Understanding Finite Pulse Effects on REDOR Experiments

Justin K Thomas

A dissertation submitted for the partial fulfilment of BS-MS dual degree in Science

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"Some inspiring quote"

Who was the person

"Yet another if you have any."

Name the guy

My humble effort I dedicate to my loving parents, whose affection,encouragement and prays make me able to get such success and honor

Certificate of Examination

This is to certify that the dissertation titled **"Understanding Finite Pulse Effects on REDOR Experiments"** submitted by **Mr. Justin K Thomas** (Reg. No. MS11039) for the partial fulfilment of BS-MS dual degree programme of the Institute, has been examined by the thesis committee duly appointed by the Institute. The committee finds the work done by the candidate satisfactory and recommends that the report be accepted.

Dr. Arijit Kumar De Dr. K. R. Shamasundar Dr. Ramesh Ramachandran

(Supervisor)

Dated:April 22,2016

Declaration

The work presented in this dissertation has been carried out by me under the guidance of Dr. Ramesh Ramachandran at the Indian Institute of Science Education and Research Mohali.

This work has not been submitted in part or in full for a degree, a diploma, or a fellowship to any other university or institute. Whenever contributions of others are involved, every effort is made to indicate this clearly, with due acknowledgement of collaborative research and discussions. This thesis is a bonafide record of original work done by me and all sources listed within have been detailed in the bibliography.

Justin K Thomas
(Candidate)

Dated: April 22, 2016

In my capacity as the supervisor of the candidate's project work, I certify that the above statements by the candidate are true to the best of my knowledge.

Dr. Ramesh Ramachandran
(Supervisor)

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Justin K Thomas

(Candidate)

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Abbreviations

NMR	**N**uclear **M**agnetic **R**esonance
REDOR	**R**otational **E**cho **D**ouble **R**esonance
SEDOR	**S**pin **E**cho **D**ouble **R**esonance
MAS	**M**agic **A**ngle **S**pinning
SSNMR	**S**olid **S**tate **N**uclear **M**agnetic **R**esonance
AHT	**A**verage **H**amiltonian **T**heory

Abstract

by AUTHOR NAME

Ever since its inception in 1992, Rotational Echo Double Resonance (REDOR) technique remains the most widely employed pulse sequence to date for measuring heteronulear dipolar interactions in solid-state NMR. In this thesis, our objective is to develop an analytic framework based on Average Hamiltonian Theory to understand its implementation at faster spinning frequencies.

Chapter 1

Introduction

Ever since the discovery of magic angle spinning (MAS)technique in 1958, the field of Solid State Nuclear Magnetic Resonance (SSNMR) has become an integral part for determining molecular structure in wide range of systems of Chemical, Physical and Biological relevance. In contrast to Solution State NMR the spectrum in Solid State NMR is broad and featureless. Consequently the number of constraints estimated is also limited in the solid state. Due to inherent rapid molecular motion, the spin interactions in the solution state are isotropic and results in a narrow spectrum. The broadening effects resulting from the dipolar and quadrupolar interactions are averaged by the molecular motion inherently present in the solution state. By contrast, the restricted mobility in the solid state renders the spin interaction anisotropy. To overcome this inherent limitation due to restricted mobility, the sample is physically rotated (in a rotor) along an axis inclined at magic angle ($\theta = 54.74^0$) with respect to the external magnetic field. Recent advances in technology and availability of faster spinning probes (spinning frequency $\approx$ 100 KHz), the resolution of the spectra in the solid state NMR has increased dramatically recent years. Interestingly the improvements in spectral resolution in MAS is accompanied by loss of structural information. In Solid State NMR the important structural constraints such as internuclear distances ($^{13}C -^{13} C,^{13} C -^{13} O$) and torsion angles are obtain only through the presence of dipolar interaction. To overcome their drawback imposed by MAS, the special class of experiment termed as 'recoupling sequences' have emerged in the last two decades. In recoupling experiments the desired anisotropic interactions are reintroduced only during certain period without compromising spectral resolution afforded by MAS. Especially the spacial averaging effect of MAS is compensated (at certain time period) through application of designed multi-pulse sequence. In particular the emergence of dipolar recoupling experiments has been quite

In general the dipolar coupling experiments are classified into broadband and selective. Depending on the nature of applications, the choice of dipolar recoupling experiments are differ. For example, in spectral assignment experiments presence of all the dipolar couplings in the system is essential to establish local connectivity. Hence in such cases broadband dipolar recoupling experiments are derived. Alternatively, in the case of experiments involving distance measurement, the dipolar interactions are reintroduced in a controlled (or selective) fashion in 'selective recoupling' experiments. In the initial stages of distance measurements, the dipolar interactions were quantified using spin-pair labeled samples. Since multiple distance constraints are required for structure determination, selectively labeled spin-pair were abandoned in favour of uniformly uniformly $^{13}C,^{15}N$ labeled samples.

To minimize multi-spin effects in uniformly labeled samples, selective recoupling experiments integrated with two-dimensional NMR has been employed for measuring internuclear distances in Homonuclear spin systems. Here in this thesis we confine our discussion to Heteronuclear recouplings experiments in Solid State NMR. Specifically, we confine our attention to rotational echo double resonance (REDOR) experiments in Solid State. In REDOR experiments, the dipolar interaction between Heteronuclear spins are reintroduced through the application of π pulses. To account for other spin interactions and relaxation effects, reference experiments along with the standard REDOR experiments are employed for quantifying the dephasing trajectories obtained in REDOR experiments. our bjective in this thesis is to reexamine the REDOR experiments and develop a theoretical framework for understanding its implementation at faster spinning frequencies. Specifically the effect of finite pulse at faster spinning frequencies will be discussed using analytic methods based on Average Hamiltonian Theory (AHT).

Chapter 2

Average Hamiltonian Theory

In the Schrodinger picture, the state of a system is described by the wave function, $\psi(t)$. The $H(t)$ is the Hamiltonian operator contain the internal spin interactions of the system. The evolution of system under a time independent Hamiltonian is given by the Schrodinger equation.

$$i\hbar\frac{d}{dt}\psi(t) = H\psi(t) \tag{2.1}$$

$$\psi(t) = e^{\frac{-iHt}{\hbar}}\psi(t) \tag{2.2}$$

When the Hamiltonian is time dependent the evolution of the system is described by a complicated expression

$$\psi(t) = e^{-i\int_0^t\frac{H(t)dt}{\hbar}}\psi(t) \tag{2.3}$$

In the case were the system (comprising of many subsystems) is described by more than one wave functions, the state of the system is represent using density matrix. The density operator $\rho(t)$ is the weighted average over the possible spin states that in the sample.

$$\rho(t) = \overline{|\psi(t) >< \psi(t)|} \tag{2.4}$$

where the bar represent the weighted average over the spin degrees of freedom present in the system.

Subsequently, the time evolution of the system is described by quantum Liouville equation.

$$i\hbar\frac{d}{dt}\rho(t) = [H(t), \rho(t)] \tag{2.5}$$

When the Hamiltonian is time independent the final solution of above equation reduces to a simple form.

$$\rho(t) = e^{\frac{-iHt}{\hbar}}\rho(0)e^{\frac{iHt}{\hbar}} \tag{2.6}$$

where $\rho(0)$ is the initial state of the system. The evolution of system under a time dependent Hamiltonian has a complicated form

$$\rho(t) = e^{-i\int_0^t \frac{H(t)dt}{\hbar}}\rho(0)e^{i\int_0^t \frac{H(t)dt}{\hbar}} \tag{2.7}$$

Since Hamiltonian in the Magic Angle Spinning (MAS) experiments are time dependent analytic description in terms of Average Hamiltonian Theory (AHT)have extensively been employed to describe the system. In AHT framework the time evolution of the system is described through a time-averaged effective/average Hamiltonian.

$$\rho(t) = e^{-i\frac{\tilde{H}_{ave}t}{\hbar}}\rho(0)e^{i\frac{\tilde{H}_{ave}t}{\hbar}} \tag{2.8}$$

Employing Magnus expansion the time-averaged Hamiltonian is expressed in a series of terms of decreasing magnitude.

$$\tilde{H}_{ave} = \tilde{H}^0 + \tilde{H}^1 + \tilde{H}^2 + \tilde{H}^3 ... \tag{2.9}$$

$$\tilde{H}^0 = \frac{1}{t}\int_0^t \tilde{H}(t')dt' \tag{2.10}$$

$$\tilde{H}^1 = -\frac{i}{2t}\int_0^t dt' \int_0^{t'} dt''[\tilde{H}(t'), \tilde{H}(t'')] \tag{2.11}$$

$$\tilde{H}^1 = \frac{1}{6t}\int_0^t dt' \int_0^{t'} dt'' \int_0^{t''} dt''([\tilde{H}(t'), \tilde{H}(t'')], \tilde{H}(t''')] + [\tilde{H}(t'''), \tilde{H}(t'')], \tilde{H}(t')] \tag{2.12}$$

In the past Average Hamiltonian Theory (AHT) has been extensively employed to describe and design experiments in Solid State NMR. A brief demonstration of this procedure in the description of RF pulse is presented in the following section.

The nuclear spin Hamiltonian under the magic angle spinning condition is represented by

$$H(t) = H_D(t) + H_{CSA}(t) + H_{ICS}(t) + H_{rf}(t) \tag{2.13}$$

In the above equation the Hamiltonian $H_D(t)$,$H_{CSA}(t)$,$H_{ICS}(t)$ and $H_{rf}(t)$ correspond to dipolar, chemical shift anisotropic, isptropic chemical shift and RF pulse respectively. In the interaction representation defined by the RF field, the Hamiltonian H(t) acquires additional time dependence. The transformation to RF interaction frame is given below

$$\tilde{H}(t) = U_{rf}^{-1} H(t) U_{rf} \tag{2.14}$$

where U_{rf} is the propagator due to the RF pulse.

The Average Hamiltonian theory for the effective Hamiltonian is based Magnus expansion. The Magnus expansion is a perturbative solution to exponential of a time varying operator. According to AHT, the evolution of the spin system under the time dependent Hamiltonian represent in the form of time independent effective Hamiltonian.

$$\rho(t) = e^{-i\int_0^t \frac{\tilde{H}dt}{\hbar}} \rho(0) e^{i\int_0^t \frac{\tilde{H}dt}{\hbar}} \tag{2.15}$$

$$\rho(t) = e^{-i\frac{\tilde{H}_{ave}t}{\hbar}} \rho(0) e^{i\frac{\tilde{H}_{ave}t}{\hbar}} \tag{2.16}$$

$$\tilde{H}_{ave} = \frac{1}{t}\int_0^t \tilde{H}dt \tag{2.17}$$

The effective Hamiltonian $\tilde{H}_{ave}$ derived in AHT is based on Magnus expansion.

$$e^{-i\int_0^t \frac{\tilde{H}t}{\hbar}} = e^{-i\frac{\tilde{H}_{ave}t}{\hbar}}$$

The effective time independent Hamiltonian $\tilde{H}_{ave}$ can be expanded in a series of terms of increasing order of time by Magnus expansion

$$\tilde{H}_{ave} = \tilde{H}^0 + \tilde{H}^1 + \tilde{H}^2 + \tilde{H}^3 ...$$

$$e^{-i\frac{\tilde{H}_{ave}t}{\hbar}} = e^{-it\frac{\tilde{H}^0 + \tilde{H}^1 + \tilde{H}^2 + \tilde{H}^3 ...}{\hbar}}$$

$$\tilde{H}^0 = \frac{1}{t}\int_0^t \tilde{H}(t')dt' \tag{2.18}$$

$$\tilde{H}^1 = -\frac{i}{2t}\int_0^t dt' \int_0^{t'} dt'' [\tilde{H}(t'), \tilde{H}(t'')] \tag{2.19}$$

$$\tilde{H}^1 = \frac{1}{6t}\int_0^t dt' \int_0^{t'} dt'' \int_0^{t''} dt'' ([\tilde{H}(t'), \tilde{H}(t'')], \tilde{H}(t''')] + [\tilde{H}(t'''), \tilde{H}(t'')], \tilde{H}(t')]) \tag{2.20}$$

The AHT applicable in pulse sequence that consist block of RF irradiation that is repeated many times as in case of recoupling experiments of SSNMR.The following condition should met for the application of AHT to the pulse sequence with cycle time t_c.

$$H_{rf}(t_c + t) = H_{rf}(t) \tag{2.21}$$

$$H_D(t_c + t) = H_D(t) \tag{2.22}$$

$$H_{CSA}(t_c + t) = H_{CSA}(t) \tag{2.23}$$

The convergence of Magnus expansion ensured only when $\|H\|t_c \ll 1$

2.0.1 Effect of Finite Pulse

The response of a nuclear spin system in NMR spectroscopy is determined through the application of radio frequency (RF) pulses.A typical pulse sequence ranges from a single pulse experiment to a complicated pulse sequence for extracting specific interactions. Depending on the duration of the pulse and relative magnitude of the amplitude of the pulse with respect to internal spin interactions, the RF pulses are classified to (a) hard (b) soft pulses.In the hard pulse limit, the amplitude of RF pulse exceeds the magnitude of internal spin interactions. Consequently during a pulse, the evolution of system is solely governed by RF Hamiltonian (it is independent of the internal interactions).By contrast, in the soft pulse limit the evolution of the system depends on both the RF Hamiltonian and internal spin Hamiltonian. The important characteristic of a soft pulse includes its shape, amplitude and duration. The pulse shape is correlated with shape of the excitation profile, the pulse amplitude with flip angle and pulse duration with the selectivity. Such pulses are known as soft pulses or selective pulses which specifically shaped in order to tailor their excitation profile.

The pulse sequence like REDOR contain a train of pulse cycles which consumes an appreciable part of the entire pulse sequence. Comparing the duration of pulse with respect to duration of pulse sequence the pulses can be classified to delta pulse and finite pulse.

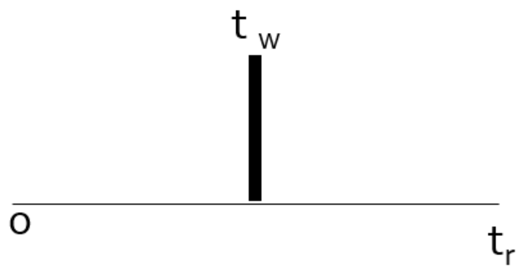

FIGURE 2.1: Delta pulse
(**Schematic depiction of Delta Pulse**:t_w and t_r are the duration of pulse and pulse sequence respectively)

In the case a of delta pulse the duration of pulse (t_w) is very short when compared to the duration of the entire pulse sequence such the ratio of $\frac{t_w}{t_r} \approx 0$. The duration of pulse is negligible such that the evolution of the system in this small interval is insignificant. By contrast, the finite pulse are the pulses which occupy an appreciable amount of time in the pulse sequence. The approximation $\frac{t_w}{t_r} \approx 0$ is no more valid in such cases. The system evolves during the duration of the pulse. The schematic representation is given in figure 2.2.

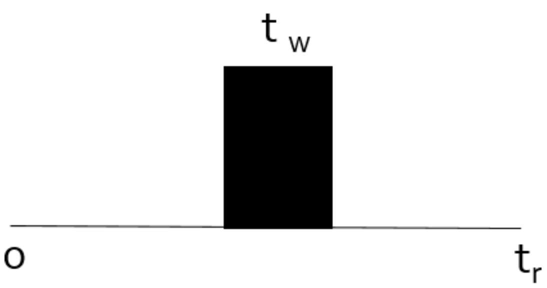

FIGURE 2.2: **Schematic depiction of Finite Pulse**
t_w and t_r are the duration of pulse and pulse sequence respectively. $\frac{t_w}{t_r} \neq 0$

SSNMR experiments routinely used for the analysis of weak coupling interaction between the spins which in important for the structural changes like tertiary and quarternery structure of proteins. The analysis of signal using appropriate mathematical expression have high impact. The small errors can effect the experimental results drastically. As part of methodological development the figuring out the suitable conditions for the experiment also equally important. The analysis of effects of finite pulse in large pulse sequence like REDOR always exciting. The AHT gives a platforms to figure out the effects of finite pulse in the signal. Before describing the finite pulse effect in REDOR pulse sequence let's try to find the effect of the finite pulse in chemical shift Hamiltonian of pulse sequence given in Figure 3.3 using AHT.

