High harmonic generation by femtosecond pulses: Numerical simulations and experimental setup design

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Certificate of Examination

This is to certify that the dissertation titled "High harmonic generation by femtosecond pulses: Numerical simulations and experimental setup design" submitted by Mr. Shashank Yadav (Reg. No. MS08046) for the partial fulfillment of BS-MS dual degree programme of the Institute, has been examined by the thesis committee duly appointed by the Institute. The committee finds the work done by the candidate satisfactory and recommends that the report be accepted.

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Declaration

The work presented in this dissertation has been carried out by me under the guidance of Dr. Kamal P. Singh at the Indian Institute of Science Education and Research, Mohali.

This work has not been submitted in part or in full for a degree, a diploma, or a fellowship to any other university or institute. Whenever contributions of others are involved, every effort is made to indicate this clearly, with due acknowledgement of collaborative research and discussions. This thesis is a bonafide record of original work done by me and all sources listed within have been detailed in the bibliography.

> Shashank Yadav Dated: August 14th, 2013

In my capacity as the supervisor of the candidate's project work, I certify that the above statements by the candidate are true to the best of my knowledge.

> Dr. Kamal P. Singh (Supervisor)

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Abstract

High harmonic generation (HHG) is a process in which a noble gas is irradiated by an intense femtosecond laser pulse at frequency ω_0 and the gas atoms emit radiation of higher frequencies which are odd multiples of ω_0 . We can achieve upto more than 200^{th} harmonics in this process. In the first part of this thesis we will discuss about the different aspects and the theory behind the phenomenon and focus on the 'Three step model' which is an intuitive approach towards the explanation of the mechanism of this process.

In the second and the major part of this thesis we will reproduce so called 'Semiclassical calculations' which provide validation to the 'Three step model' also known as the 'Common man model'. These calculations can not only determine the cutoff on the frequencies of the harmonics which are generated but can also predict the temporal characteristics of the emitted light and its frequency modulation in time.

The third and the final chapter of the thesis focuses on the designing of an experimental setup for the process. We will discuss in particular how to separate different wavelengths (of the harmonics), the use of diffraction grating for the same purpose and the issues involved. Finally after determining all the physical attributes of the different components of the setup, namely the converging mirror, the vacuum chamber, the grating and the detection unit we will demonstrate a preliminary design for the setup.

Chapter 1 Introduction

Electrons are responsible for the intra-atomic forces in a molecule and they move and change their distribution in the attosecond regime which is 10^{-18} sec. Where as, heavier atoms move in the femtosecond (10^{-15} sec) timescale. To capture the temporal evolution of electronic processes such as the relaxation of an inner-shell vacancy we have to use attosecond pulses [4] as probes. To produce attosecond pulses there are some fundamental conditions which we have to satisfy. A limitation on the pulse duration for a light source comes from the period of the carrier component of the radiation i.e the frequency of the radiation. Laser pulses which are in the visible or infrared spectral region (frequencies ranging from 300 GHz to 790 THz) are limited to pulse duration of few femtoseconds. We need much shorter wavelengths to reach the attosecond regime.

Along with the requirement of high frequencies, attosecond pulses require phase locked [16] frequencies over a large bandwidth. We know two of this kind of processes which can provide a coherent spectra with bandwidth broad enough to create attosecond pulses. One is based on Raman scattering [9] [15] in which a broad spectrum UV pulse is obtained by phase modulation from impulsively exciting coherent vibration of molecules. This method can produce sub-femtosecond sub-cycle pulses. For instance we can create periodic train of 6 fs pulses [13] and single 4 fs [1] pulse at 400 nm.

