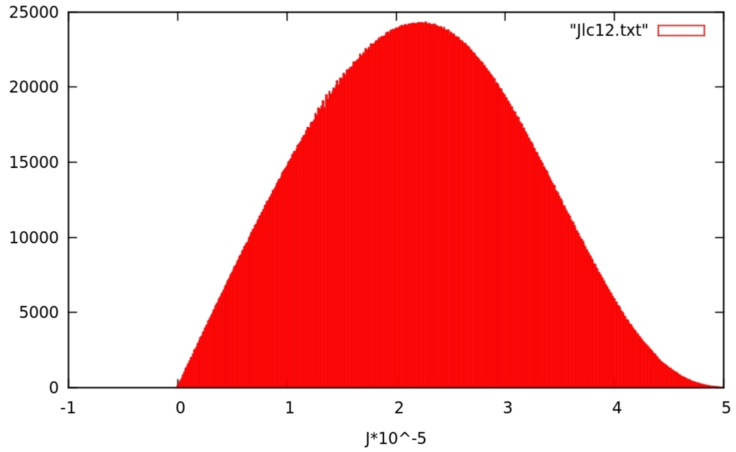
The value of δ calculated using (3.9) and the quadrature error propagation formula (3.7,3.8) is,

$$\delta = (69 \pm 61)^{\circ}$$

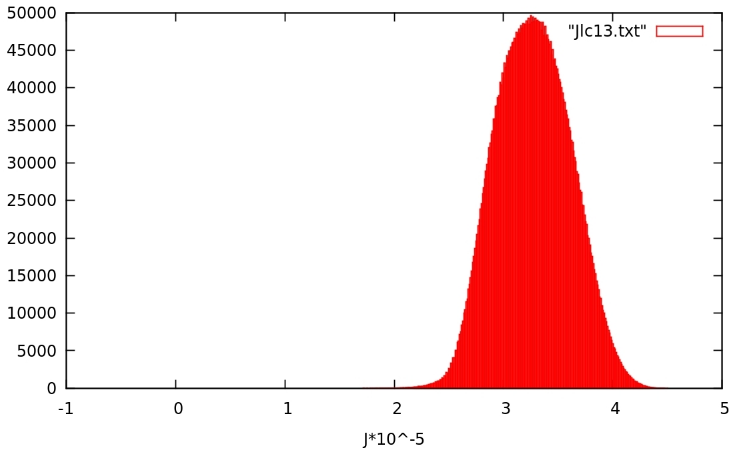
The value of δ calculated using the values of sine of angles from the PDG table,(2.2) and its quadrature error propagation formula is

$$\delta = (80 \pm 40)^{\circ}$$

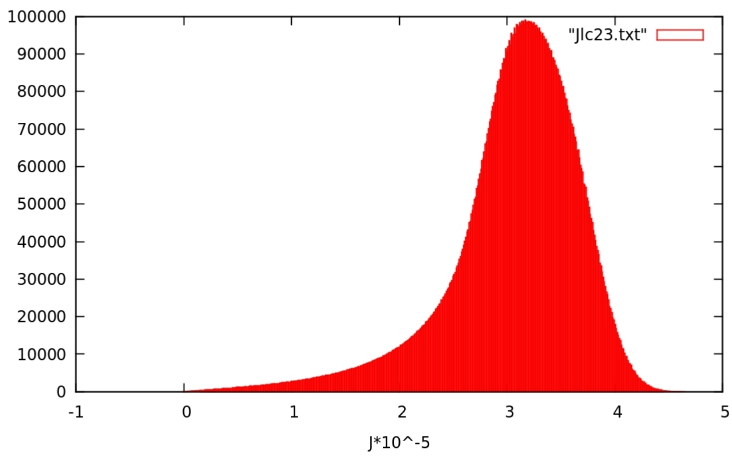
Plot for delta taking three sigma value of J(3.9) is shown in figure(3.6). The mean value gets shifted to left and the spread increases.