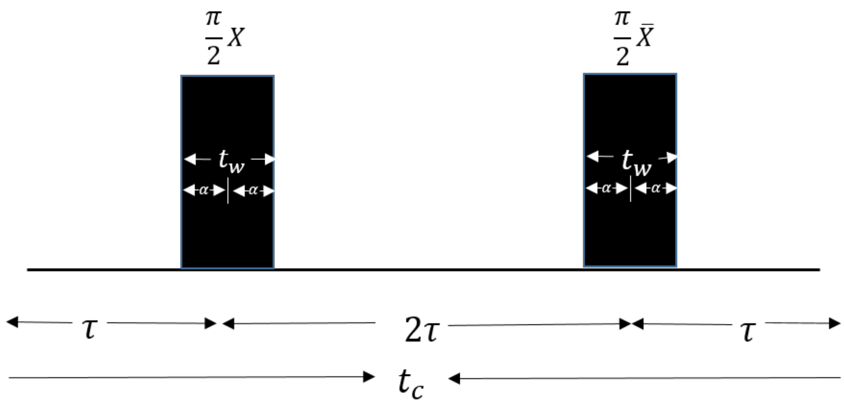

FIGURE 2.3: The pulse sequence contain $\frac{\pi}{2}X$ and $\frac{\pi}{2}\overline{X}$ pulses with pulse duration $t_w = 2\alpha$. The cycle time of this sequence $t_c = 4\tau$

Consider the following pulse two sequence comprising of $\frac{\pi}{2}X$ and $\frac{\pi}{2}\overline{X}$ finite pulses. The duration of each pulse is $t_w = 2\alpha$ and the cycle time of the sequence is $t_c = 4\tau$. The general transformation over the cycle from 0 to t_n is given by the propagator $U(t_n, 0)$.

$$U(t_n, 0) = U(t_n, t_{n-1})U(t_{n-1}, t_{n-2})...U(t_1, t_0) \tag{2.24}$$

To simplify the description, let the pulse sequence be applied to to an isolated spin system, the internal Hamiltonian consist of Chemical Shift Interaction.

$$H = \Delta\omega I_z \tag{2.25}$$

In the RF interaction frame the chemical shift Hamiltonian is time dependent. Employing Average Hamiltonian Theory (AHT) the effect of the finite pulses on the chemical shift Hamiltonian is derived,

$$\widehat{I}_z(\tau+\alpha) = U^{\dagger}(\tau+\alpha,0)I_zU(\tau+\alpha,0) \tag{2.26}$$

$$U^{\dagger}(\tau+\alpha,0) = U^{\dagger}(\tau-\alpha,0)U^{\dagger}(\tau+\alpha,\tau-\alpha) \tag{2.27}$$

$$U(\tau+\alpha,0) = U(\tau+\alpha,0)U(\tau-\alpha,0) \tag{2.28}$$

There is no RF pulse from 0 to $\tau-\alpha$ so the propagator $U(\tau-\alpha,0)$ and $U^{\dagger}(\tau-\alpha,0)$ is one in this case.

$$U(\tau-\alpha,0) = 1 \tag{2.29}$$

$$U^{\dagger}(\tau-\alpha,0)) = 1 \tag{2.30}$$

The $\frac{\pi}{2}$ X pulse applied for the time interval from $\tau-\alpha$ to $\tau+\alpha$. The propagator correspond the time interval is $U^{\dagger}(\tau+\alpha) = exp(-i\theta(t)I_x)$ where the $\theta(t)$ is zero when time $t=\tau-\alpha$ and it is $\frac{\pi}{2}$ at time $t=\tau+\alpha$. From the above conditions the functional form of $\theta(t)$ is derived

$$\theta(t) = \frac{\pi}{4}(\frac{t-(\tau-\alpha)}{\alpha})$$

Subsequently, the $I_z(\tau+\alpha)$ will transform to $\widehat{I}_z(\tau+\alpha)$ in the toggling frame by the action of the propagator defined for the time interval.

$$\widehat{I}_z(\tau+\alpha) = U^{\dagger}(\tau-\alpha,0)U^{\dagger}(\tau+\alpha,\tau-\alpha)I_zU(\tau+\alpha,\tau-\alpha)U(\tau-\alpha,0) \tag{2.31}$$

$$= I_z\cos\theta(t) - I_y\sin\theta(t) \tag{2.32}$$

For the time interval from $\tau + \alpha$ to $3\tau - \alpha$ the $\widehat{I}_z(3\tau - \alpha)$ given by the following transformation.

$$\widehat{I}_z(3\tau-\alpha) = U^\dagger(\tau-\alpha,0)U^\dagger(\tau+\alpha,\tau-\alpha)U^\dagger(3\tau-\alpha,\tau+\alpha)I_z U(3\tau-\alpha,\tau+\alpha)U(\tau+\alpha,\tau-\alpha)U(\tau-\alpha,0)$$

Where $U^\dagger(\tau - \alpha, 0)$ and $U^\dagger(3\tau - \alpha, \tau + \alpha)$ is one because no pulse applied applied during the given time interval. The $\theta(t)$ of the propagator $U^\dagger(\tau + \alpha, \tau - \alpha)$ is $\frac{\pi}{2}$ at time $t = \tau + \alpha$.

$$\widehat{I}_z(3\tau - \alpha) = exp(-i\frac{\pi}{2}I_x)I_z exp(i\frac{\pi}{2}I_x) = -I_y$$

For an interval from $3\tau - \alpha$ to $3\tau + \alpha$ a $\frac{\pi}{2}$ pulse applied in -X direction.

$$\begin{aligned}
\widehat{I}_z(3\tau + \alpha) &= U^\dagger(\tau - \alpha, 0)...U^\dagger(3\tau + \alpha, 3\tau - \alpha)I_z U(3\tau + \alpha, 3\tau - \alpha)...U(\tau - \alpha, 0) \\
&= exp(-i\frac{\pi}{2}I_x)exp(i\theta(t)I_x)I_z exp(-i\theta(t)I_x)exp(i\frac{\pi}{2}I_x) \\
&= -I_y \cos\theta(t) + I_z \sin\theta(t)
\end{aligned}$$

where the $\theta(t) = \frac{\pi}{4}(\frac{t - (3\tau - \alpha)}{\alpha})$
There is no pulse applied during the time interval from $3\tau - \alpha$ to 4τ.

$$\widehat{I}_z(3\tau - \alpha) = U^\dagger(\tau - \alpha, 0)...U^\dagger(4\tau, 3\tau + \alpha)I_z U(4\tau, 3\tau + \alpha)...U(\tau - \alpha, 0) = I_z$$

The effective Hamiltonian for the pulse sequence in figure 2.3 is calculated using Average Hamiltonian Theory. To zeroth order

$$\overline{H}_0^0 = \frac{1}{t_c}\int_0^{t_c} \tilde{H}_0(t)dt$$

t_c is the cycle time of the pulse sequence. The effective Hamiltonian of the pulse sequence is the sum of effective Hamiltonians of the the intervals in pulse sequence.

$$\overline{H}_0^0 = \frac{\Delta\omega}{t_c}[\int_0^{\tau-\alpha} I_z dt + \int_{\tau-\alpha}^{\tau+\alpha} (I_z \cos\theta(t) - I_y \sin\theta(t))dt + \int_{\tau+\alpha}^{3\tau-\alpha} (-I_y)dt +$$

$$\int_{3\tau-\alpha}^{3\tau+\alpha} (I_z(\sin\theta(t) - I_y \cos\theta(t))dt + \int_{3\tau+\alpha}^{4\tau} (-I_z)dt]$$

$$\overline{H}_0^0 = \frac{\Delta\omega}{2}[I_z - I_y] + \frac{2t_w}{t_c}(\frac{4}{\pi} - 1)[I_z - I_y]$$

In the next chapter, the effect of finite pulses in REDOR experiment will be discussed.

Chapter 3

The Effect of Finite pulses in REDOR Experiment

From a theoretical perspective, the spin interactions under MAS conditions are time-dependent and periodic. As described earlier, important structural constraints are encoded in the anisotropic interactions and are often averaged under MAS. To recover anisotropic interactions (or part of it), periodic modulations in the form of multiple-pulse schemes are introduced to compensate the periodic time modulation imposed by MAS. Here in this chapter, we confine our discussion to heteronuclear dipolar recoupling of dipolar interactions through REDOR experiments. The REDOR experiment is essentially derived from SEDOR experiment. A detailed description of those pulse sequences in the Average Hamiltonian framework described in the following section.

3.1 Spin Echo Double Resonance-SEDOR

In 1968, Kaplan and Hahn introduced the SEDOR (Spin Echo Double Resonance) technique to reintroduce heteronuclear dipolar interactions in static solids. The Hamiltonian in NMR is generally expressed as product of spacial and spin parts. Since SEDOR is a stationary experiment, the spacial part of the Hamiltonian is constant through out the experiment. Hence SEDOR sequence is designed to recouple heteronuclear dipolar interaction using pulses that affect only the spin part of the Hamiltonian. Consider a model system comprising of an isolated heteronuclear spin pair I and S.

The basic SEDOR experiment comprises of two pulse sequences, commonly referred to as reference and dephasing experiments.The schematic depiction of the pulse schemes corresponding to dephasing and reference experiments are depicted.

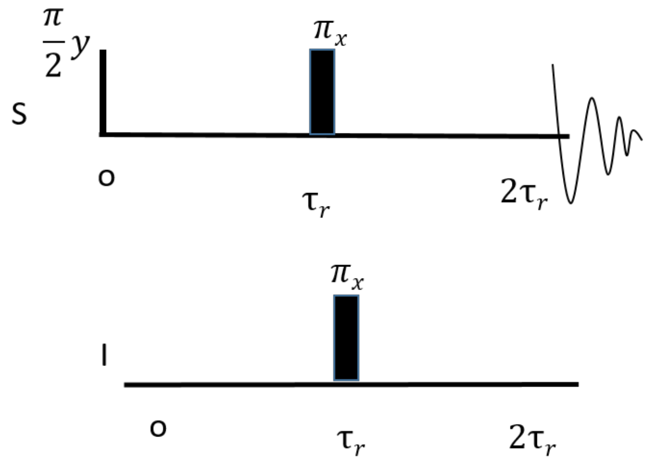

FIGURE 3.1: **Schematic depiction of dephasing experiment of SEDOR**: The spin echo pulse sequence consist of $\frac{\pi}{2}$ pulse applied along Y direction to S spin. A π pulse applied at exactly middle of the pulse sequence to both spins along X direction. signal collected from S spin at time $2\tau_r$.

Dephasing pulse experiment begins with a $(\frac{\pi}{2})_y$ pulse on the S channel. After time τ_r a π pulse (X-direction) is applied on both the channels.Subsequently the signal on S channel is detected after a time $2\tau_r$ Since it is a dipolar recoupling sequence, all the other spin interactions are refocused. The dipolar Hamiltonian for a stationary samples is represented by,

$$H_D^{IS} = 2\omega_D I_z S_z \tag{3.1}$$

The initial $\frac{\pi}{2}$ Y pulse on S-channel,converts the initial density operator $\rho(0) = S_z$ to S_x

$$\rho(t_p) = e^{-i\frac{\pi}{2}S_y} S_z e^{i\frac{\pi}{2}S_y} = S_x \tag{3.2}$$

After the pulse the system evolves under the dipolar Hamiltonian from time 0 to τ_r. The state of the system after time τ_r represented by

$$\rho(t_p + \tau_r) e^{-i\omega_D I_z S_z} S_x e^{i\omega_D I_z S_z} = \cos(\omega_D \tau_r) S_x + \sin(\omega_D \tau_r) 2S_y I_z \tag{3.3}$$

At time $t = \tau_r$, a $(\pi)_X$ pulse is applied on both channels. The effect of π pulses is calculated and represented below,

$$e^{-i\pi S_x} e^{-i\pi I_x} (\cos(\omega_D \tau_r) S_x + \sin(\omega_D \tau_r) 2S_y I_z) e^{i\pi S_x} e^{i\pi I_x} = \cos(\omega_D \tau_r) S_x + \sin(\omega_D \tau_r) 2S_y I_z \tag{3.4}$$

From the time $t = \tau_r$ to $t = 2\tau_r$ the system evolves under dipolar Hamiltonian. Final state of the system is represented by

$$e^{-i\omega_D I_z S_z} (\cos(\omega_D \tau_r) S_x + \sin(\omega_D \tau_r) 2S_y I_z) e^{i\omega_D I_z S_z} = \cos(2\omega_D \tau_r) S_x + \sin(2\omega_D \tau_r) 2S_y I_z \tag{3.5}$$

The signal after time $t = 2\tau_r$ along the S-channel is represented through the given expression

$$S_r(2\tau_r) = S_i \cos(2\omega_D \tau_r) e^{\frac{-2\tau_r}{T}} \tag{3.6}$$

In the above expression the term $e^{\frac{-2\tau_r}{T}}$ relaxation and other indirect effects. To get rid of all other factors responsible for dephasing of the signal reference experiments are performed.

The reference experiment begins with a $\frac{\pi}{2}$ Y pulse on S-channel.After time τ_r a $(\pi)_x$ pulse applied on the S spin channel and signal is collected from S-channel at $2\tau_r$. The sequence refocus all interactions a gives back initial magnetization produced exactly after the $\frac{\pi}{2}y$ pulse. The important steps are illustrated below.

$$\rho(t_{p1}) = e^{-i\frac{\pi}{2}S_y} S_z e^{i\frac{\pi}{2}S_y} = S_x \tag{3.7}$$

$$\rho(t_{p1} + \tau_r) = e^{-i\omega_D I_z S_z} S_x e^{i\omega_D I_z S_z} = \cos(\omega_D \tau_r) S_x + \sin(\omega_D \tau_r) 2 S_y I_z \tag{3.8}$$

At time τ_r a π pulse applied at exactly middle of the pulse sequence to S spins along X direction . The effect of the pulses on the state of spins found by

$$\begin{aligned} \rho(t_{p1} + \tau_r + t_{p2}) &= e^{-i\pi S_x}(\cos(\omega_D \tau_r) S_x + \sin(\omega_D \tau_r) 2 S_y I_z) e^{i\pi S_x} \quad (3.9) \\ &= \cos(\omega_D \tau_r) S_x - \sin(\omega_D \tau_r) 2 S_y I_z \quad (3.10) \end{aligned}$$

The system evolves under the dipolar Hamiltonian from time τ_r to $2\tau_r$. The final state of the system given by

$$\rho(2\tau_r) = e^{-i\omega_D I_z S_z}(\cos(\omega_D \tau_r) S_x + \sin(\omega_D \tau_r) 2 S_y I_z) e^{i\omega_D I_z S_z} = \cos(\omega_D \tau_r)\cos(\omega_D \tau_r) S_x$$

$$+ \sin(\omega_D \tau_r)\sin(\omega_D \tau_r) S_y + \cos(\omega_D \tau_r)\sin(\omega_D \tau_r) I_z S_y + \cos(\omega_D \tau_r)\sin(\omega_D \tau_r) I_z S_y = S_x$$

The signal of the reference experiment is represented by

$$S_0(2\tau_r) = S_i e^{\frac{-2\tau_r}{T}} \tag{3.11}$$

The ratio of the signal from dephasing and reference experiment gives an expression that describes the dephasing purely resulting from dipolar interaction.

$$\frac{S_r}{S_0} = \cos(2\tau_r \omega_D) \tag{3.12}$$

The ratio only depend upon dipolar coupling and dipolar evolution time. Thus by simply measuring signal amplitudes it is possible to obtain dipolar coupling between I and S spin.

3.2 Rotational Echo Double Resonance NMR - REDOR

The REDOR experiment was introduced in 1989 by J.Shaefer and T. Gullion to recouple heteronuclear dipolar interactions under MAS condition. As described in the previous section, REDOR experiment is an extension of SEDOR experiment.The dipolar Hamiltonian under MAS condition is time-dependent

$$H_D^{IS}(t) = \sum_{m\neq 0, m=-2}^{2} \omega^m \exp(im\omega_r t) I_z S_z \tag{3.13}$$

Here ω_r represents the spinning frequency of sample.The average value of dipolar interaction over a rotor period is zero due to magic angle spinning. The RF pulses are designed to compensate the averaging effect of MAS in such a way that the dipolar interactions are recovered. As in case of SEDOR experiments, REDOR also contain reference and dephasing experiments. The reference experiment refocuses all the interactions, while dephasing experiment recouple only the dipolar interaction between heteronuclear spin pair. A schematic depiction of the pulse sequence depicted below.

The REDOR pulse sequence begins with a $\frac{\pi}{2}$ pulse applied along Y direction on the S-channel. The $(\pi)_x$ pulses are applied at $\frac{\tau_r}{2}$ and $\frac{3\tau_r}{2}$ on I channel. At τ_r a $(\pi)_x$ pulse is applied on S channel. AS described in SEDOR experiment the dephasing sequence recouple dipolar interaction at the end. The signal collected from S spin can express as $S_r(2\tau_r)$,

$$S_r(2\tau_r) = S_i \cos(2\omega_D \tau_r) e^{\frac{-2\tau_r}{T}}$$

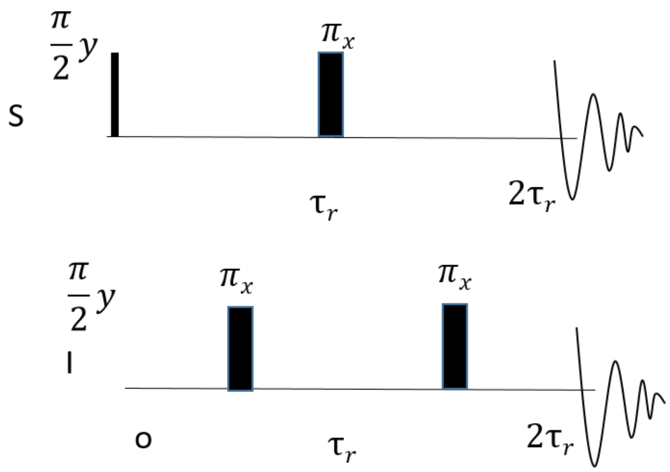

FIGURE 3.2: Pulse sequence for dephasing experiment of REDOR: The pulse sequence start with a $\frac{\pi}{2}$ pulse applied along Y direction to S spin. The π pulses applied at $\frac{\tau_r}{2}$ and $\frac{3\tau_r}{2}$ to S spin along X direction. One π pulse applied at τ_r on S spin. The signal collected from S spin at time $2\tau_r$.