The second process which we are going to emphasis in this thesis is based on high order harmonic generation (HHG), which can provide attosecond emission in the XUV



(a) short duration pulse generation by superpo- (b) random phased harmonics don't add-up effisition of phase locked harmonics ciently

Figure 1.1: Wave superposition

spectral range. This process was first observed in 1987 in Chicago by Mcpherson [14] $(17^{th} \text{ harmonic of KrF laser})$. In HHG we can create light which consists of a comb of many VUV lines which are equally spaced and are of almost equal amplitude. The spectral range that we get is broader than 100 eV. Such broad spectral range can support the generation of very short duration pulse. An advantage of this process is that the harmonics are phase locked i.e they are coherent in nature which is a must required condition for the generation of isolated attosecond pulse or any isolated short duration pulse as such. To illustrate this point we have generated two plots (Fig 1.1) for the superposition of odd harmonics (wave pulses with frequencies which are odd multiples of some original frequency). We have taken harmonics of the order 1 to 29 coming from arbitrary fundamental frequency. In the first plot the harmonics are phase locked while the other has pulses which have random phase. We can clearly see that high intensity peaks which are of very short duration can be obtained when the pulses are coherent. But the intensities can't add up efficiently when the phases are generated randomly.

1.1 High Harmonic Generation

When light passes through a material, the electric field of the light polarizes the material and the charges within the medium are redistributed in response to the polarization. Then they start oscillating with the same frequency as of electric field. Now these charges due to acceleration emit radiation with the same frequency/wavelength



Figure 1.2: HHG

as the original radiation and that's how the light propagates through a medium. This is the linear model for light propagation and it assumes that the charges are free to follow the electric field and oscillate sinusoidally. This model works well when the intensity of the light and the corresponding field strength is low in comparison to the intra-atomic field. When the strength of the electric field increases we start to see higher order frequency terms in the induced polarization. This leads to radiation of the frequencies which are multiple of the incident frequency. Usually the efficiency of such a process is small and is inversely proportional to the order of the process. HHG is an example of such non-linear process.

In the experimental setup (Fig 1.2 (a)) we take a near infrared laser pulse, very short in duration and very high in intensity and focus it into a gas and the out come of the process is the generation of very high harmonics. We can get, not just up to the second or third harmonic radiations but even upto 200^{th} [3] harmonics. A whole string of frequencies is obtained which is called the harmonics comb [10] (Fig 1.2 (b)). In a typical harmonic spectrum the intensity drops rapidly for the first few harmonics. This decrease is what one would expect from a perturbative approach for which the efficiency of the process rapidly decreases with order. What cannot be explained by this approach is the far-reaching plateau in which the harmonics have almost constant intensity, ending in an abrupt cutoff. These harmonics belong to the EUV region and in some cases they lie even in soft X-ray region. The duration of the generated light pulses is very short and lies mostly in the femtosecond region

that makes them highly advantageous in dynamical studies. The energy per pulse is not so high although it's increasing each year as the advancement of the research area.

Usually inert gases are used because they are not easily ionized and plasma is not created when the intense electric field interacts with the atoms. However the conditions which are created by the focusing of the laser pulses provide the possibilities of plasma creation. Delay of a few femtoseconds can create a lot of free electrons but the duration of the incident pulse is not enough for that. The coherent propagation and even the fundamental pulse gets affected due to the significant optical effects of the plasma. Calculations for the intensity in a typical experimental setup shows that the peak field strength can reach up to few GV per square centimeter. As an example 50 fs laser pulse with energy of 1 mJ provides a peak power of 20 GW. If this pulse is focused to a spot of area 100 μm^2 the peak intensity will be 0.2 PW/cm^2 which corresponds to a field strength of $0.4 \ GV/cm$. Electric field inside the atom is around $1 \ GV/cm$. These kind of electric fields are capable of ejecting core electrons from the atoms of gases. The reason for which plasma is not created and we get to see high harmonics is the duration of the incident pulse. Before the emission of whole lot of free electrons and the starting of plasma effects to be seen harmonics are formed. That is the reason why the incident pulses should be of high intensities and short duration.

1.1.1 Semi-classical picture of HHG

The electric field of the laser pulse is strong enough to pull not only valence electrons but also the core electrons from the gas atoms. So the atoms get ionized and free electrons are formed. These electrons experience a really strong electric field in the free space between the ions and they start moving away from the parent atom. The electric field is sinusoidal in nature (with a very high frequency) so it changes its direction every half of the period of radiation. The pulled off electron moves away from the atom until the field reverses its direction, then it turns around and comes flying back in the general direction of the leftover ion it was emitted from.