(a) Plot for '$\nu_1\nu_2$' triangle

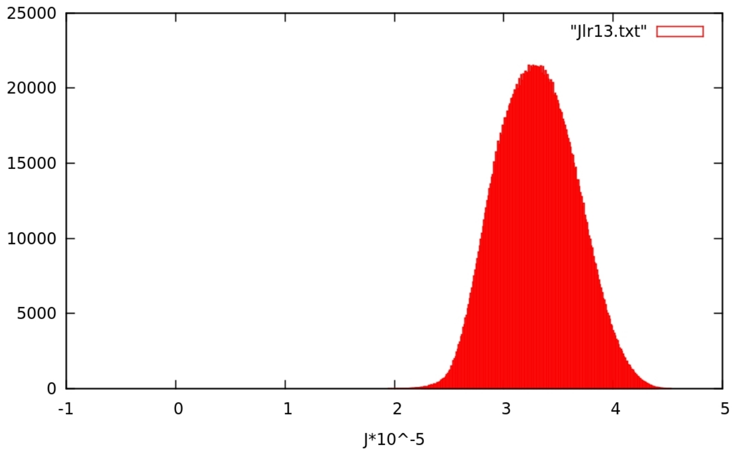


(b) Plot for $\nu_1\nu_3$ triangle

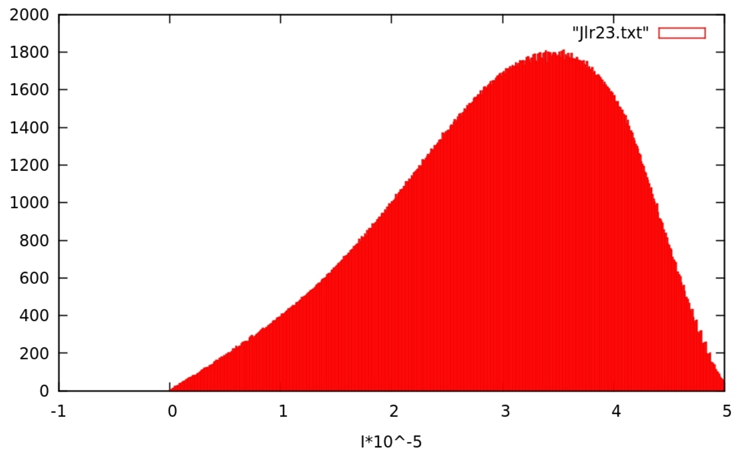


(c) Plot for $\nu_2\nu_3$ triangle

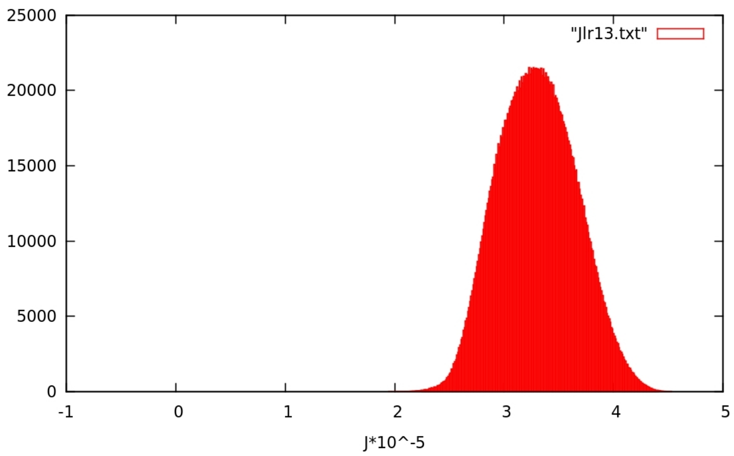
Figure 3.4: Histogram plots for Jarlskog invariant(J) in Leptonic sector