As in case of SEDOR experiments dephasing the signal is proportional to the dipolar coupling and evolution time. The referance sequence refocus all interactions as shown SEDOR experiment. The pulse sequence for the reference is given below,

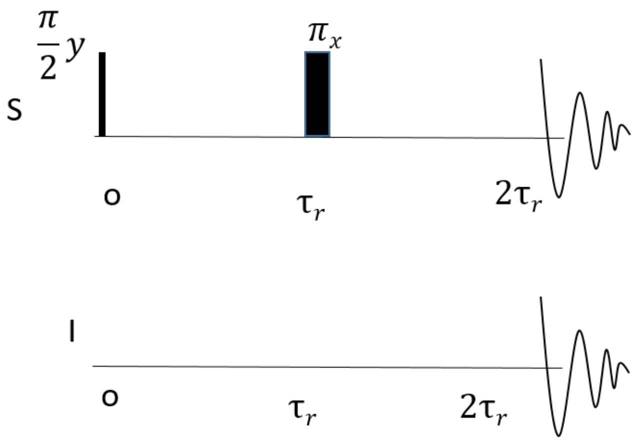

FIGURE 3.3: Pulse sequence for reference experiment of REDOR: The spin echo pulse sequence start with a $\frac{\pi}{2}$ pulse applied along Y direction to S spin. One π pulse applied at τ_r on S spin. The signal acquisition begins at $2\tau_r$ from S spin.

The signal from the reference experiment along the S-channel is described by,

$$S_0(2\tau_r) = S_i e^{\frac{-2\tau_r}{T}} \tag{3.14}$$

Similar to the SEDOR experiment, the ratio of the signal from dephasing and reference experiment

$$\frac{S_r}{S_0} = \cos(2\tau_r\omega_D)$$

The ratio only depend upon dipolar coupling and dipolar evolution time. The ratio independent on T, the relaxation constant and all other interactions.

In the above description the pulses has been considered to be ideal, that is the duration of the pulse is extremely short (Delta pulse approximation). However in real MAS experiments the duration of pulses is significant and the explicit evolution under MAS conditions have to considered.To compensate pulse imperfections and other errors, REDOR XX-4 is commonly employed in SSNMR.The detailed analysis of this pulse sequence within the Average Hamiltonian framework is presented in the following sections.

Following the description in chapter-2 the dipolar Hamiltonian in the toggling frame is derived through the description presented below

In the interval from α to $\frac{\tau}{2}-\alpha$,there is no pulse. The propagator corresponding to the interval is $U(\frac{\tau}{2}-\alpha,\alpha)$ is 1. The Hamiltonian in toggling frame is represented by,

$$U^{\dagger}(\frac{\tau}{2}-\alpha,\alpha)H_D(t)^{IS}U(\frac{\tau}{2}-\alpha,\alpha) = \sum_{m\neq 0,m=-2}^{2} \omega^m \exp(im\omega_r t)I_zS_z$$

The Hamiltonian in toggling frame is time dependent. The effective Hamiltonian is given by the integration over the interval α to $\frac{\tau}{2}-\alpha$.

$$\sum_{m\neq 0,m=-2}^{2}\int_{\alpha}^{\frac{\tau}{2}-\alpha} \omega^m \exp(im\omega_r t)dtI_zS_z = I_zS_z\{\frac{[-\omega^{-2}-\omega^{2}]}{\omega_r}\sin 2\omega_r\alpha + \frac{[\omega^{-1}-\omega^{1}]}{i\omega_r}2\cos\omega_r\alpha\}$$

A π pulse applied along X axis for finite duration from $\frac{\tau}{2}-\alpha$ to $\frac{\tau}{2}+\alpha$.Subsequently, the Hamiltonian in toggling frame get altered

$$U^{\dagger}(\frac{\tau}{2}+\alpha,\frac{\tau}{2}-\alpha)H_D^{IS}U(\frac{\tau}{2}+\alpha,\frac{\tau}{2}-\alpha)=\sum_{m\neq0,m=-2}^{2}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z$$

Where $\theta(t)=\frac{\pi}{2}[\frac{t-\frac{\tau}{2}+\alpha}{\alpha}]$
the Hamiltonian for the the same duration given by

$$\sum_{m\neq0,m=-2}^{2}\int_{\frac{\tau}{2}-\alpha}^{\frac{\tau}{2}+\alpha}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z dt$$

$$=-iI_zS_z\{\frac{\omega_r(\omega^{-2}-\omega^2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos2\omega_r\alpha+\frac{\omega_r(\omega^1-\omega^{-1})}{\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos\omega_r\alpha\}$$

$$-I_yS_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}2\cos2\omega_r\alpha+\frac{\frac{\pi}{\alpha}(-\omega^1-\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

For the time interval from $\frac{\tau}{2}+\alpha$ to $\tau-\alpha$ the propagator $U(\frac{\tau}{2}+\alpha,\tau-\alpha)$ is one. The Hamiltonian under transformation is represented by

$$U^{\dagger}(\frac{\tau}{2}+\alpha,\tau-\alpha)H_D^{IS}(t)U(\frac{\tau}{2}+\alpha,\tau-\alpha)=-\sum_{m\neq0,m=-2}^{2}\omega^m\exp(im\omega_r t)I_zS_z$$

$$\sum_{m\neq0,m=-2}^{2}\int_{\frac{\tau}{2}+\alpha}^{\tau-\alpha}\omega^m\exp(im\omega_r t)dtI_zS_z=-I_zS_z\{\frac{[-\omega^{-2}-\omega^2]}{\omega_r}\sin2\omega_r\alpha+\frac{[-\omega^{-1}+\omega^1]}{i\omega_r}2\cos\omega_r\alpha\}$$

For duration $\tau-\alpha$ to $\tau+\alpha$ a π pulse applied along X axis. The Hamiltonian corresponding to above duration is represented

$$U^{\dagger}(\tau+\alpha,\tau-\alpha)H_D^{IS}(t)U((\tau+\alpha,\tau-\alpha))=-\sum_{m\neq0,m=-2}^{2}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z$$

Where $\theta(t) = \frac{\pi}{2}[\frac{t-\tau+\alpha}{\alpha}]$

$$-\sum_{m\neq 0,m=-2}^{2}\int_{\tau-\alpha}^{\tau+\alpha}\omega^m \exp(im\omega_r t)(I_z\cos\theta(t) - I_y\sin\theta(t))S_z dt$$

$$= iI_zS_z\{\frac{\omega_r(\omega^{-2}-\omega^2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha + \frac{\omega_r(-\omega^1+\omega^{-1})}{\omega_r^2-(\frac{\pi}{2\alpha})^2}2\cos\omega_r\alpha\}$$

$$I_yS_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha + \frac{\frac{\pi}{\alpha}(\omega^1+\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

For the duration of $\tau+\alpha$ to $\frac{3\tau}{2}-\alpha$ no pulses applied. The transformation to toggling frame results

$$U^\dagger(\frac{3\tau}{2}-\alpha,\tau+\alpha)H_D^{IS}(t)U(\frac{3\tau}{2}-\alpha,\tau+\alpha) = \sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_r t)I_zS_z$$

the Hamiltonian corresponding to the interval is

$$\sum_{m\neq 0,m=-2}^{2}\int_{\tau+\alpha}^{\frac{3\tau}{2}-\alpha}\omega^m\exp(im\omega_r t)dtI_zS_z = I_zS_z\{\frac{[-\omega^{-2}-\omega^2]}{\omega_r}\sin 2\omega_r\alpha + \frac{[\omega^{-1}-\omega^1]}{i\omega_r}2\cos\omega_r\alpha\}$$

A π pulse along X direction consists in interval $\frac{3\tau}{2}-\alpha$ to $\frac{3\tau}{2}+\alpha$. The Hamiltonian in toggling frame is calculated to be

$$U^\dagger(\frac{3\tau}{2}+\alpha,\frac{3\tau}{2}-\alpha)H_D^{IS}(t)U((\frac{3\tau}{2}-\alpha,\frac{3\tau}{2}-\alpha)) = \sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z$$

$$\theta(t) = \frac{\pi}{2}[\frac{t-\frac{3\tau}{2}+\alpha}{\alpha}]$$

The effective Hamiltonian for the same interval given by

$$\sum_{m\neq 0,m=-2}^{2}\int_{\frac{3\tau}{2}-\alpha}^{\frac{3\tau}{2}+\alpha}\omega^{m}\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z dt$$

$$= iI_zS_z\{\frac{\omega_r(-\omega^2+\omega^{-2})}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha+\frac{\omega_r(\omega^1-\omega^{-1})}{\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos\omega_r\alpha\}$$

$$-I_yS_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha+\frac{\frac{\pi}{\alpha}(-\omega^1-\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

There is no RF pulse applied during the interval $\frac{3\tau}{2}+\alpha$ to $2\tau-\alpha$. The Hamiltonian in toggling frame is represented by

$$U^{\dagger}(2\tau-\alpha,\frac{3\tau}{2}+\alpha)H_D^{IS}(t)U(2\tau-\alpha,\frac{3\tau}{2}+\alpha)=-\sum_{m\neq 0,m=-2}^{2}\omega^{m}\exp(im\omega_r t)I_zS_z$$

$$\sum_{m\neq 0,m=-2}^{2}\int_{\frac{3\tau}{2}+\alpha}^{2\tau-\alpha}\omega^{m}\exp(im\omega_r t)dtI_zS_z=-I_zS_z\{\frac{[-\omega^{-2}-\omega^2]}{\omega_r}\sin 2\omega_r\alpha+\frac{[-\omega^{-1}+\omega^1]}{i\omega_r}2\cos\omega_r\alpha\}$$

The interval $2\tau-\alpha$ to $2\tau+\alpha$ consist a π pulse along X direction. The transformation of Hamiltonian to toggling frame results

$$U^{\dagger}(2\tau+\alpha,2\tau-\alpha)H_D^{IS}(t)U((2\tau+\alpha,2\tau-\alpha))=-\sum_{m\neq 0,m=-2}^{2}\omega^{m}\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z$$

Where $\theta(t)=\frac{\pi}{2}[\frac{t-2\tau+\alpha}{\alpha}]$

the Hamiltonian for the interval given by

$$-\sum_{m\neq 0,m=-2}^{2}\int_{2\tau-\alpha}^{2\tau+\alpha}\omega^{m}\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z dt$$

$$=-iI_zS_z\{\frac{\omega_r(-\omega^2+\omega^{-2})}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha+\frac{\omega_r(-\omega^1+\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}2\cos\omega_r\alpha\}$$

$$I_yS_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^{-2})}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha+\frac{\frac{\pi}{\alpha}(\omega^1+\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

The interval $2\tau+\alpha$ to $3\tau-\alpha$ consist no pulse. The Hamiltonian corresponding this interval is

$$U^\dagger(3\tau-\alpha,2\tau+\alpha)H_D^{IS}(t)U(3\tau-\alpha,2\tau+\alpha)=\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_rt)I_zS_z$$

$$\sum_{m\neq 0,m=-2}^{2}\int_{2\tau+\alpha}^{3\tau-\alpha}\omega^m\exp(im\omega_rt)dtI_zS_z=I_zS_z\{\frac{-[\omega^{-2}-\omega^2]}{\omega_r}\sin 2\omega_r\alpha+\frac{[-\omega^{-1}-\omega^1]}{\omega_r}2\cos\omega_r\alpha\}$$

A π pulse applied along X direction at exactly half of the cycle to S spin. The duration of the pulse is $3\tau-\alpha$ to $3\tau+\alpha$. The transformation to toggling frame is

$$U^\dagger(3\tau+\alpha,3\tau-\alpha)H_D^{IS}(t)U((3\tau+\alpha,3\tau-\alpha))=\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_rt)(S_z\cos\theta(t)-S_y\sin\theta(t))I_z$$

Where $\theta(t)=\frac{\pi}{2}[\frac{t-3\tau+\alpha}{\alpha}]$

$$\sum_{m\neq 0,m=-2}^{2}\int_{3\tau-\alpha}^{3\tau+\alpha}\omega^m\exp(im\omega_rt)(S_z\cos\theta(t)-S_y\sin\theta(t))I_zdt$$

$$=iI_zS_z\{\frac{\omega_r(\omega^2-\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha+\frac{\omega_r(\omega^1-\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos\omega_r\alpha\}$$

$$-S_yI_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha+\frac{\frac{\pi}{\alpha}(\omega^1+\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

From the interval from $3\tau+\alpha$ to $4\tau-\alpha$ no pulses applied. The Hamiltonian in toggling is represented by

$$U^{\dagger}(4\tau-\alpha,3\tau+\alpha)H_D^{IS}(t)U(4\tau-\alpha,3\tau+\alpha)=-\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_r t)I_zS_z$$

$$\sum_{m\neq 0,m=-2}^{2}\int_{3\tau+\alpha}^{4\tau-\alpha}\omega^m\exp(im\omega_r t)dtI_zS_z=-I_zS_z\{\frac{[-\omega^{-2}-\omega^{2}]}{\omega_r}\sin 2\omega_r\alpha+\frac{[-\omega^{-1}-\omega^{1}]}{\omega_r}2\cos\omega_r\alpha\}$$

The π pulse applied along X direction for the duration of $4\tau-\alpha$ to $4\tau+\alpha$ to I spin. The Hamiltonian corresponding to interval is

$$U^{\dagger}(4\tau+\alpha,4\tau-\alpha)H_D^{IS}(t)U(4\tau+\alpha,4\tau-\alpha)=-\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z$$

Where $\theta(t)=\frac{\pi}{2}[\frac{t-4\tau+\alpha}{\alpha}]$

$$\begin{aligned}&-\sum_{m\neq 0,m=-2}^{2}\int_{4\tau-\alpha}^{4\tau+\alpha}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_zdt\\&=iI_zS_z\{\frac{\omega_r(\omega^2-\omega^{-2})}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha+\frac{\omega_r(\omega^1-\omega^{-1})}{\omega_r^2-(\frac{\pi}{2\alpha})^2}2\cos\omega_r\alpha\}\\&\quad I_yS_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha+\frac{\frac{\pi}{\alpha}(\omega^1+\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}\end{aligned}$$

The interval from $4\tau+\alpha$ to $\frac{9\tau}{2}-\alpha$ consist no pulse. The Hamiltonian in toggling frame is

$$U^{\dagger}(\frac{9\tau}{2}-\alpha,4\tau+\alpha)H_D^{IS}(t)U(\frac{9\tau}{2}-\alpha,4\tau+\alpha)=\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_r t)I_zS_z$$

the effective Hamiltonian is

$$\sum_{m\neq 0,m=-2}^{2}\int_{4\tau+\alpha}^{\frac{9\tau}{2}-\alpha}\omega^{m}\exp(im\omega_{r}t)dtI_{z}S_{z}=I_{z}S_{z}\{\frac{[-\omega^{-2}-\omega^{2}]}{\omega_{r}}\sin 2\omega_{r}\alpha+\frac{[\omega^{-1}-\omega^{1}]}{i\omega_{r}}2\cos\omega_{r}\alpha\}$$

The interval $\frac{9\tau}{2}-\alpha$ to $\frac{9\tau}{2}+\alpha$ contain a *pi* pulse in X direction. The Hamiltonian in toggling frame and effective Hamiltonian given below

$$U^{\dagger}(\frac{9\tau}{2}+\alpha,\frac{9\tau}{2}-\alpha)H_{D}^{IS}(t)U(\frac{9\tau}{2}+\alpha,\frac{9\tau}{2}-\alpha)=\sum_{m\neq 0,m=-2}^{2}\omega^{m}\exp(im\omega_{r}t)(I_{z}\cos\theta(t)-I_{y}\sin\theta(t))S_{z}$$

Where $\theta(t)=\frac{\pi}{2}[\frac{t-\frac{9\tau}{2}+\alpha}{\alpha}]$

$$\sum_{m\neq 0,m=-2}^{2}\int_{\frac{9\tau}{2}-\alpha}^{\frac{9\tau}{2}+\alpha}\omega^{m}\exp(im\omega_{r}t)(I_{z}\cos\theta(t)-I_{y}\sin\theta(t))S_{z}dt$$

$$=-iI_{z}S_{z}\{\frac{\omega_{r}(\omega^{2}-\omega^{-2})}{4\omega_{r}^{2}-(\frac{\pi}{2\alpha})^{2}}4\cos 2\omega_{r}\alpha+\frac{\omega_{r}(\omega^{1}-\omega^{-1})}{\omega_{r}^{2}-(\frac{\pi}{2\alpha})^{2}}2\cos\omega_{r}\alpha\}$$

$$-I_{y}S_{z}\{\frac{\frac{\pi}{\alpha}(\omega^{2}+\omega^{-}2)}{4\omega_{r}^{2}-(\frac{\pi}{2\alpha})^{2}}\cos 2\omega_{r}\alpha+\frac{\frac{\pi}{\alpha}(-\omega^{1}-\omega^{-}1)}{\omega_{r}^{2}-(\frac{\pi}{2\alpha})^{2}}\cos\omega_{r}\alpha\}$$