A reasonable amount of times the electron comes close enough to the ion. Although it doesn't come back every time and some of the times it flies away from the core ion depending upon the initial conditions of its motion. The electron sees the nuclear charge which is screened with the valence electrons. If it has sufficient kinetic energy to overcome the repulsion from the electrons around the ion it reemerges with the nucleus and gets a very sharp acceleration. And this acceleration leads to radiation and depending upon the time duration of the interaction we can estimate the frequencies it's going to emit. The period of the laser pulse for a typical case is around 25 fs [6] and the electron takes approximately 12 - 20 femtoseconds in a round trip from the atom. But the collision time can be very short. So in the Fourier space it leads to generation of very high frequencies. All of it happens to all the atoms simultaneously which are exposed to the laser field. Each atom emits different frequencies but remains coherent with each other and the incident laser pulse.

The energetics of the process allow very high frequencies. Usually inert gases such as helium or argon are used in the gas jet. For the case of helium the ionization potential for the ground state is 24.6 eV. The electron which recombines with the parent ion is expected to emit a photon of a minimum 24.6 eV of energy. But in addition to this the electron has gained some kinetic energy in the free space by interacting with the laser electric field. The kinetic energy or speed of the electron when it reemerges with the ion depends upon its phasing with the sinusoidal field when it was emitted initially. The kinetic energy the electron emits in the collision is around $325 \ eV$ that is the kinetic energy plus the ionization potential for the ground state. This gives an idea of the energies which are involved in the process. Valence electrons also participate in the process but they lack high ionization potential so they contribute to the lower harmonics. Highest level harmonics come from the interaction with the core electrons.

All the atoms in the gas jet experience the electric field at the same time and the radiations emitted is in phase with each other in the forward direction and the process is coherent by mechanism. The 100^{th} harmonic has a wavelength 50 times shorter than the 2^{nd} harmonic but both are coherent to each other. Although the harmonics start in phase and if they travel in space they will remain in phase but the gas itself as an optical medium affects the propagation and influence the laser pulse and the harmonics differently. So something has to be done to maintain the coherency. To get efficient energy transfer into the high harmonics we have to induce quasi phase matching. This is a real challenge to make the process efficient along with the problem

of low intensities of the harmonics.

From an atom which is experiencing the incident laser field, the emitted electron oscillates in the field and every time it goes past the nucleus it gives out radiation. And it does it twice per pulse of the incident field. That means the electron passes through the nucleus twice per the time period of the electric field. A laser pulse passes through all the atoms that comes into its line, so every time it passes through an atom the harmonics add up and get bigger and bigger in amplitude. In an experimental setup these harmonics and the original laser pulse go through a spectrometer which has a grating that spreads them by the wavelengths. These different wavelengths are detected by a CCD camera and a spectrum of lots of harmonics of shorter wavelengths is obtained. An incident pulse of 800 nm can give rise to wavelengths up to 8 nm.

As a consequence of time–energy uncertainty principle, in order to produce a light pulse with a given duration it is necessary to use a broad enough spectral bandwidth. For instance a Gaussian-shaped pulse lasting for one picosecond (10^{-12} sec) has a minimum spectral bandwidth of 441 MHz ($\Delta \omega = 4.41 \times 10^{11} Hz$). These short duration harmonics give rise to broad bandwidth. The process starts with very high intensity laser field but the outgoing harmonics are of very low intensities. Nowadays people are using addition techniques to get increased bandwidth.

One important feature of the spectrum (Fig 1.2 (b)) which is obtained in the process is, we are getting only odd harmonics. Due to the periodic nature of the process the electrons have a high probability to be emitted when the sinusoidal electric field comes close to maximum. Also the gas is isotropic in space, so there is no difference in the two cases where the electron tunnels out when the electric field is $\pm E_0$. So it makes the time period of the process to be T/2 where T is the period of the laser pulse. Hence the periodicity of the process in the frequency domain is $2\omega_0$. This implies that the harmonics are emitted each half cycle with an alternating phase, i.e., field direction in such a way that the harmonic field $E_h(t)$ can be expressed in

the following form [7]:

$$E_h(t) = \dots + F_h(t + 2\pi/\omega_0) - F_h(t + \pi/\omega_0) + F_h(t) - F_h(t - \pi/\omega_0) + F_h(t - 2\pi/\omega_0) - \dots$$
(1.1)

The Fourier transform of this equation takes nonzero values only at odd multiples of ω_0 . This observation explains why the harmonic spectrum is composed of only odd order components. This argument is as mathematical as it gets and does well without the understanding of the physics of the process. There is a limitation on the frequencies obtained by the Fourier analysis of the periodic function which depends upon the intensity of the laser field and the ionization potential of the gas atoms.