(a) Plot for '$e\mu'$ triangle

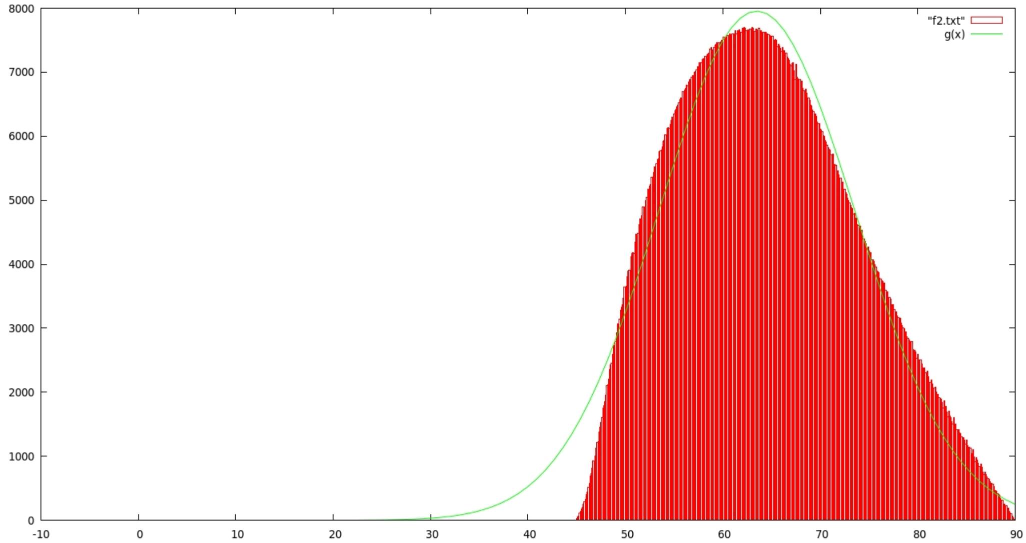


(b) Plot for '$\mu\tau'$ triangle

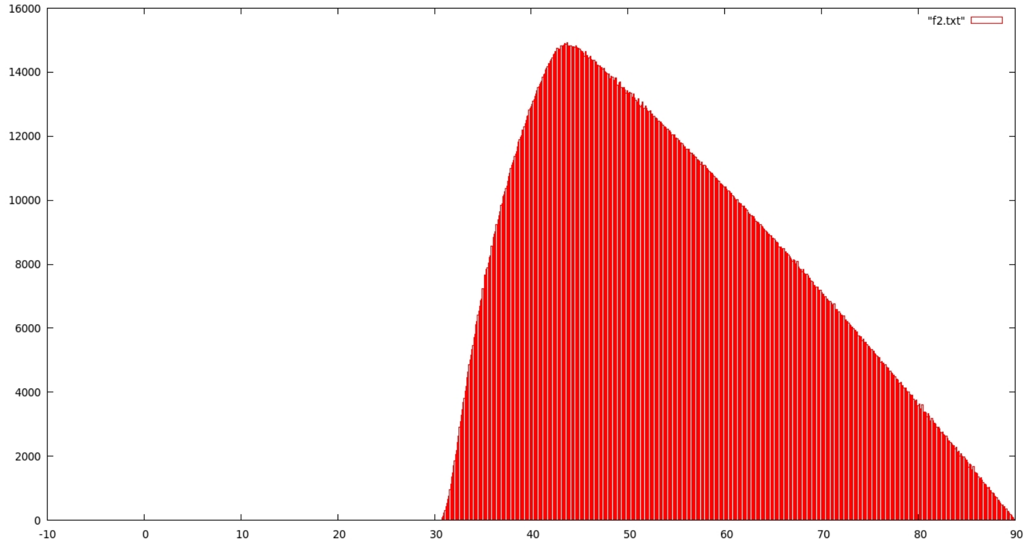


(c) Plot for '$e\tau'$ triangle

Figure 3.5: Histogram plots for Jarlskog invariant(J) in Leptonic sector



(a) Plot using one sigma error in J



(b) Plot using three sigma error in J

Figure 3.6: Histogram plots for cp violating phase(δ) in leptonic sector

Chapter 4

Conclusion

Recent measurements of the reactor mixing angle θ_{13} in the context of ν oscillations restores the parallelism between quark mixing and lepton mixing. Relatively large value of θ_{13} suggests the possibility of CP violation in leptonic sector.The purpose of the project on one hand, is to understand CP violation phenomenon in CKM paradigm for the case of quark mixing matrix while on the other hand, to use the experience gained in the quark sector to explore CP violation phenomenon in leptonic sector.

To this end, first we have exclusively discussed the formalism of CP violation within the CKM paradigm including relationship of Jarlskog invariant to the area of any of the unitarity triangle.In the case of quark sector we are able to find the value of Jarlskog invariant and then the magnitude of CP violating phase which is quite compatible with the PDG group and in the case of leptonic sector our value suggests the possibility of CP violation.

The experimental values of CKM matrix suggests that three generation unitarity is almost preserved leaving a small possibility for fourth generation. In case of fourth generation the way to parameterise the quark mixing matrix and how to find the invaiant quantities is also investigated.

Further studying about beyond standard model process of CP violation and more theoretical frame work of mass matrices would be interesting

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