The effective Hamiltonian for the duration $\frac{9\tau}{2}+\alpha$ to $5\tau-\alpha$

$$U^{\dagger}(5\tau-\alpha,\frac{9\tau}{2}+\alpha)H_{D}^{IS}(t)U(5\tau-\alpha,\frac{9\tau}{2}+\alpha)=-\sum_{m\neq 0,m=-2}^{2}\omega^{m}\exp(im\omega_{r}t)I_{z}S_{z}$$

the effective Hamiltonian is

$$\sum_{m\neq 0,m=-2}^{2}\int_{\frac{9\tau}{2}+\alpha}^{5\tau-\alpha}\omega^{m}\exp(im\omega_{r}t)dtI_{z}S_{z}=-I_{z}S_{z}\{\frac{[-\omega^{-2}-\omega^{2}]}{\omega_{r}}\sin 2\omega_{r}\alpha+\frac{[-\omega^{-1}+\omega^{1}]}{i\omega_{r}}2\cos\omega_{r}\alpha\}$$

There is a π pulse applied along X direction on I spin.The duration of the pulse is from $5\tau - \alpha$ to $5\tau + \alpha$. The effective Hamiltonian described as

$$U^{\dagger}(5\tau+\alpha, 5\tau-\alpha)H_D^{IS}(t)U(5\tau+\alpha, 5\tau-\alpha) = -\sum_{m\neq 0, m=-2}^{2} \omega^m \exp(im\omega_r t)(I_z \cos\theta(t) - I_y \sin\theta(t))S_z$$

Where $\theta(t) = \frac{\pi}{2}[\frac{t-5\tau+\alpha}{\alpha}]$

$$\begin{aligned}
&-\sum_{m\neq 0, m=-2}^{2} \int_{5\tau-\alpha}^{5\tau+\alpha} \omega^m \exp(im\omega_r t)(I_z \cos\theta(t) - I_y \sin\theta(t))S_z dt \\
&= iI_z S_z\{\frac{\omega_r(-\omega^2\omega^{-2})}{4\omega_r^2 - (\frac{\pi}{2\alpha})^2} 4\cos 2\omega_r\alpha + \frac{\omega_r(-\omega^1 + \omega^{-1})}{\omega_r^2 - (\frac{\pi}{2\alpha})^2} 2\cos\omega_r\alpha\} \\
&\quad I_y S_z\{\frac{\frac{\pi}{\alpha}(\omega^2 + \omega^-2)}{4\omega_r^2 - (\frac{\pi}{2\alpha})^2} \cos 2\omega_r\alpha + \frac{\frac{\pi}{\alpha}(\omega^1 + \omega^-1)}{\omega_r^2 - (\frac{\pi}{2\alpha})^2} \cos\omega_r\alpha\}
\end{aligned}$$

For the interval of $5\tau + \alpha$ to $\frac{11\tau}{2} - \alpha$ consist no pulse. The effective Hamiltonian can found as

$$U^{\dagger}(\frac{11\tau}{2} - \alpha, 5\tau + \alpha)H_D^{IS}(t)U(\frac{11\tau}{2} - \alpha, 5\tau + \alpha) = \sum_{m\neq 0, m=-2}^{2} \omega^m \exp(im\omega_r t)I_z S_z$$

$$\sum_{m\neq 0, m=-2}^{2} \int_{5\tau+\alpha}^{\frac{11\tau}{2}-\alpha} \omega^m \exp(im\omega_r t)dt I_z S_z = I_z S_z\{\frac{[-\omega^{-2} - \omega^2]}{\omega_r} \sin 2\omega_r\alpha + \frac{[\omega^{-1} - \omega^1]}{i\omega_r} 2\cos\omega_r\alpha\}$$

A π pulse applied along X direction on I spin for a duration of $\frac{11\tau}{2} - \alpha$ to $\frac{11\tau}{2} + \alpha$. The effective Hamiltonian is

$$U^{\dagger}(\frac{11\tau}{2}+\alpha, \frac{11\tau}{2}-\alpha)H_D^{IS}(t)U(\frac{11\tau}{2}+\alpha, \frac{11\tau}{2}-\alpha) = \sum_{m\neq 0, m=-2}^{2} \omega^m \exp(im\omega_r t)(I_z \cos\theta(t) - I_y \sin\theta(t))$$

Where $\theta(t) = \frac{\pi}{2}[\frac{t-\frac{11\tau}{2}+\alpha}{\alpha}]$

$$\sum_{m\neq 0,m=-2}^{2}\int_{\frac{11\tau}{2}-\alpha}^{\frac{11\tau}{2}+\alpha}\omega^m \exp(im\omega_r t)(I_z\cos\theta(t) - I_y\sin\theta(t))S_z dt$$

$$= -iI_zS_z\{\frac{\omega_r(-\omega^2+\omega^{-2})}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha + \frac{\omega_r(\omega^1-\omega^{-1})}{\omega_r^2-(\frac{\pi}{2\alpha})^2}2\cos\omega_r\alpha\}$$

$$-I_yS_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^{-2})}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha + \frac{\frac{\pi}{\alpha}(\omega^1+\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

The effective Hamiltonian for the interval $\frac{11\tau}{2}+\alpha$ to $6\tau-\alpha$ is represented as

$$U^\dagger(6\tau-\alpha, \frac{11\tau}{2}+\alpha)H_D^{IS}(t)U(6\tau-\alpha, \frac{11\tau}{2}+\alpha) = -\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_r t)I_zS_z$$

$$\sum_{m\neq 0,m=-2}^{2}\int_{\frac{11\tau}{2}+\alpha}^{6\tau-\alpha}\omega^m\exp(im\omega_r t)dtI_zS_z = -I_zS_z\{\frac{[-\omega^{-2}-\omega^2]}{\omega_r}\sin 2\omega_r\alpha + \frac{[-\omega^{-1}+\omega^1]}{i\omega_r}2\cos\omega_r\alpha\}$$

The effective Hamiltonian for REDOR XX-4 pulse sequence is derived by subtracting the effective Hamiltonian obtained from dephasing experiment from the reference experiment. The pulse sequence for the reference experiment consist a π pulse along X direction at exactly middle of the sequence on S spin and no pulses applied on I spin. A $\frac{\pi}{2}$ Y pulse applied at the starting of the pulse sequence on S spin and Signal collected at the end of the cycle time from S spin.

The detailed description REDOR - XX4 pulse sequence is given below

The effective Hamiltonian for the reference experiment is derived using Average Hamiltonian Theory (AHT).The important steps involved in the calculation are summarized below

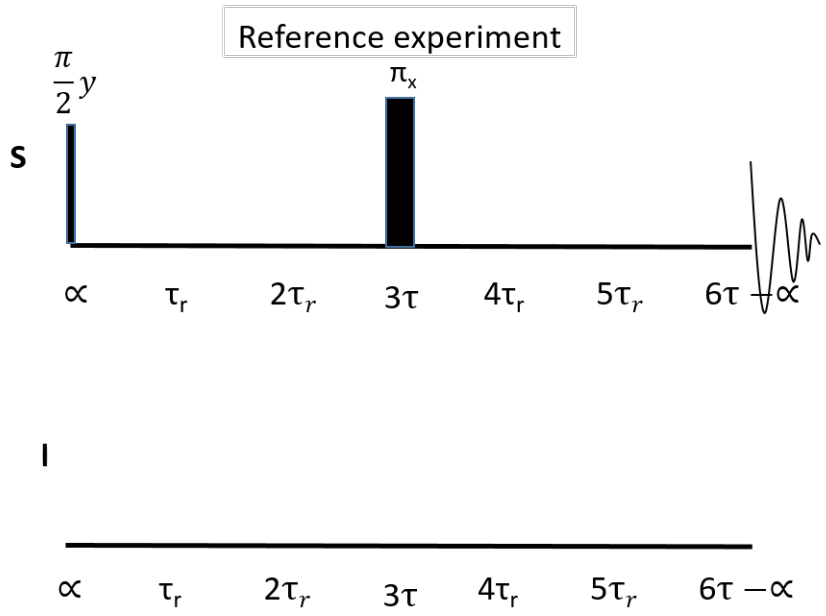

FIGURE 3.4: **Schematic depiction ofreference experiment of REDOR XX-4** *The pulse sequence for the reference experiment of REDOR XX-4 as follows.The sequence starting with a $\frac{\pi}{2}y$ pulse on S spin. A πx pulse apply exactly middle of the pulse cycle. The cycle time of the sequence is 6τ and the duration of the π pulse is t_w where the $t_w = 2\alpha$. The spin echo signal is collected at S spin at $6\tau - \alpha$.*

$$U^\dagger(3\tau+\alpha, 3\tau-\alpha)H_D^{IS}(t)U((3\tau+\alpha, 3\tau-\alpha)) = \sum_{m\neq 0, m=-2}^{2} \omega^m \exp(im\omega_r t)(S_z\cos\theta(t) - S_y \sin\theta(t))I_z$$

Where $\theta(t) = \frac{\pi}{2}[\frac{t-3\tau+\alpha}{\alpha}]$

$$\sum_{m\neq 0, m=-2}^{2} \int_{3\tau-\alpha}^{3\tau+\alpha} \omega^m \exp(im\omega_r t)(S_z\cos\theta(t) - S_y\sin\theta(t))I_z dt$$

$$= iI_zS_z\{\frac{\omega_r(\omega^2-\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha + \frac{\omega_r(\omega^1-\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos\omega_r\alpha\}$$

$$-S_yI_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha + \frac{\frac{\pi}{\alpha}(\omega^1+\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

Employing the Magnus formula, average Hamiltonian is calculated as the average of interaction frame Hamiltonian over the cycle time t_c of the pulse sequence.

$$\overline{\widehat{H}}^{1}_{IS} = \frac{1}{t_c}\int_{t_0}^{t_o+t_c} \widehat{H}_{IS}(t)dt$$

Calculation of the first-order Average Hamiltonian for REDOR XX-4 with finite pulse results,

$$\overline{\widehat{H}}^{1}_{IS} = -\frac{\cos\frac{\pi}{2}\phi}{3\pi(1-\phi^2)} i(\omega^{-1}-\omega^{1})I_zS_z - 8\phi(\omega^{-1}-\omega^{1})I_yS_z$$

Where $\phi = \frac{2t_w}{\tau}$ is the fraction of rotor period occupied by RF pulses defined range of $o \leq \phi \leq 1$,t_w is the time duration of π pulse and τ is the rotor period.

$\overline{\widehat{H}}^{1}_{IS}$ can be rewrite as

$$\overline{\widehat{H}}^{1}_{IS} = -CI_zS_z - DI_yS_z$$

, Where $C = \frac{\cos\frac{\pi}{2}\phi}{3\pi(1-\phi^2)} i(\omega^{-1}-\omega^{1})$ and $D = 8\phi(\omega^{-1}-\omega^{1})$

The finite pulse inclusion in effective Hamiltonian calculation leads to a conclusion that the finite pulse plays some role in the experiment. Since t_w is considerably large such that $\phi = \frac{2t_w}{\tau} \neq 0$, the terms correspond to I_yS_z comes to picture. The next task is find the effect of finite pulse in the signal.

In REDOR XX-4 the signal is collected from S spin. The signal $\langle S_x \rangle$ defined as Trace $\langle S_x\rho(t)\rangle$.

$$S(\tau) = Trace\langle S_x\rho(t)\rangle$$

$$\rho(t) = e^{\frac{-i\overline{\widehat{H}}^{1}_{IS}t}{\hbar}} \rho(0) e^{\frac{i\overline{\widehat{H}}^{1}_{IS}t}{\hbar}}$$

The $\rho(0)$ is initial density matrix which is S_x in REDOR XX-4. The $\rho(t)$ found from BCH expansion.

$$\rho(t) = S_x(\cos\frac{\sqrt{C^2+D^2}}{2}\tau) + \frac{2}{\sqrt{C^2+D^2}}(\cos\frac{\sqrt{C^2+D^2}}{2}\tau)(CI_zS_z + DI_yS_z)$$

The signal $S(t) = Trace\langle S_x\rho(t)\rangle$

$$S(\tau) = \cos(\frac{\sqrt{C^2+D^2}}{2})\tau$$

The effect of finite pulse is clearly visible from the expression of signal. The frequency of the signal is $\frac{\sqrt{C^2+D^2}}{2}$. The C and D are depend on nature of the pulse by the variable ϕ. If the pulse is delta that is $\phi = 0$, the term D will be zero. As ϕ increases contribution from D also increases. The similar dependence also appears in C. The ϕ dependence in C is coming from $\cos\frac{\pi}{2}\phi$ term. The analytic expression of the signal compared with numerical simulations using Simpson. The of signal REDOR XX-4 pulse sequence from simpson is given below,

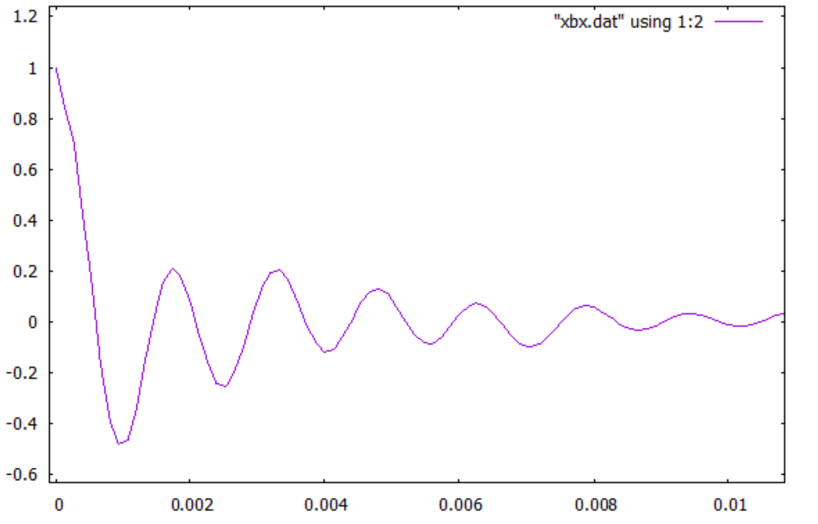

FIGURE 3.5: The numerically simulated signal for REDOR XX-4 pulse sequence is obtained from Simpson. The pulses in this sequence approximated as Delta pulse. The signal behaves as expected from the calculations.The signal starting from maximum amplitude and decays to zero.

The finite pulse effect in REDOR XX-4 pulse sequence is visualized by comparing signals from pulse sequences by varying magnitude of ϕ. As changing ϕ resulted The numerical simulation for different ϕ values implies that the signal depend on the ratio of pulse length to cycle time. The numerical simulations are given below,

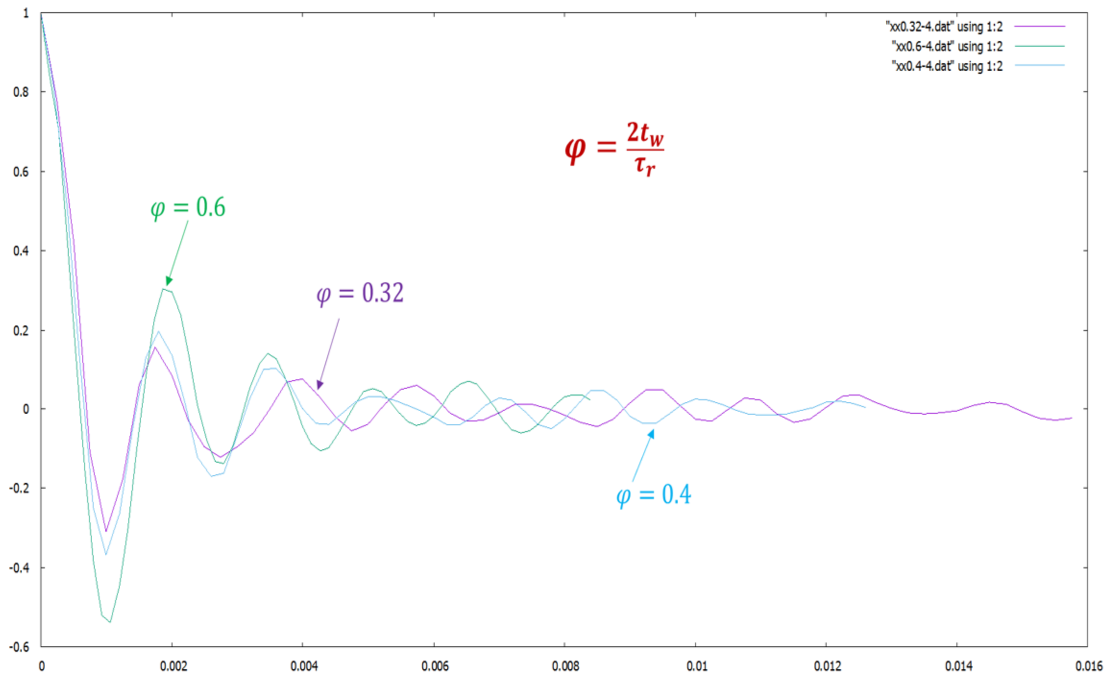

FIGURE 3.6: The numerically simulated signal of REDOR XX-4 pulse sequence for different ϕ values.The ϕ is $\frac{2t_w}{t_r}$, where t_w is duration of the pulse and τ_r is the cycle time. The purple, green and blue graphs correspond to $\phi = 0.32$, 0.6 and 0.4 respectively

The numerically simulated signal for REDOR XX-4 pulse sequence for different ϕ.The ϕ is the ratio $\frac{2t_w}{t_r}$, where t_w is duration of the pulse and τ_r is the cycle time. The purple, green and blue correspond to $\phi = 0.32$, 0.6 and 0.4 respectively. The numerical simulation for different ϕ values clearly indicate that the signal is varying with ϕ. When ϕ increases the frequency of evolution also increase, the frequency of evolution is higher in case of ϕ= 0.6(green line) where frequency is lower for $\phi = 0.32$ (blue). The signal decays fast in case of higher ϕ values.