To sum-up the main attributes of the harmonics which are observed are:

- 1. Harmonics of order > 200 and energy $> 500 \ eV$ [12] can be obtained.
- 2. Coherent ultra-short pulses with low divergence.
- 3. Process is too fast for plasma to form.
- 4. Cutoff frequency increases as the ionization potential, I_p gets bigger.
- 5. Cutoff frequency is inversely proportional to the fundamental frequency.
- 6. The smaller pulse duration, the higher the cutoff frequency.

By looking the process in terms of photons one can see that the emitted photons has much higher energy than the photons from the fundamental laser light. So its clear that to get one photon of higher energy we have to supply multiple photons of lower energy. That's why the process is highly nonlinear in nature.

Chapter 2

Three step model: Semi-classical approach

In the semi classical model [8] [2] we neglect the photonic nature of the light and simply assume its a wave with electric field which is varying in space and time. We assume that the atom is an ionic core and an electron. The ionic core has a positive coulomb potential in which the electron evolves. The laser light consists of an electric field of sinusoidal nature i.e $E = E_0 \sin(\omega t)$. When the atom is irradiated by the light the total potential Fig(2.1) which the electron gets is the sum of the coulombic potential and the electric interaction potential, V = qE.r. It leads to a potential barrier in the direction of the laser electric field. If the strength of the electric field is comparable to the intra-atomic electric field the electron no longer remains in the bound state during the interaction. There is some probability that the electron will tunnel through the said potential barrier and leave the ionic core. After that the fate of the electron is solely in the hands of the electric field until it comes back in the vicinity of the ionic core again.

The mechanism of the process can be broken down into three steps:

1. An atom exposed to a strong laser field can be ionized at some time t_i , creating a positive ion (the parent ion) and a free electron with no initial kinetic energy.

2. The electron is accelerated by the laser electric field. When the electric field changes sign, it may (or not) bring back the electron in the vicinity of the parent atom.



(a) Distorted potential [10]: When the external electric field is near its maximum, the total potential of the atom (dotted line) and the laser field (dashed line) forms a barrier (solid line) through which the electron may ionize by tunneling. When in the continuum the electron is accelerated by the oscillating electric field, gaining kinetic energy. When the field changes sign, the electron may be accelerated back to the vicinity of the ion core where it recombines, and a photon is emitted.

Figure 2.1: Three step model

3. If the electron clashes with the parent ion, the ion and electron can then recombine and the total extra energy (kinetic energy, E_{kin} of the electron plus the binding energy (ionization potential) of the atom, I_p) is released by emitting a photon of energy, $h\omega = E_{kin} + I_p$.

The maximum amount of energy that it could gain is $h\omega = I_p + 3.2U_p$, where U_p is known as ponderomotive energy which is; $U_p = e^2 E_0^2 / 4m\omega^2$ where e is the charge of the electron, E_0 is the maximum amplitude of the laser field, ω is the laser angular frequency and m is the mass of the electron.

Ponderomotive energy is the average energy that the electron could gain in some applied electric field. Its averaged over a cycle of the laser field so it has a factor of $1/_2$ along with another $1/_2$ that comes from the kinetic energy calculation. The factor 3.2 comes from the phasing of the ejection of the electron with the laser field.

2.1 Classical calculations

The temporal profile of the emitted light depends upon the dynamics of the electron in the continuum interacting with the laser field. We assume that the evolution of the electron is purely classical in nature and we have performed simulations regarding the trajectory and the energy of the electron. These calculations are from an article which was published in American journal of physics in 2009 [10]. These classical calculations [10](simulations) help a lot in understanding the process. They not only provide an intuitive picture of the process but also the results from the simulations are fairly consistent with the the experimental values [1]. We can determine the value of cut-off frequency upto a fair accuracy by simply calculating the kinetic energy of an electron evolving in the field. The temporal characteristics of the emitted light and its frequency modulation (chirp) in time can also be predicted. It comes out that kinetic energy of the electron depends upon the trajectory of the electron. That leads to the existence of short and long trajectories.



(a) The classical electron trajectories (blue lines) for different initial phases relative to the IR laser field (red line). The electron returns to the atom when it again crosses the horizontal black line

Figure 2.2: Classical electron trajectories

2.1.1 Trajectories of the electron in the laser field

The atom is irradiated by a laser field of sinusoidal nature i.e $E = E_0 \sin(\omega t)$. The electron tunnels through the resultant potential barrier and it's released into the

continuum. We assume that the velocity of the electron at that initial time $(t = t_i)$ is zero. It starts with zero kinetic energy and moves in the influence of the force from the laser electric field F(t) = -eE(t). We neglect any effects from the atomic potential after it exits the atom and the force from the field is assumed to be the only active force. We solve for the position of the electron in one dimension assuming these conditions and as a function of time (t > ti) it comes out to be:

$$x(t) = \frac{eE_0}{m\omega^2} [\sin(\omega t) - \sin(\omega t_i) - \omega(t - t_i)\cos(\omega t_i)]$$
(2.1)

By plotting the position of the electron it is possible to determine the trajectories (Fig 2.2) for which the electron returns to the core. In turn we can determine the tunneling times (t_i) for such trajectories. By numerically looking at the zero- crossing of the function x(t) for a range of t desired tunneling times can be obtained. It is known that the process is periodic with time period T/2 (T is the time period of the incident laser field). That means the periodicity is 2ω in frequency space. So its adequate to observe the function x(t) for tunneling times only belonging to the range $0 < t_i < \pi$. One can observe that there are trajectories for which there are multiple return times. In reality the first zero- crossing of x(t) i.e the first return time is the only significant one. The reason is that the probability of emitting harmonics from electron that have spent more than one period in the laser electric field is low because of the spreading of the wave-packet.

2.1.2 Kinetic energy and return time

The electron starts with zero velocity. Its time of return and trajectory depends upon the phase of the field at that time. We have obtained the tunneling times for which the electron returns to the ion core, so we have got a set of pairs (t_i, t_r) , each pair identifying a trajectory of importance. We can now calculate the velocity of the returning electron by differentiating position w.r.t time. From this we can arrive at the kinetic energy of the electron for different trajectories. It comes out as a function of t_i and t_r

$$E_{kin} = 2U_p(\cos(\omega t_r) - \cos(\omega t_i))^2$$
(2.2)

We have plotted (Fig 2.3) the returning K.E as function of return time (t_r) . It can be observed from the graph that there is a maxima in the energy that corresponds to the cut-off frequency. The maximum kinetic energy from the graph equals to $3.2U_p$ which is consistent with the experimental value. Given the value of E_0 , we can determine the order of harmonics for different values of return times. The cut-off energy i.e $I_p + 3.2U_p$ corresponds to tunneling time $t_i = 0.3T$ and return time $t_r = 0.95T$. The corresponding trajectory is highlighted in the plot of position vs time (Fig 2.2)



(a) The energy gained by the electron in the continuum as a function of the time of return to the core. This calculation shows the existence of short and long trajectories separated by the cut-off, as well as the frequency variation in time.

Figure 2.3: Kinetic energy as a function of return time

We can see from the graph that for each kinetic energy there are two different return times with corresponding tunneling times (except the maximum K.E trajectory). That means that return energy is same for two different trajectories. It comes out that one of these trajectories is longer than the other. The longer trajectory has its return time greater than 0.95T and the shorter one has return time less than 0.95T. Also the excursion time of the electron is different for the two cases. In the long trajectories the electron remains in the continuum longer than 0.65T(0.95T - 0.3T = 0.65T) and for short trajectories the excursion time $(t_r - t_i)$ is less than 0.65T.

The two different trajectory classes differ in continuum dynamics because of the different excursion times. This leads to difference in the properties of the emitted

light. Also from the plot we can see that for long trajectories when we increase the return time then the frequency (K.E in the plot) of the emitted light decreases whereas for the short trajectories the frequency increases as the return increases. That defines the chirp [18] of the pulse. Long trajectories have negatively chirped pulses whereas the short ones have positively chirped light.