The analysis of effects of finite pulse in REDOR XX-4 sequences conclude that the ϕ $(\frac{2t_w}{\tau_r})$ depend on signal. The analytic expression and numerical simulations hold the same result. The calculation extending to REDOR $X\overline{X}-4$ to conclude the effect finite pulse.

3.3 REDOR $X\overline{X}-4$

The effect of finite pulse on REDOR $X\overline{X}-4$ calculated using Average Hamiltonian theory. Compared to REDOR XX-4 pulse sequence REDOR $X\overline{X}-4$ is much more robust. The π pulses applied alternatively from X and $\overline{X}$ to S spin which reduces experimental errors. The motivation of this calculation is to compare the result with REDOR XX-4 and generalize the effect of finite pulse on REDOR experiments.

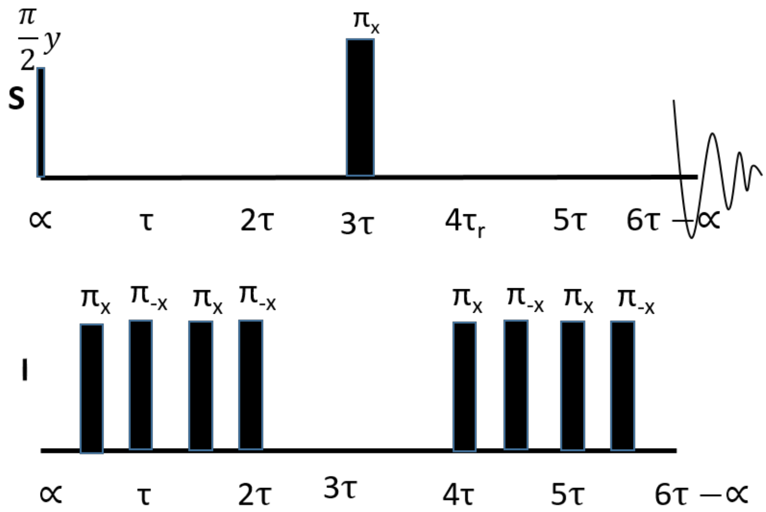

FIGURE 3.7: **Reference experiment of REDOR** $X\overline{X}-4$ *The pulse sequence for the dephasing experiment of REDOR* $X\overline{X}-4$ *as follows.The sequence starting with a* $\frac{\pi}{2}y$ *pulse on S spin. A* πx *pulses applied exactly middle of the pulse cycle.The* π *applied to I spin from X and -X directions alternatively. The cycle time of the sequence is* 6τ *and the duration of the* π *pulse is* t_w *where the* $t_w = 2\alpha$*.The* πx *pulses applied on I spin which creates spin echo. The spin echo signal is collected at S spin at* $6\tau - \alpha$*.*

The dipolar Hamiltonian under Magic angle spinning conditions can be represented as

$$H_D^{IS}(t) = \sum_{m\neq 0, m=-2}^{2} \omega^m \exp(im\omega_r t) I_z S_z$$

Average Hamiltonian Theory provide suitable framework for effective Hamiltonian. The effective time independent Hamiltonian $\tilde{H}_{ave}$ can be expanded in a series of terms of increasing order of time by Magnus expansion

$$\tilde{H}_{ave} = \tilde{H}^0 + \tilde{H}^1 + \tilde{H}^2 + \tilde{H}^3 ...$$

$$e^{-i\frac{\tilde{H}_{ave}t}{\hbar}} = e^{-it\frac{\tilde{H}^0 + \tilde{H}^1 + \tilde{H}^2 + \tilde{H}^3 ...}{\hbar}}$$

The Hamiltonians for small intervals were calculated and added to get the effective Hamiltonian. The Hamiltonian become time dependent because of magic angle spinning and RF pulse irradiation.The Hamiltonian $H_D^{IS}(t)$ in toggling frame or interaction frame is given by the following transformation

$$\widehat{H}_D^{IS}(t) = U^\dagger(t) H_D^{IS} U(t)$$

The $U(t)$ is the propagator corresponding to the interval t.

There is no RF pulse applied during interval from α to $\frac{\tau}{2} - \alpha$. The propagator corresponding to the interval is $U(\frac{\tau}{2} - \alpha, \alpha)$ is 1. The Hamiltonian in toggling frame given by the frame transformation

$$U^\dagger(\frac{\tau}{2} - \alpha, \alpha) H_D(t)^{IS} U(\frac{\tau}{2} - \alpha, \alpha) = \sum_{m\neq 0, m=-2}^{2} \omega^m \exp(im\omega_r t) I_z S_z$$

The Hamiltonian in toggling frame is time dependent. The effective Hamiltonian is given by the integration over the interval α to $\frac{\tau}{2} - \alpha$.

$$\sum_{m\neq 0, m=-2}^{2} \int_{\alpha}^{\frac{\tau}{2}-\alpha} \omega^m \exp(im\omega_r t) dt I_z S_z = I_z S_z \{ \frac{[-\omega^{-2} - \omega^2]}{\omega_r} \sin 2\omega_r \alpha + \frac{[\omega^{-1} - \omega^1]}{i\omega_r} 2 \cos \omega_r \alpha \}$$

A π pulse applied along X axis on I spin for finite duration from $\frac{\tau}{2}-\alpha$ to $\frac{\tau}{2}+\alpha$. The Hamiltonian in toggling frame can be written as

$$U^{\dagger}(\frac{\tau}{2}+\alpha,\frac{\tau}{2}-\alpha)H_D^{IS}U(\frac{\tau}{2}+\alpha,\frac{\tau}{2}-\alpha)=\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z$$

Where $\theta(t)=\frac{\pi}{2}[\frac{t-\frac{\tau}{2}+\alpha}{\alpha}]$
the Hamiltonian for the the same duration given by

$$\sum_{m\neq 0,m=-2}^{2}\int_{\frac{\tau}{2}-\alpha}^{\frac{\tau}{2}+\alpha}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z dt$$

$$=-iI_zS_z\{\frac{\omega_r(\omega^{-2}-\omega^2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha+\frac{\omega_r(\omega^1-\omega^{-1})}{\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos\omega_r\alpha\}$$

$$-I_yS_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}2\cos 2\omega_r\alpha+\frac{\frac{\pi}{\alpha}(-\omega^1-\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

For a time interval $\frac{\tau}{2}+\alpha$ to $\tau-\alpha$ the propagator $U(\frac{\tau}{2}+\alpha,\tau-\alpha)$ is one. The Hamiltonian under transformation can be represent as

$$U^{\dagger}(\frac{\tau}{2}+\alpha,\tau-\alpha)H_D^{IS}(t)U(\frac{\tau}{2}+\alpha,\tau-\alpha)=-\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_r t)I_zS_z$$

$$\sum_{m\neq 0,m=-2}^{2}\int_{\frac{\tau}{2}+\alpha}^{\tau-\alpha}\omega^m\exp(im\omega_r t)dtI_zS_z=-I_zS_z\{\frac{[-\omega^{-2}-\omega^2]}{\omega_r}\sin 2\omega_r\alpha+\frac{[-\omega^{-1}+\omega^1]}{i\omega_r}2\cos\omega_r\alpha\}$$

For duration $\tau-\alpha$ to $\tau+\alpha$ a π pulse applied along -X axis. The Hamiltonian corresponding to above duration is represented as

$$U^{\dagger}(\tau+\alpha,\tau-\alpha)H_D^{IS}(t)U((\tau+\alpha,\tau-\alpha)) = -\sum_{m\neq 0,m=-2}^{2}\omega^m \exp(im\omega_r t)(I_z\cos\theta(t)+I_y\sin\theta(t))S_z$$

Where $\theta(t) = \frac{\pi}{2}[\frac{t-\tau+\alpha}{\alpha}]$

$$-\sum_{m\neq 0,m=-2}^{2}\int_{\tau-\alpha}^{\tau+\alpha}\omega^m \exp(im\omega_r t)(I_z\cos\theta(t) - I_y\sin\theta(t))S_z dt$$

$$= iI_zS_z\{\frac{\omega_r(\omega^{-2}-\omega^2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha + \frac{\omega_r(-\omega^1+\omega^{-1})}{\omega_r^2-(\frac{\pi}{2\alpha})^2}2\cos\omega_r\alpha\}$$

$$-I_yS_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha + \frac{\frac{\pi}{\alpha}(\omega^1+\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

For the duration of $\tau+\alpha$ to $\frac{3\tau}{2}-\alpha$ no pulses applied. The transformation to toggling frame results

$$U^{\dagger}(\frac{3\tau}{2}-\alpha,\tau+\alpha)H_D^{IS}(t)U(\frac{3\tau}{2}-\alpha,\tau+\alpha) = \sum_{m\neq 0,m=-2}^{2}\omega^m \exp(im\omega_r t)I_zS_z$$

the Hamiltonian corresponding to the interval is

$$\sum_{m\neq 0,m=-2}^{2}\int_{\tau+\alpha}^{\frac{3\tau}{2}-\alpha}\omega^m \exp(im\omega_r t)dtI_zS_z = I_zS_z\{\frac{[-\omega^{-2}-\omega^2]}{\omega_r}\sin 2\omega_r\alpha + \frac{[\omega^{-1}-\omega^1]}{i\omega_r}2\cos\omega_r\alpha\}$$

A π pulse along X direction consists in interval $\frac{3\tau}{2}-\alpha$ to $\frac{3\tau}{2}+\alpha$. The Hamiltonian in interaction frame is calculated to be

$$U^{\dagger}(\frac{3\tau}{2}+\alpha,\frac{3\tau}{2}-\alpha)H_D^{IS}(t)U((\frac{3\tau}{2}-\alpha,\frac{3\tau}{2}-\alpha)) = \sum_{m\neq 0,m=-2}^{2}\omega^m \exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z$$

$$\theta(t) = \frac{\pi}{2}[\frac{t - \frac{3\tau}{2} + \alpha}{\alpha}]$$

The effective Hamiltonian for the same interval represented as

$$\sum_{m\neq 0, m=-2}^{2} \int_{\frac{3\tau}{2}-\alpha}^{\frac{3\tau}{2}+\alpha} \omega^m \exp(im\omega_r t)(I_z \cos\theta(t) - I_y \sin\theta(t))S_z dt$$

$$= iI_zS_z\{\frac{\omega_r(-\omega^2+\omega^{-2})}{4\omega_r^2 - (\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha + \frac{\omega_r(\omega^1-\omega^{-1})}{\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos\omega_r\alpha\}$$

$$-I_yS_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha + \frac{\frac{\pi}{\alpha}(-\omega^1-\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

There is no RF pulse applied during the interval $\frac{3\tau}{2}+\alpha$ to $2\tau-\alpha$. The Hamiltonian in toggling frame can be represent as

$$U^\dagger(2\tau-\alpha, \frac{3\tau}{2}+\alpha)H_D^{IS}(t)U(2\tau-\alpha, \frac{3\tau}{2}+\alpha) = -\sum_{m\neq 0, m=-2}^{2} \omega^m \exp(im\omega_r t)I_zS_z$$

$$\sum_{m\neq 0, m=-2}^{2} \int_{\frac{3\tau}{2}+\alpha}^{2\tau-\alpha} \omega^m \exp(im\omega_r t)dtI_zS_z = -I_zS_z\{\frac{[-\omega^{-2}-\omega^2]}{\omega_r}\sin 2\omega_r\alpha + \frac{[-\omega^{-1}+\omega^1]}{i\omega_r}2\cos\omega_r\alpha\}$$

The interval $2\tau-\alpha$ to $2\tau+\alpha$ consist a π pulse along -X direction on I spin. The transformation of Hamiltonian to toggling frame results

$$U^\dagger(2\tau+\alpha, 2\tau-\alpha)H_D^{IS}(t)U((2\tau+\alpha, 2\tau-\alpha)) = -\sum_{m\neq 0, m=-2}^{2} \omega^m \exp(im\omega_r t)(I_z\cos\theta(t) - I_y\sin\theta(t))S_z$$

Where $\theta(t) = \frac{\pi}{2}[\frac{t-2\tau+\alpha}{\alpha}]$

the Hamiltonian for the interval represented as

$$\sum_{m\neq 0,m=-2}^{2}\int_{2\tau-\alpha}^{2\tau+\alpha}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z dt$$

$$=-iI_zS_z\{\frac{\omega_r(-\omega^2+\omega^{-2})}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha+\frac{\omega_r(-\omega^1+\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}2\cos\omega_r\alpha\}$$

$$-I_yS_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^{-2})}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha+\frac{\frac{\pi}{\alpha}(\omega^1+\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

The interval $2\tau+\alpha$ to $3\tau-\alpha$ consist no pulse. The Hamiltonian corresponding this interval is

$$U^\dagger(3\tau-\alpha,2\tau+\alpha)H_D^{IS}(t)U(3\tau-\alpha,2\tau+\alpha)=\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_r t)I_zS_z$$

$$\sum_{m\neq 0,m=-2}^{2}\int_{2\tau+\alpha}^{3\tau-\alpha}\omega^m\exp(im\omega_r t)dtI_zS_z=I_zS_z\{\frac{-[\omega^{-2}-\omega^2]}{\omega_r}\sin 2\omega_r\alpha+\frac{[-\omega^{-1}-\omega^1]}{\omega_r}2\cos\omega_r\alpha\}$$

A π pulse applied along X direction at exactly half of the cycle to S spin. The duration of the pulse is $3\tau-\alpha$ to $3\tau+\alpha$. The transformation to toggling frame is

$$U^\dagger(3\tau+\alpha,3\tau-\alpha)H_D^{IS}(t)U((3\tau+\alpha,3\tau-\alpha))=\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_r t)(S_z\cos\theta(t)-S_y\sin\theta(t))I_z$$

Where $\theta(t)=\frac{\pi}{2}[\frac{t-3\tau+\alpha}{\alpha}]$

$$\sum_{m\neq 0,m=-2}^{2}\int_{3\tau-\alpha}^{3\tau+\alpha}\omega^m\exp(im\omega_r t)(S_z\cos\theta(t)-S_y\sin\theta(t))I_z dt$$

$$=iI_zS_z\{\frac{\omega_r(\omega^2-\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha+\frac{\omega_r(\omega^1-\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos\omega_r\alpha\}$$

$$-S_yI_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha+\frac{\frac{\pi}{\alpha}(\omega^1+\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

The interval from $3\tau+\alpha$ to $4\tau-\alpha$ consist no pulses. The Hamiltonian in toggling is

$$U^{\dagger}(4\tau-\alpha,3\tau+\alpha)H_D^{IS}(t)U(4\tau-\alpha,3\tau+\alpha)=-\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_rt)I_zS_z$$

The Hamiltonian corresponding to this interval

$$\sum_{m\neq 0,m=-2}^{2}\int_{3\tau+\alpha}^{4\tau-\alpha}\omega^m\exp(im\omega_rt)dtI_zS_z=-I_zS_z\{\frac{[-\omega^{-2}-\omega^2]}{\omega_r}\sin 2\omega_r\alpha+\frac{[-\omega^{-1}-\omega^1]}{\omega_r}2\cos\omega_r\alpha\}$$

The π pulse applied along X direction for the duration of $4\tau-\alpha$ to $4\tau+\alpha$ to I spin. The Hamiltonian corresponding to interval is

$$U^{\dagger}(4\tau+\alpha,4\tau-\alpha)H_D^{IS}(t)U(4\tau+\alpha,4\tau-\alpha)=-\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_rt)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z$$

Where $\theta(t)=\frac{\pi}{2}[\frac{t-4\tau+\alpha}{\alpha}]$

$$-\sum_{m\neq 0,m=-2}^{2}\int_{4\tau-\alpha}^{4\tau+\alpha}\omega^m\exp(im\omega_rt)(I_z\cos\theta(t)-I_y\sin\theta(t))S_zdt$$

$$=iI_zS_z\{\frac{\omega_r(\omega^2-\omega^{-2})}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha+\frac{\omega_r(\omega^1-\omega^{-1})}{\omega_r^2-(\frac{\pi}{2\alpha})^2}2\cos\omega_r\alpha\}$$

$$I_yS_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha+\frac{\frac{\pi}{\alpha}(\omega^1+\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