Chapter 3

Experimental setup designing : Basic calculations and spectrometer model

When the harmonics are generated in the gas-cell they travel all together with each other and the original laser frequency in a common direction along a common axis. To detect different harmonics we have to remove the original frequency and separate different wavelengths. This can be done with the help of a diffraction grating [17]. Grating is an optical component with grooves on it. It also has special material such as gold as the coating because its hard to generate sufficient reflection in the XUV range. When light falls on a grating it splits the light into several directions depending upon the wavelengths of the different components of the spectrum and the spacing between the grooves. These separated harmonics can be detected by the MCP detectors [5]. The basic equation [11] for the reflection of the light from a grating is:

$$\sin(\alpha) + \sin(\beta) = \frac{m\lambda}{d} \tag{3.1}$$

where α is the angle of incidence, β is the angle of reflection, λ is the wavelength and d is the groove spacing. We have plotted this equation as wavelength(λ) v/s angle of reflection(β) (Fig 3.1). Here d is chosen to be $^{1}/_{600}$ mm and the angle of incidence is taken as 80⁰ (grazing angle makes sure greater reflection). And the allowed order of reflection are 1, 2, 3, 4 and 5. From this plot we can see that different wavelengths are coming on some common angle for different orders of reflection. It means that different harmonics can get diffracted at a common angle towards the MCP screen by any standard grating. In fact the wavelengths which are integer multiple of some original wavelength appear at the same reflection angle. But that defies the purpose of the grating all together.



Figure 3.1: Wavelength v/s angle of reflection for different orders

To check this overlapping of wavelengths we have to choose an angular range of reflection such that any two or more different wavelengths won't come together at any angle belonging to that range. This causes restriction on the choice of wavelength range also. For instance if we choose to capture an angular range of $69^{0}(1.21 \text{ rad})$ to $74^{0}(1.29 \text{ rad})$ then any wavelengths which are greater than 42 nm and less than 84 nm do not overlap. For a fundamental laser pulse having wavelength of 800 nm the $11^{th}, 13^{th}, 15^{th}, 17^{th}$ and 19^{th} harmonics belong to the same wavelength window. That is how different harmonics are separated in the spectrometer and the position of the wavelength detection screen(MCP screen) is decided based on the selected angular range. Also the knowledge of the generated harmonics from the theory is crucial to select the wavelength window. For our spectrometer the angular range that's needed is 69^{0} to 76^{0} .

3.1 Design of the spectrometer

An emblematical ray diagram of the path of the harmonics inside the spectrometer looks like Fig 3.2. Harmonics start from the gas jet and they get reflected from a



Figure 3.2: Schematic ray diagram of path of light inside the spectrometer

converging mirror to fall an a grating which diffracts them over a MCP screen. The purpose of the mirror is to converge the spectrum over the detecting screen. The grating in between doesn't affect the converging process except different (angularly separated) harmonics get converged at different points on the MCP screen. The distance between the gas jet and the mirror is set as 28 cm and the mirror is placed such that the angle of incidence is 80[°] (grazing angle for high reflectivity). The focal length of the mirror has to be such that its tangential focus lies on the MCP screen. For tangential incidence the focal length is $\cos(\alpha)$ times the original focal length, where α is the angle of incidence which is 80[°] in this case. The screen is (40 + 8) cm apart from the mirror. As we know that the angular range of the harmonics which are reflected from the grating is 69[°] to 76[°] degree, the area (length) of the MCP screen should be enough to capture all the harmonics. A 4.884 cm long screen works well for this angular range if placed at a distance of 40 cm from the grating.

We have created a preliminary design (Fig 3.3) of the spectrometer in the Solidworks software based on these chosen geometrical parameters. The lengths and the angles are not shown in the design for the sake of clarity. We have a laser system of 25 fs and 2 mJ laser pulses. These pulses fall on the gas jet filled with argon gas kept at a pressure of approx. 65 torr [19]. Then the generated harmonics get reflected by a mirror which has 276.5 cm of focal length. Then after diffraction by a $25 \times 25 \text{ mm}^2$ grating which has grove distance of $1/_{600} \text{ mm}$ they get detected on the MCP screen. There is a processing unit attached to the screen which creates the spectrum on the computer screen by analyzing different wavelengths. All the different components are put inside a vacuum chamber to preserve the coherent nature of the harmonics. The shape and size of the chamber can be altered depending upon the engineering issues that might appear during the actual setup. Removable glass windows must be established to adjust and calibrate the setup.