The interval from $4\tau+\alpha$ to $\frac{9\tau}{2}-\alpha$ consist no pulse. The Hamiltonian in toggling frame is

$$U^{\dagger}(\frac{9\tau}{2}-\alpha,4\tau+\alpha)H_D^{IS}(t)U(\frac{9\tau}{2}-\alpha,4\tau+\alpha)=\sum_{m\neq0,m=-2}^{2}\omega^m\exp(im\omega_r t)I_zS_z$$

the effective Hamiltonian is

$$\sum_{m\neq0,m=-2}^{2}\int_{4\tau+\alpha}^{\frac{9\tau}{2}-\alpha}\omega^m\exp(im\omega_r t)dtI_zS_z=I_zS_z\{\frac{[-\omega^{-2}-\omega^2]}{\omega_r}\sin 2\omega_r\alpha+\frac{[\omega^{-1}-\omega^1]}{i\omega_r}2\cos\omega_r\alpha\}$$

The interval $\frac{9\tau}{2}-\alpha$ to $\frac{9\tau}{2}+\alpha$ contain a *pi* pulse in -X direction on I spin. The Hamiltonian in toggling frame and effective Hamiltonian represented as

$$U^{\dagger}(\frac{9\tau}{2}+\alpha,\frac{9\tau}{2}-\alpha)H_D^{IS}(t)U(\frac{9\tau}{2}+\alpha,\frac{9\tau}{2}-\alpha)=\sum_{m\neq0,m=-2}^{2}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)+I_y\sin\theta(t))S_z$$

Where $\theta(t)=\frac{\pi}{2}[\frac{t-\frac{9\tau}{2}+\alpha}{\alpha}]$

$$\sum_{m\neq0,m=-2}^{2}\int_{\frac{9\tau}{2}-\alpha}^{\frac{9\tau}{2}+\alpha}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_zdt$$
$$=-iI_zS_z\{\frac{\omega_r(\omega^2-\omega^{-2})}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha+\frac{\omega_r(\omega^1-\omega^{-1})}{\omega_r^2-(\frac{\pi}{2\alpha})^2}2\cos\omega_r\alpha\}$$
$$+I_yS_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha+\frac{\frac{\pi}{\alpha}(-\omega^1-\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

The effective Hamiltonian for the duration $\frac{9\tau}{2}+\alpha$ to $5\tau-\alpha$

$$U^{\dagger}(5\tau-\alpha,\frac{9\tau}{2}+\alpha)H_D^{IS}(t)U(5\tau-\alpha,\frac{9\tau}{2}+\alpha)=-\sum_{m\neq0,m=-2}^{2}\omega^m\exp(im\omega_r t)I_zS_z$$

the effective Hamiltonian is

$$\sum_{m\neq 0,m=-2}^{2}\int_{\frac{9\tau}{2}+\alpha}^{5\tau-\alpha}\omega^m\exp(im\omega_r t)dtI_zS_z=-I_zS_z\{\frac{[-\omega^{-2}-\omega^2]}{\omega_r}\sin 2\omega_r\alpha+\frac{[-\omega^{-1}+\omega^1]}{i\omega_r}2\cos\omega_r\alpha\}$$

There is a π pulse applied along X direction on I spin.The duration of the pulse is from $5\tau-\alpha$ to $5\tau+\alpha$. The effective Hamiltonian described as

$$U^\dagger(5\tau+\alpha,5\tau-\alpha)H_D^{IS}(t)U(5\tau+\alpha,5\tau-\alpha)=-\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z$$

Where $\theta(t)=\frac{\pi}{2}[\frac{t-5\tau+\alpha}{\alpha}]$

$$-\sum_{m\neq 0,m=-2}^{2}\int_{5\tau-\alpha}^{5\tau+\alpha}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_zdt$$
$$=iI_zS_z\{\frac{\omega_r(-\omega^2\omega^{-2})}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha+\frac{\omega_r(-\omega^1+\omega^{-1})}{\omega_r^2-(\frac{\pi}{2\alpha})^2}2\cos\omega_r\alpha\}$$
$$I_yS_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^-2)}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha+\frac{\frac{\pi}{\alpha}(\omega^1+\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

For the interval of $5\tau+\alpha$ to $\frac{11\tau}{2}-\alpha$ consist no pulse. The effective Hamiltonian can found as

$$U^\dagger(\frac{11\tau}{2}-\alpha,5\tau+\alpha)H_D^{IS}(t)U(\frac{11\tau}{2}-\alpha,5\tau+\alpha)=\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_r t)I_zS_z$$

$$\sum_{m\neq 0,m=-2}^{2}\int_{5\tau+\alpha}^{\frac{11\tau}{2}-\alpha}\omega^m\exp(im\omega_r t)dtI_zS_z=I_zS_z\{\frac{[-\omega^{-2}-\omega^2]}{\omega_r}\sin 2\omega_r\alpha+\frac{[\omega^{-1}-\omega^1]}{i\omega_r}2\cos\omega_r\alpha\}$$

A π pulse applied along -X direction on I spin for a duration of $\frac{11\tau}{2}-\alpha$ to $\frac{11\tau}{2}+\alpha$. The effective Hamiltonian is

$$U^{\dagger}(\frac{11\tau}{2}+\alpha,\frac{11\tau}{2}-\alpha)H_D^{IS}(t)U(\frac{11\tau}{2}+\alpha,\frac{11\tau}{2}-\alpha)=\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)+I_y\sin\theta(t))$$

Where $\theta(t)=\frac{\pi}{2}[\frac{t-\frac{11\tau}{2}+\alpha}{\alpha}]$

$$\sum_{m\neq 0,m=-2}^{2}\int_{\frac{11\tau}{2}-\alpha}^{\frac{11\tau}{2}+\alpha}\omega^m\exp(im\omega_r t)(I_z\cos\theta(t)-I_y\sin\theta(t))S_z dt$$

$$=-iI_zS_z\{\frac{\omega_r(-\omega^2+\omega^{-2})}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}4\cos 2\omega_r\alpha+\frac{\omega_r(\omega^1-\omega^{-1})}{\omega_r^2-(\frac{\pi}{2\alpha})^2}2\cos\omega_r\alpha\}$$

$$I_yS_z\{\frac{\frac{\pi}{\alpha}(\omega^2+\omega^{-2})}{4\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos 2\omega_r\alpha+\frac{\frac{\pi}{\alpha}(\omega^1+\omega^-1)}{\omega_r^2-(\frac{\pi}{2\alpha})^2}\cos\omega_r\alpha\}$$

The effective Hamiltonian for the interval $\frac{11\tau}{2}+\alpha$ to $6\tau-\alpha$ is given as

$$U^{\dagger}(6\tau-\alpha,\frac{11\tau}{2}+\alpha)H_D^{IS}(t)U(6\tau-\alpha,\frac{11\tau}{2}+\alpha)=-\sum_{m\neq 0,m=-2}^{2}\omega^m\exp(im\omega_r t)I_zS_z$$

$$\sum_{m\neq 0,m=-2}^{2}\int_{\frac{11\tau}{2}+\alpha}^{6\tau-\alpha}\omega^m\exp(im\omega_r t)dtI_zS_z=-I_zS_z\{\frac{[-\omega^{-2}-\omega^2]}{\omega_r}\sin 2\omega_r\alpha+\frac{[-\omega^{-1}+\omega^1]}{i\omega_r}2\cos\omega_r\alpha\}$$

The effective Hamiltonian for REDOR $X\overline{X}-4$ pulse sequence is found by subtracting the effective Hamiltonian of dephasing experiment from the reference experiment as did in REDOR XX-4. The pulse sequence for the reference experiment consist a π pulse along X direction at exactly middle of the sequence on S spin and no pulses applied on I spin. A $\frac{\pi}{2}$ Y pulse applied at the starting of the pulse sequence on S spin and Signal collected at the end of the cycle time from S spin.

The detailed description REDOR - XX4 pulse sequence is given below

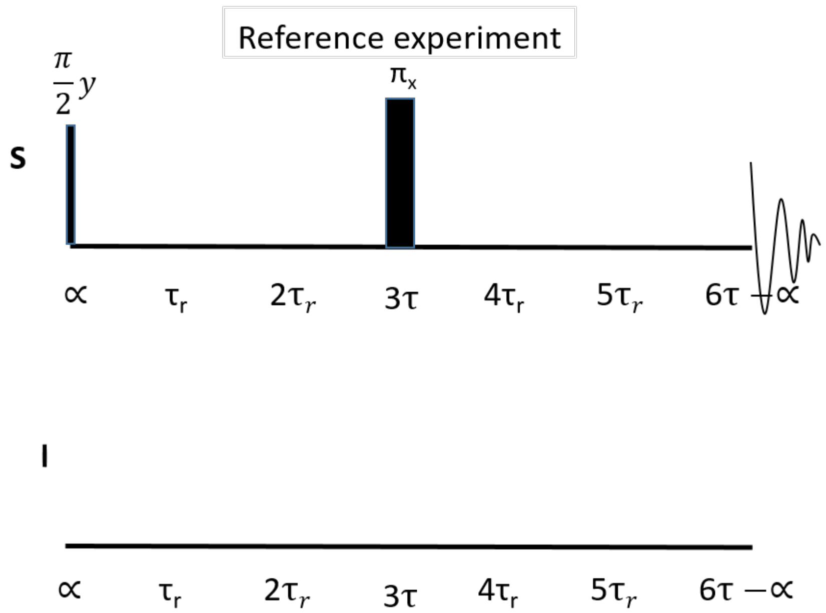

FIGURE 3.8: **Reference experiment of REDOR** $X\overline{X}-4$ *The pulse sequence for the reference experiment of REDOR* $X\overline{X}-4$*-4 as follows.The sequence starting with a* $\frac{\pi}{2}y$ *pulse on S spin. A* πx *pulse apply exactly middle of the pulse cycle. The cycle time of the sequence is* 6τ *and the duration of the* π *pulse is* t_w *where the* $t_w = 2\alpha$*. The spin echo signal is collected at S spin at* $6\tau - \alpha$*.*

The effective Hamiltonian for the reference pulse sequence found using Average Hamiltonian Theory (AHT). The magic angle spinning will average out all second order interactions. Application of finite pulse contribute on effective Hamiltonian of reference experiment.

$$U^{\dagger}(3\tau+\alpha, 3\tau-\alpha)H_D^{IS}(t)U((3\tau+\alpha, 3\tau-\alpha)) = \sum_{m\neq 0, m=-2}^{2} \omega^m \exp(im\omega_r t)(S_z \cos\theta(t) - S_y \sin\theta(t))I_z$$

Where $\theta(t) = \frac{\pi}{2}[\frac{t-3\tau+\alpha}{\alpha}]$

$$\sum_{m\neq 0,m=-2}^{2}\int_{3\tau-\alpha}^{3\tau+\alpha}\omega^{m}\exp(im\omega_{r}t)(S_{z}\cos\theta(t)-S_{y}\sin\theta(t))I_{z}dt$$

$$= iI_{z}S_{z}\{\frac{\omega_{r}(\omega^{2}-\omega^{-}2)}{4\omega_{r}^{2}-(\frac{\pi}{2\alpha})^{2}}4\cos 2\omega_{r}\alpha+\frac{\omega_{r}(\omega^{1}-\omega^{-}1)}{\omega_{r}^{2}-(\frac{\pi}{2\alpha})^{2}}4\cos\omega_{r}\alpha\}$$

$$-S_{y}I_{z}\{\frac{\frac{\pi}{\alpha}(\omega^{2}+\omega^{-}2)}{4\omega_{r}^{2}-(\frac{\pi}{2\alpha})^{2}}\cos 2\omega_{r}\alpha+\frac{\frac{\pi}{\alpha}(\omega^{1}+\omega^{-}1)}{\omega_{r}^{2}-(\frac{\pi}{2\alpha})^{2}}\cos\omega_{r}\alpha\}$$

The first-order average Hamiltonian is calculated as the average of interaction frame Hamiltonian over the cycle time t_c of the pulse sequence.

$$\overline{\widehat{H}}_{IS}^{1}=\frac{1}{t_c}\int_{t_0}^{t_o+t_c}\widehat{H}_{IS}(t)dt$$

Calculation of the first-order Average Hamiltonian for REDOR $X\overline{X}-4$ with finite pulse yields,

$$\overline{\widehat{H}}_{IS}^{1}=\frac{4}{3\pi}\cos(\frac{\pi}{2}\phi)\frac{i(\omega^{1}-\omega^{-1})}{2(1-\phi^{2})}I_{z}S_{z}+\frac{(\omega^{2}-\omega^{-2})\phi}{1-4\phi^{2}}\cos(\pi\phi)I_{y}S_{z}$$

Where $\phi=\frac{2t_w}{\tau}$ is the fraction of rotor period occupied by RF pulses defined range of $o \leq \phi \leq 1$,t_w is the time duration of π pulse and τ is the rotor period.

$\overline{\widehat{H}}_{IS}^{1}$ can be rewrite as

$$\overline{\widehat{H}}_{IS}^{1}=-CI_{z}S_{z}-DI_{y}S_{z}$$

, Where $C=\frac{4}{3\pi}\cos(\frac{\pi}{2}\phi)\frac{i(\omega^{1}-\omega^{-1})}{2(1-\phi^{2})}$ and $D=\frac{(\omega^{2}-\omega^{-2})\phi}{1-4\phi^{2}}\cos(\pi\phi)$

The finite pulse inclusion in effective Hamiltonian calculation leads to a conclusion that the finite pulse plays important role in the experiment. Since t_w is considerably large such that $\phi=\frac{2t_w}{\tau}\neq 0$, the terms correspond to I_yS_z comes to picture. The next task is find the effect of finite pulse in signal.

In REDOR $X\overline{X}-4$ the signal is collected from S spin. The signal $\langle S_x\rangle$ defined as Trace $\langle S_x\rho(t)\rangle$.

$$S(\tau) = Trace\langle S_x \rho(t) \rangle$$

$$\rho(t) = e^{\dfrac{-i\overline{\widehat{H}}^{1}_{IS} t}{\hbar}} \rho(0) e^{\dfrac{i\overline{\widehat{H}}^{1}_{IS} t}{\hbar}}$$

The $\rho(0)$ is initial density matrix which is S_x in REDOR $X\overline{X}-4$. The $\rho(t)$ found from BCH expansion.

$$\rho(t) = S_x(\cos\frac{\sqrt{C^2+D^2}}{2}\tau) + \frac{2}{\sqrt{C^2+D^2}}(\cos\frac{\sqrt{C^2+D^2}}{2}\tau)(CI_zS_z + DI_yS_z)$$

The signal $S(t) = Trace\langle S_x \rho(t) \rangle$

$$S(\tau) = \cos(\frac{\sqrt{C^2+D^2}}{2})\tau$$

Where $C = \frac{4}{3\pi}\cos(\frac{\pi}{2}\phi)\frac{i(\omega^1-\omega^{-1})}{2(1-\phi^2)}$ and $D = \frac{(\omega^2-\omega^{-2})\phi}{1-4\phi^2}\cos(\pi\phi)$

The effect of finite pulse is clearly visible from the expression of signal. The frequency of the signal is $\frac{\sqrt{C^2+D^2}}{2}$. The C and D are depend on nature of the pulse by the variable ϕ. If the pulse is delta that is $\phi = 0$, the term D will be zero. As ϕ increases contribution from D also increases. The similar dependence also appears in C. The ϕ dependence in C is coming from $\cos\frac{\pi}{2}\phi$ term. The analytic expression of the signal compared with numerical simulations. The numerical simulations are given by Simpson. The of signal REDOR $X\overline{X}-4$ pulse sequence from simpson is given below,

The numerical simulation qualitatively matching with analytic expression of the signal. When signal acquisition starts with maximum amplitude and decays to zero

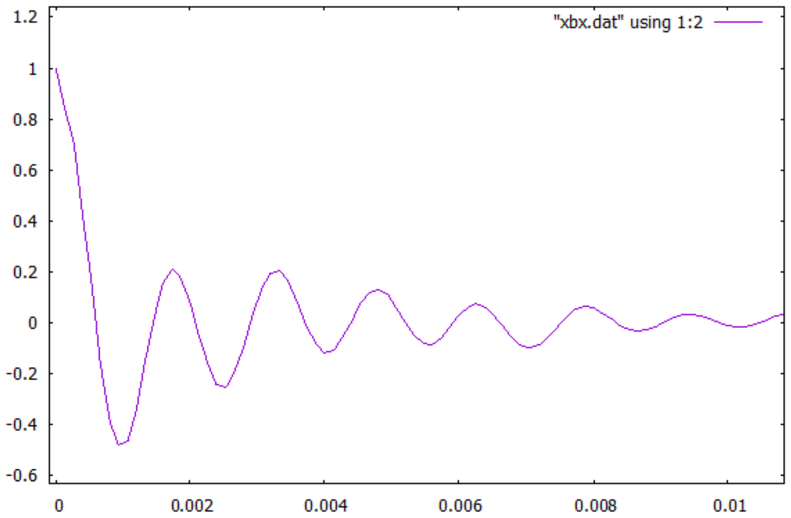

FIGURE 3.9: The numerically simulated signal for REDOR $X\overline{X}-4$ pulse sequence is obtained from Simpson. The pulses in this sequence approximated as Delta pulse. The signal behaves as expected from the calculations.The signal starting from maximum amplitude and decays to zero.

as time increases. The decaying nature of the signal is because of spin relaxation. The signal has functional form of cos and the frequency of the evolution changes according to change in spin parameters. The numerical simulation for different ϕ values implies that the signal depend on the ratio of pulse length to cycle time. The numerical simulations are given below,

The numerically simulated signal for REDOR $X\overline{X}-4$ pulse sequence for different ϕ.The ϕ is the ratio $\frac{2t_w}{t_r}$, where t_w is duration of the pulse and τ_r is the cycle time. The purple, green and blue correspond to ϕ = 0.32, 0.5 and 0.25 respectively. The numerical simulation for different ϕ values clearly indicate that the signal is varying with ϕ. When ϕ increases the frequency of evolution also increase, the frequency of evolution is higher in case of ϕ= 0.5(green line) where frequency is lower for $\phi = 0.25$ (blue). The signal decays fast in case of higher ϕ values.