Figure 3.3: Proposed preliminary experimental setup

Conclusions

It's been only about 25 years since the discovery of this phenomenon. So this is a pretty new field of science and so much work has been done in a short period. And a lot has to be done for better understanding of the phenomenon and thus improving the experimental techniques. High harmonic generation can be a good source of attosecond pulses because the obtained spectrum from the process is composed of higher phase locked frequencies over a large bandwidth. We discussed the coherence condition for the generation of short duration pulses in the first chapter and understood the semi-classical model for the process. We performed the classical calculations which can calculate the value of cutoff that is consistent with the experimental values. Although this model has some limitations such as we can't explain the typical plateau that is visible in any harmonic spectra but the model is commonly excepted and supplies valuable intuitive and quantitative inputs to the theory.

Gratings are an important component of any spectrometer. Nowadays we can eliminate the converging mirror from the design as there is a type of grating which is available in the market known as the 'concave holographic grating' which can act as both the dispersive element and the focusing element. We have virtually made the setup in the Solidworks software. And it's expected that we'll bring the actual setup into existence within a year. The process of purchasing different components is under progress

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Appendix A

Matlab Programs

TRAJECTORIES OF THE ELECTRON FOR DIFFERENT TUNNELING TIMES(Ti)

```
t = 2*pi*(0:0.01:1); % define the t values
figure, hold on % open a new figure window with overlay
% contours are reflected about origin
for Ti = 0:0.1:pi % Ti is tunneling time
    syms t
    ezplot(sin(Ti)-sin(-t)+(-t-Ti)*cos(Ti)) % trajectories
end
ezplot(2*sin(-t))
ezplot(0*t)
t=[0 0]; x=[-100 100]; plot(t,x)
axis([-2*pi, 2*pi, -5, 5])
hold off
```

RETURNING KINETIC ENERGY AS A FUNCTION OF RETURNING TIME (${\rm Tr}\,)$

```
clear all
figure, hold on
for Ti = 0:0.001:pi % Ti is tunneling time
x = @(t) sin(t)-sin(Ti)-(t-Ti)*cos(Ti); % position
E = 2(cos(Tr)-cos(Ti))^2; % Kinetic Energy in units of ponderomotive energy
Ti;
Tr;
plot(Tr/(2*pi),E)
end
Tr=[0.95 0.95]; E=[-10 10]; plot(Tr,E)
hold off, axis([0.4, 1.3, 0, 2])
```

SUPERPOSITION OF HARMONICS HAVING RANDOM PHASES

syms t hold on

```
Y = 0;
for W = 1:2:29 % W is angular frequency
    ph = 2*pi*rand(1); % random phase
    Y = Y + cos(W*t + ph); % wave superposition
end
    ezplot(Y*Y); % Intensity
    ezplot(0*t)
hold off
```

SUPERPOSITION OF PHASE LOCKED HARMONICS

syms t
hold on
Y = 0;
for W = 1:2:29 % W is angular frequency; phase difference is zero
Y = Y + cos(W*t); % superposition of waves
end
ezplot(Y*Y); % Intensity

```
ezplot(0*t)
hold off
```

GRATING: OVERLAPPING WAVELENGTHS AS A FUNCTION OF ANGLE OF REFLECTION

```
figure , hold on % open a new figure window with overlay
% contours are reflected about origin
for m = 1:1:5
    d = 1000000/600 ; % grove spacing is 1/600 mm
    syms B
    ezplot((d/m)*(sin((160/360)*pi)-sin(B))); % Alpha is 80 deg
end
ezplot(0*B)
B=[0 0]; x=[-10 10]; plot(B,x)
B=[1.2043 1.2043]; x=[-100 200]; plot(B,x)
B=[1.3265 1.3265]; x=[-100 200]; plot(B,x)
B=[0 1.4]; x=[28 28]; plot(B,x)
B=[0 1.4]; x=[84 84]; plot(B,x)
B=[0 1.4]; x=[42 42]; plot(B,x)
```

```
hold off
```