The ratio $(\frac{2t_w}{\tau_r})$ can be change two ways, by changing the pulse duration t_w and by changing rotor period τ_r

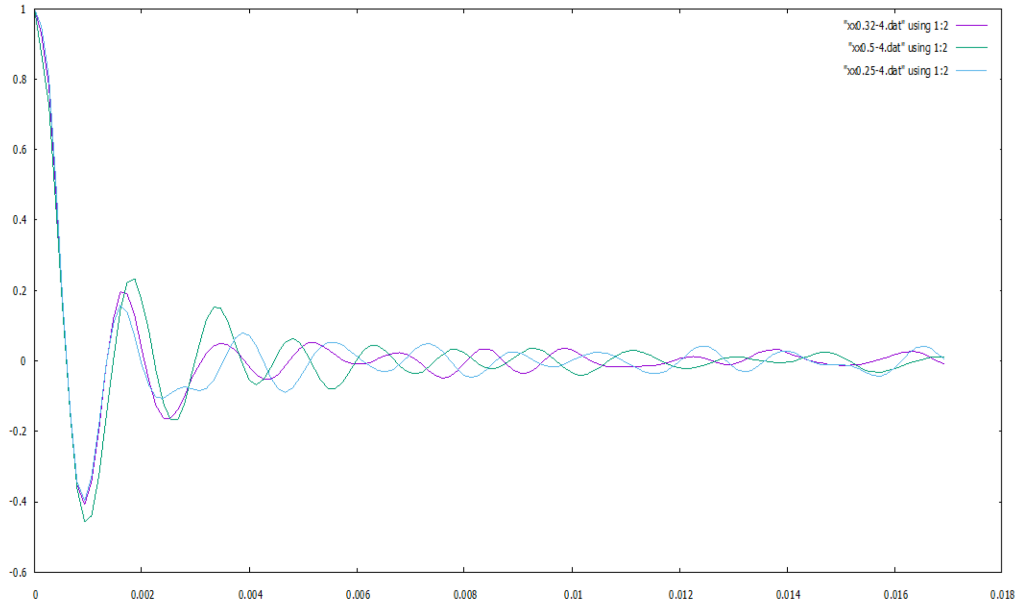

FIGURE 3.10: The numerically simulated signal of REDOR $X\overline{X}-4$ pulse sequence for different ϕ values.The ϕ is $\frac{2t_w}{t_r}$, where t_w is duration of the pulse and τ_r is the cycle time. The purple, green and blue graphs correspond to $\phi =$ 0.32, 0.5 and 0.25 respectively

. The figure at left side correspond to for different ϕ values such that rotor period (τ) put constant and pulse duration (t_w) changed. the figure on right side correspond to signal for different ϕ such that pulse duration (t_w) put constant and rotor period (τ_r) varied. The purple, green and blue graphs correspond to $\phi =$ 0.32, 0.5 and 0.25 respectively. The conclusion is the nature of the signal depend on the the ratio ϕ. The ϕ can change in two ways, but the signal only depend on magnitude of ϕ.

Analysis of finite pulse effect in REDOR XX-4 and REDOR $X\overline{X}-4$ states that the ratio of pulse duration with cycle time $(\frac{2t_w}{t_r})$ has inevitable dependency on signal. The analytic expression for the signal indicate that as increase in ϕ values the frequency of the signal also increases. The same conclusion shown in numerical simulations. The effect of finite pulse inclusion is very important in REDOR experiments. Since the ϕ values are depend on frequency of acquired signal,finite

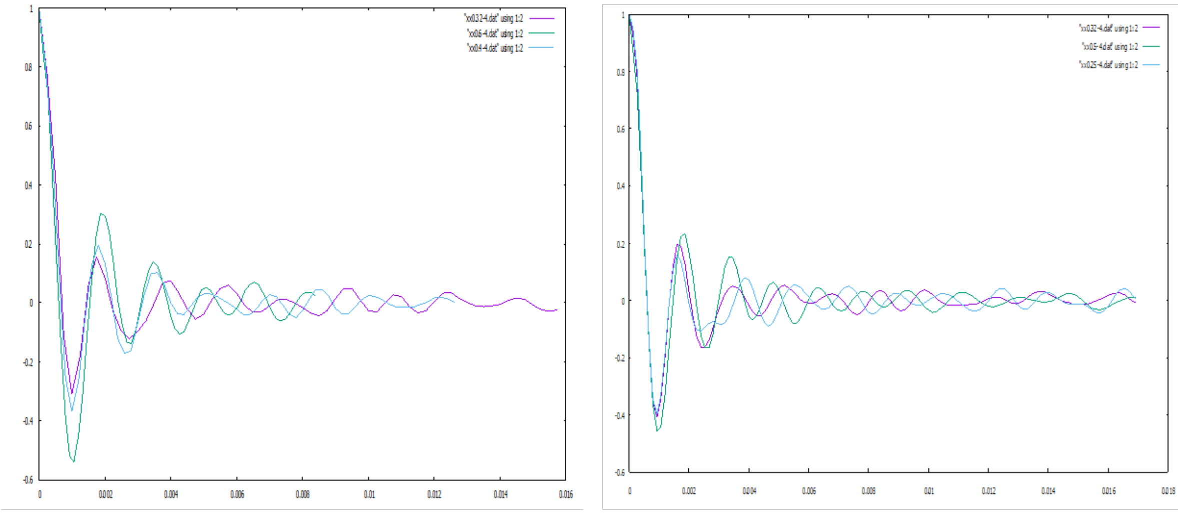

FIGURE 3.11: The numerically simulated signal of REDOR XX-4 . The figure at left side correspond to for different ϕ values such that rotor period (τ) put constant and pulse duration (t_w) changed. the figure on right side correspond to signal for different ϕ such that pulse duration (t_w) put constant and rotor period (τ_r) varied. The purple, green and blue graphs correspond to $\phi = 0.32$, 0.5 and 0.25 respectively

pulse effects can change the accuracy of measurements. REDOR is used to measure weak dipolar intercation which causes tertiary and quaternary structure of proteins. The internuclear distance accuracy has great significance in structure determination. So effect of finite pulse should include in signal analysis.

Chapter 4

REDOR in Multispin System

Dipolar coupling measurements using solid state NMR is very important to structural determination of complex system such as membrane proteins, large enzymes and photochemical reaction intermediates. The accurate internuclear distance measurement is useful to three dimensional structural determination. The resolution and sensitivity of the SSNMR experiment optimized by the implementation of magic angle spinning condition. The recoupling sequences like REDOR are used to reintroduce dipolar interaction under spinning conditions.

REDOR commonly called rotational echo double resonance NMR used to recouple heteronuclear interactions between isolated spin pair. REDOR provides accurate internulear distance between isolated low γ spin pairs. So extending REDOR experiments to larger spin systems has high significance. Before going to extension of REDOR into larger spin systems, it is very important to understand about signal analysis in REDOR experiment. REDOR signal from multispin system contains frequencies correspond to multiple dipolar coupling. The Fourier transformation is the general method used in all kind of spectroscopy to extract individual frequencies from time domain signal. In the Fourier transformation $e^{nD_i\tau_r}$ used as the Kernal to transform time domain signal to frequency domain.

$$\int_0^{\infty} A_i S(nD_i\tau_r)e^{nD_i\tau_r} = A_i\delta(D - D_i)$$

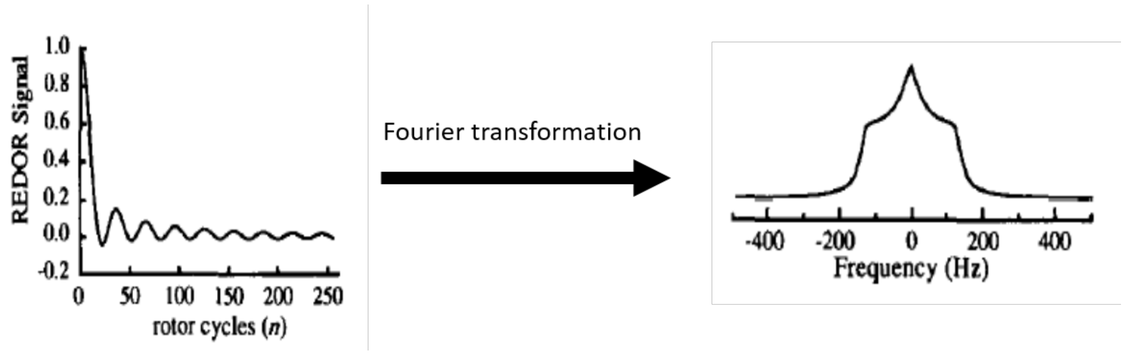

FIGURE 4.1: The Fourier transformation of time domain REDOR signal to frequency domain domain shown in the figure. Fourier transformation lead spectrum with spectral width proportional to the dipolar coupling.

The width of the peak in frequency domain spectrum is proportional to the dipolar coupling. The accurate measurement of dipolar coupling is not possible with Fourier transformation because peaks are not sharp enough to give exact value of dipolar coupling. The accurate measurement of internuclear distance is very important in structural determination.

The Fourier transformed spectrum of multispin system shown below. The multispin system has more than one dipolar coupling frequencies.

The Fourier transformed spectrum of REDOR in multispin system is very broad. The individual frequency determination from this broad spectrum is impossible. This is the major disadvantage of Fourier transformation method in REDOR experiments. The another drawback is frequency determination of weak interactions. when dipolar interaction is weak the Fourier transformed spectrum become very broad. Accurate determination of dipolar frequency in such cases are impossible.

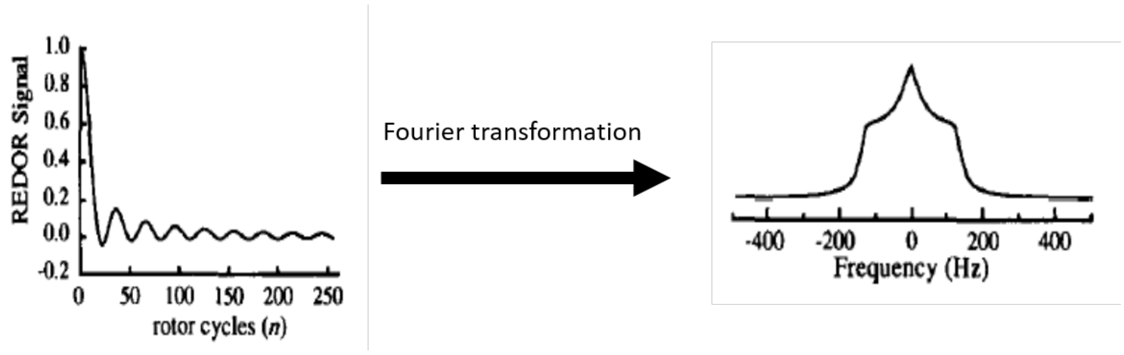

Figure 4.2: The Fourier transformation of time domain REDOR signal from a multispin sytem to frequency domain domain shown in the figure. Fourier transformation lead spectrum with spectral width proportional to the dipolar coupling.

In this cases numerical simulations gives better results. But numerical method is not applicable in multispin systems.

The REDOR transform replaces Fourier transformation in REDOR.

4.0.1 REDOR Transfrom

The REDOR Transform is the method of direct calculation of internuclear coupling from dipolar-dephasing NMR data, proposed by K.T Mueller in 1995. The REDOR signal for a particular crystal orientation without including finite pulse effect is given by the expression,

$$\frac{S_r}{S_0} = \cos(2\tau_r \omega_D)$$

Where the ω_D is the dipolar frequency for that specific orientation can be represent as

$$\omega_D(\alpha, \beta, t) = \mp\pi D[\sin^2\beta\cos 2(\alpha + \omega_r t) - \sqrt{2}\sin 2\beta\cos(\alpha + \omega_r t)]$$

The angles α and β are the azimuthal and polar angles describing orientation of internuclear vector. The spinning frequency is ω_r. The is dipolar coupling constant,

$$D = \frac{\gamma_I \gamma_S}{r^3}$$

where the γ_I and γ_S are the gyromagnetic ratio of spins I and S.

The signal of the powder sample is given by integrating over all possible orientations. The reduced signal $\frac{S_r}{S_0}$ is,

$$\frac{S_r}{S_0} = \frac{1}{4\pi}\int_0^{2\pi} d\alpha \int_0^{\pi} \sin\beta d\beta \cos(\Delta\phi_n)$$

The $\Delta\phi_n$ is the dephasing angle which depend on number of rotor cycles

$$\Delta\phi_n = 4\sqrt{2}nD\tau_r \sin\beta\cos\beta\sin\{\alpha + \frac{2\pi t_1}{\tau_r}\}$$

The reduced signal can be rewrite by substituting $\Delta\phi_n$

$$\frac{S_r}{S_0} = \frac{1}{4\pi}\int_0^{2\pi} d\alpha \int_0^{\pi} \sin\beta d\beta \cos(4\sqrt{2}nD\tau_r \sin\beta\cos\beta\sin\{\alpha + \frac{2\pi t_1}{\tau_r}\})$$

The above integrals usually evaluated numerically to get dipolar coupling constant. The difficulty to obtain analytic solution to the integrals lies in the functional morass of trigonometric functions of trigonometric functions. The analytic solution of above integrals can be represented as Bessel functions.
Bessel functions (J_k) of first kind are solutions to Bessel equation

$$x^2\frac{d^2J_k(x)}{dx^2} + x\frac{dJ_k(x)}{dx} + (x^2 - k^2)J_k(x) = 0$$

The trigonometric functions can be rewrite using Bessel functions

$$\cos(x\sin\theta) = J_0(x) + 2\sum_{k=1}^{\infty} J_{2k}(x)\cos(2k\theta)$$

$$\sin(x\sin\theta) = 2\sum_{k=1}^{\infty} J_{2k-1}(x)\sin((2k-1)\theta)$$

$$\cos(x\cos\theta) = J_0(x) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(x)\cos(2k\theta)$$

$$\sin(x\cos\theta) = 2\sum_{k=1}^{\infty}(-1)^{k-1} J_{2k-1}(x)\cos(2k-1)\theta)$$

The analytic expression of the signal is found using Bessel functions,

$$S(nD_i\tau_i) = \frac{\sqrt{2}pi}{4} J_{\frac{1}{4}}(\sqrt{2}nD_i\tau_i) J_{\frac{-1}{4}}(\sqrt{2}nD_i\tau_i)$$

The next task is finding appropriate Kernal ($K(nD_i\tau_i)$) to transfer time domain signal to frequency domain. A general formula is available for the inverse function of the product of two spherical Bessel functions such that

$$\int_0^{\infty} j_l(\sqrt{2}nD_i\tau_i) j_{l+m}(\sqrt{2}nD_i\tau_i) g_{l,m}(\sqrt{2}nD_i\tau_i) dj_l(\sqrt{2}n\tau_i) = \delta(D - D_i)$$

The Kernal or inverse function $g_{l,m}(x)$ defined as

$$g_{l,m}(x) = \frac{8x^2}{\pi}\frac{d}{dx^2}[x^2 y_l(x) j_{l+m}(x)]$$

The Kernal in form form cylindrical Bessel functions of first and second kind is given below.

$$K(nD_i\tau_i) = 8nD_i\tau_i\{J_{\frac{1}{4}}(\sqrt{2}nD_i\tau_i)Y_{\frac{-1}{4}}(\sqrt{2}nD_i\tau_i)\}$$
$$+8\sqrt{2}(nD_i\tau_i)^2\{J_{\frac{-3}{4}}(\sqrt{2}nD_i\tau_i)Y_{\frac{-1}{4}}(\sqrt{2}nD_i\tau_i) + J_{\frac{1}{4}}(\sqrt{2}nD_i\tau_i)Y_{\frac{5}{4}}(\sqrt{2}nD_i\tau_i)\}$$

The exact analytic solution of REDOR signal found using Bessel functions. Kernal for the REDOR transformation also available in literature as inverse Bessel function. The introduction of REDOR transform in analysis has significant role in calculating accurate internuclear distances. The accurate distance measurement popularized REDOR over other Solid State NMR techniques. The comparison of Fourier transformation with REDOR transformation given below,

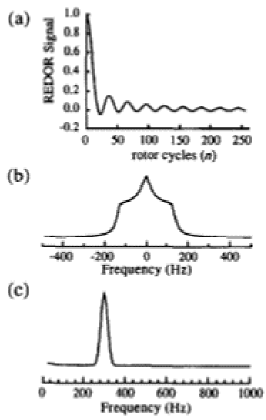

FIGURE 4.3: The Fourier transformation and REDOR transform of time domain REDOR signal.The figure (a) correspond to the inverted REDOR dephasing curve. Figure (b) correspond to Fourier transformation of REDOR siganl.The Fourier transformation results spectrum with spectral width proportional to the dipolar coupling. Figure (c) correspond to REDOR transformation of REDOR siganl.The REDOR transformation results spectrum with a single line at dipolar coupling frequency.

The Fourier transformation of the REDOR signal resulted a spectrum with broad peaks. the width of the peak correspond to the dipolar frequency. The drawback of Fourier transformation is that the accurate determination of the dipolar frequecy is not possible sue to broad nature of the peaks. The REDOR transform of the same signal is given in (c), In REDOR transformation a sharp line at interested dipolar frequency. The determination of dipolar coupling frequency is very accurate in this kind of spectrum. This example clearly indicate that REDOR transformation gives highly resolved spectrum.

The analysis of REDOR signal for multispin system which contain three dipolar coupling. The figure (a) is the inverted REDOR signal which conatain three dipolar couplings. The Fourier transformation of inverted signal resulted a broad

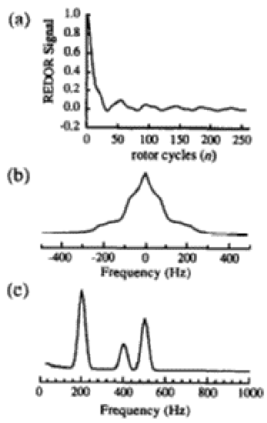

FIGURE 4.4: REDOR analysis in multispin system. Figure (a) is the inverted REDOR curve with three dipolar couplings.The figure (2)is Fourier transformation of time domain inverted REDOR curve to frequency domain. Fourier transformation lead spectrum with spectral width proportional to the dipolar coupling. The figure (c) is REDOR transformation of the same inverted REDOR curve.

peak. The individual determination of dipolar frequencies are impossible in Fourier transform spectrum. The REDOR transformation of the same inverted signal is given in figure (c). The REDOR transformation results the spectrum with three individual sharp peaks. The peaks are no longer broad and frequency determination of all three couplings are possible.

The Fourier transformation of REDOR signal results broad peaks. The accurate determination of the frequencies are not possible with Fourier transformation. The description of multiapin systems even difficult using Fourier transformation, since determination of individual frequencies are not possible. The way to solve these difficulties is the finding of appropriate mathematical method to analysis inverted REDOR. The Bessel finction are used to describe analytic form of inverted REDOR signal. The Kernal for the transformation selected from inverse Bessel functions. The REDOR transform gives high resolved spectrum compared to Fourier transformation method. The individual determination of frequencies are possible in case of multiple spin systems. The weak couplings also clearly observable in REDOR transformation.

4.0.1.1 Analysis of IS_2 spin system

Consider a IS_2 spin system, The dipolar Hamiltonian correspond to the spin system is

$$H_D = \omega_{D1} S_z I_{z1} + \omega_{D2} S_z I_{z2}$$

The magnitude of dipolar coupling between S_z and I_{z1} spins is ω_{D1} and between S_z and I_{z2} spins is ω_{D2}. The REDOR pulse sequence recouples Homonuclear dipolar interaction between S_{z1} and S_{z2}. During the REDOR pulse sequence the system evolve under the Heteronulear dipolar Hamiltonian. The signal collected from S spin by taking S_x as initial state. The REDOR signal for a individual orientation is given by

$$S(2n\tau_r) = \cos(2n\omega_{D1}\tau_r)\cos(2n\omega_{D2}\tau_r)$$

As expected signal expression contains terms ω_{D1} and ω_{D2} correspond to dipolar interaction between spin pairs. The observed signal is the integral of all signals corresponding to possible all possible orientation.

$$S(2n\tau_r : D_1, D_2) = \int d\Gamma_1 \int d\Gamma_2 \cos(2n\omega_{D1}[\Gamma_1]\tau_r)\cos(2n\omega_{D2}[\Gamma_2]\tau_r)d\Gamma_1 d\Gamma_2$$

The Γ_1 and Γ_2 correspond to the angle sets of dipole vectors. The integral over molecular orientation involves one set of unconstrained angular parameters. The two sets of angles Γ_1 and Γ_2 are related by via a single set of Euler angles. The REDOR transformation of the observed signal gives dipolar couplings between the spin pairs. But to extract dipolar coupling information by REDOR transformation one have know relative orientation between doplar vectors. predetermined structure can provide relative orientation.The conclusion is REDOR transformation on multispin system works only if relative orientation between doples has predetermined.This need is difficult to meet and comes as the major limitation of REDOR experiments in multispin system. The one method to solve this difficulty is by labeling the spins selectively. All nuclei are not NMR active so selective labeling is possible by introducing NMR active nucleus. Preparation of multiple specifically labeled samples is the straight way to apply REDOR in multispin systems. In specific labeling at a time single pair of heteronuclear system will NMR active and rest of them will be inactive. Next time other pair of spins will be NMR active.The drawback of this kind of experiment is preparation of sample in a selective manner is time consuming and laborious.

4.0.1.2 Θ-REDOR

Inspection of REDOR experiments on multispin system reveals that the signal analysis using REDOR transformation is impossible such cases where relative orientation between dipole vectors are unknown. But it is possible to extract dipolar coupling using REDOR transformation if signal represented by sum of two dipolar couplings.

$$S(2n\tau_r : D_1, D_2) = \int \cos(2n\omega_{D1}[\Gamma_1]\tau_r)d\Gamma_1 + \int \cos(2n\omega_{D2}[\Gamma_2]\tau_r)d\Gamma_2$$

Here the dipole-dipole coupling is evolving as sum of two separate dipolar coupling instead of product of dipolar couplings. The constraints from relative orientation is no more present in this situation. The idea to extract dipolar coupling from multispin system can be achieved by expressing REDOR signal as sum of dipole-dipole coupling. This is the basic idea of Θ-REDOR.The dipolar couplings from multispin system found using Θ-REDOR. Before going to description $\Theta - REDOR$ in detail, here explains how the REDOR signal varies with pulse applied on S Spin. The REDOR signal from multispin system can be written as

$$S_\Theta(2n\tau_r) = C_1^2C_2^2 + \cos\Theta S_1^2C_2^2 + \cos\Theta C_1^2S_2^2 + \cos\Theta S_1^2S_2^2$$

$$C_1 = \cos(n\omega_{D1}\tau_R), C_2 = \cos(n\omega_{D2}\tau_R)$$

$$S_1 = \sin(n\omega_{D1}\tau_R), S_2 = \sin(n\omega_{D2}\tau_R)$$

The Θ is pulse angle applied to S Spin. There is no pulse applied on to S spin during reference pulse sequence of REDOR experiment. The angle Θ is zero in case of reference pulse sequence. The signal acquired from S spin during reference experiment is,

$$S_\Theta(2n\tau_r) = C_1^2C_2^2 + \cos\Theta S_1^2C_2^2 + \cos\Theta C_1^2S_2^2 + \cos\Theta S_1^2S_2^2$$

No pulse applied to S spin, the Θ is zero in reference pulse sequence and cos(Θ) is one in this case,

$$S_\Theta(2n\tau_r) = C_1^2C_2^2 + S_1^2C_2^2 + C_1^2S_2^2 + S_1^2S_2^2 = 1$$

the as expected dipolar interactions refocused in reference sequence. Let's consider dephasing sequence of REDOR pulse sequence. A π pulse applied to S spin in dephasing Sequence, here Θ is π ($\cos\Theta = -1$). The signal from the dephasing pulse sequence is

$$S(2n\tau_r : D_1, D_2) = \cos(2n\omega_{D1}\tau_r)\cos(2n\omega_{D2}\tau_r)$$

The REDOR transformation requires relative orientation to extract dipolar frequencies in this signal expression. The Θ - REDOR ultimately provides a signal expression that contain sum of dipolar frequency terms instead of product.

The Θ-REDOR introduced by Pennington and Gullion in 1998. The experiment achieves desired deconvolution by breaking IS_n spin networks down into collection of IS spin system. As compared to standard REDOR experiments Θ-REDOR contains a Θ ($\Theta \ll \pi$) pulse on S spin rather than a standard π pulse. If Θ is sufficiently small there is a high probability that only single S spin is flipped and dephasing signal contain dipolar coupling frequency of flipped spin to the I spin. In other words at this condition phase accumulation due to $\overline{H}_D$ is associated with only a single coupling at a time.The schematic depiction of the pulse sequence for the Θ− REDOR is given below,

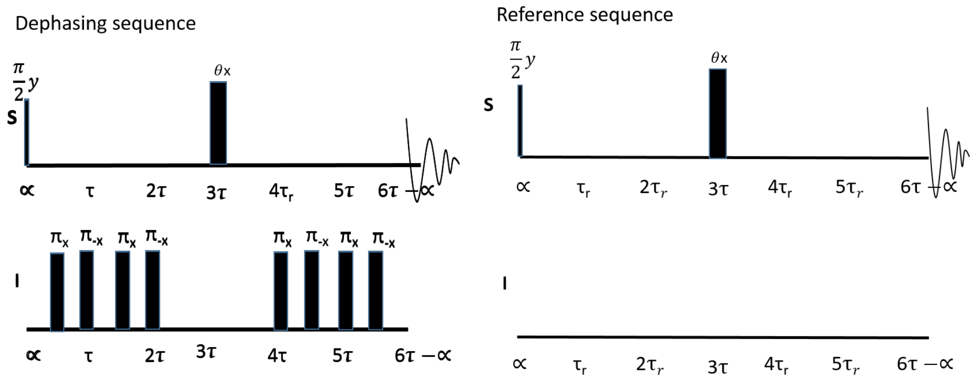

Figure 4.5: **Schematic depiction of $\Theta - REDOR$ XX-4 pulse sequence**. As in standard REDOR XX-4 pulse sequence $\Theta - REDOR$ XX-4 contains reference and dephasing pulse sequence. The difference from the standard REDOR pulse sequence is that Θ-REDOR pulse sequence consist a Θ pulse on S spin at middle of reference and dephaing pulse sequence.

when $\Theta \leqslant 1$ then the $\cos\Theta \approx 1 - \frac{\Theta^2}{2} + \frac{\Theta^4}{24}$ and $\cos^2\Theta \approx 1 - \Theta^2 + \frac{\Theta^4}{3}$. Substituting this approximation to the general equation for the signal

$$S_\Theta(2n\tau_r) = C_1^2C_2^2 + \cos\Theta S_1^2C_2^2 + \cos\Theta C_1^2S_2^2 + \cos\Theta S_1^2S_2^2$$

gives the analytic expression for Θ-REDOR signal.

$$S_\Theta(2n\tau_r) = (1 - \frac{\Theta^2}{2} + \frac{\Theta^4}{24})[C_1^2C_2^2 + S_1^2C_2^2 + C_1^2S_2^2 + S_1^2S_2^2]$$

$$+(\frac{\Theta^2}{2} - \frac{\Theta^4}{6})[C_1^2C_2^2 - S_1^2S_2^2] + \frac{\Theta^6}{16}[C_1^2C_2^2 - S_1^2C_2^2 - C_1^2S_2^2 + S_1^2S_2^2]$$

the final expression of the signal $S_\Theta(2n\tau_r)$ is

$$S_\Theta(2n\tau_r) = \{1 - \frac{\Theta^2}{2} + \frac{5\Theta^4}{48}\}$$

$$+\{\frac{\Theta^2}{2} - \frac{\Theta^4}{6}\}[\cos(n\omega_{D1}\tau_R) + \cos(n\omega_{D2}\tau_R)] + \frac{\Theta^4}{16}\cos(n\omega_{D1}\tau_R)\cos(n\omega_{D2}\tau_R)$$

The signal expression of REDOR contains three terms, the first and largest term is $(1-\frac{\Theta^2}{2}+\frac{5\Theta^4}{48}$) contain no dipolar dephasing part. This term arising from molecules that have no spin flip during application of Θ pulse on S spin. The second term of the signal is desired signal function $(\frac{\Theta^2}{2} - \frac{\Theta^4}{6}[\cos(n\omega_{D1}\tau_R) + \cos(n\omega_{D2}\tau_R)])$. This term correspond to the case were only S spin flipped during the Θ pulse and dipolar coupling arises as the sum of individual dipolar coupling frequencies. The last term is $\frac{\Theta^4}{16}\cos(n\omega_{D1}\tau_R)\cos(n\omega_{D2}\tau_R)$ contains errors where the signal function appears in the same form as found in the standard REDOR experiments. The the magnitude of third term is very small because Θ is small and Θ^4 will be negligible.

While the leading term in the dipolar evolution under Θ-REDOR correspond to signals of the form $[\cos(n\omega_{D1}\tau_R) + \cos(n\omega_{D2}\tau_R)]$. this simplification comes about at the cost of a large fraction of unused signal $(1 - \frac{\Theta^2}{2} + \frac{5\Theta^4}{48}$). The important fact is Θ - REDOR yields the desired simplified analysis in IS_n system at the price of a overall signal to noise ratio. The extension of this sequence to uniformly labeled peptides is not straightforward. Accurate measurements of weak dipolar couplings gives no satisfactory results using Θ-REDOR.

4.0.2 Frequency selective REDOR

The REDOR in multi-spin system is challenging because REDOR transformation depend on relative orientation of doplar vectors .Several modified versions of REDOR have been developed to overcome the dependence of dipolar dephasing curves in coupled spin clusters on multiple coupling and their relative orientations. The frequency selective REDOR (FS-REDOR) is a modified REDOR sequence to encodes dipolar coupling frequencies from IS_n spin system.The basic priciple of frequency REDOR experiment is to retain certain interactions while refocusing other by using frequency selective pulses. The RF pulses used in NMR can classify to two classes, hard pulse and soft pulse. The hard pulses are the common class of pulses used in NMR. The hard pulse excite the entire spectral width uniformly. The soft pulse excite selective regions. The factors determine the nature of the pulse are shape, amplitude and length of the pulse. Pulse shape is correlated with shape of the excitation profile, the amplitude proportional to the flip angle and pulse length gives the selectivity. The schematic depiction of pulse sequence for dephasing frequency selective REDOR is given below

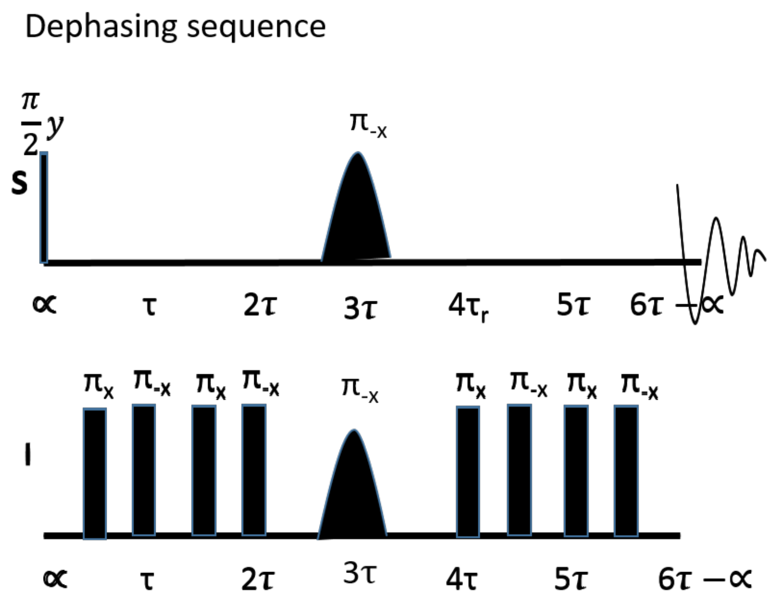

FIGURE 4.6: **Schematic depiction of frequency selective REDOR pulse sequence**: FS-REDOR pulse sequence start with a $\frac{\pi}{2}$y pulse on S spin. The signal acquired from spin ant the end of the pulse sequence as in case of standard REDOR experiment. At the middle of the pulse sequence a Gaussian π pulse applied to both of the spins which recouple selective spin pairs

The FS-REDOR contain Gaussian pulses at exactly half of the pulse cycle on S and I spins rest of the pulse sequence is exactly same as the standard REDOR sequence.The Gaussian pulse falls of quickly. There is no side lobes in frequency domain help to fulfill high degree of accuracy. The Gaussian used such a way that maxima of the Gaussian coincide with midpoint. A frequency selective spin echo generated by the application of simultaneous rotor synchronized selective π pulses on S and I spin. The selective irradiation accomplishes refocusing of selected spin pairs.