DYNAMICAL EFFECTS OF BLINKING CONNECTIONS

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Certificate of Examination

This is to certify that the dissertation titled **Dynamical Effects of Blinking Connections** submitted by **Manish Yadav** (Reg. No. MS13045) for the partial fulfilment of BS-MS dual degree program of Indian Institute of Science Education and Research (IISER) Mohali, has been examined by the committee duly appointed by the Institute. The committee finds the work done by the candidate satisfactory and recommends that the report be accepted.

Dr. Abhishek Chaudhury	Dr. Rajeev Kapri	Prof. Sudeshna Sinha
		(Supervisor)

Dated: April 20, 2018

Declaration

The work presented in this dissertation has been carried out by me under the guidance of Prof. Sudeshna Sinha at the Indian Institute of Science Education and Research Mohali.

This work has not been submitted in part or in full for a degree, a diploma, or a fellowship to any other university or institute. Whenever contributions of others are involved, every effort is made to indicate this clearly, with due acknowledgement of collaborative research and discussions. This thesis is a bonafide record of original work done by me and all sources listed within have been detailed in the bibliography.

Manish Yadav
(Candidate)

Dated: April 20, 2018

In my capacity as the MS thesis supervisor of the candidate, I certify that the above statements by the candidate are true to the best of my knowledge.

Prof. Sudeshna Sinha
(Supervisor)

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Abstract

This thesis focusses on oscillation revivals in networks of nonlinear systems mediated by a common environment. Specifically, we consider groups of Landau-Stuart (LS) oscillators, in similar or distinct dynamical states, connected indirectly via a common environment. Such an environment was shown to aid the revival of suppressed oscillations at sufficiently high coupling strengths [1]. We extend this study further by considering the dynamical effects of single and multiple blinking connections. First, we consider a single blinking oscillator-environment connection in a network comprised of two groups, with two oscillators in each group. We explore different combinations of dead and oscillatory group/s. We find that when both the groups are initially in the steady-state (OD) regime, their oscillations revive when one of the connections blinks on-off. The amplitude of these oscillations increases with increasing time-period t_{pd} of blinking. When one of the groups is initially in the oscillatory regime, the revived oscillations display distortions in the waveforms of their time series due to the connection switching on-off. Further, the bifurcation diagram for local minima and maxima which showed only one minima and one maxima for static connections, now exhibits a continuum or band of minimas and maximas. For multiple blinking connections we first investigate the scenario where one group had blinking connections, while the other group has all static connections. We then go on to study the case where all oscillator-environment connections are blinking. There were two distinct cases we consider here. First we consider the links to switch on-off together (i.e. in-phase blinking connections) and secondly, the links switch on-off alternately (i.e. out of phase blinking connections). When connections of one group are blinking in-sync, the oscillations do not revive till $\epsilon \simeq 1.4$. On the other hand, the oscillations revive quickly if the connections alternately blink on-off.

Chapter 1

Introduction

Mean-field diffusive coupling in nonlinear oscillators leads to a transition from oscillatory to amplitude death (AD) regime in the parameter space of the coupling strength and mean-field control parameter [2]. A network containing such various groups of nonlinear oscillators in different dynamical states (each group with different control parameters) can be defined. A common environment is used to connect these groups of oscillators (G_1 and G_2) thereby making a complete network structure [1]. The entire network can be represented by Eq.(1.1).

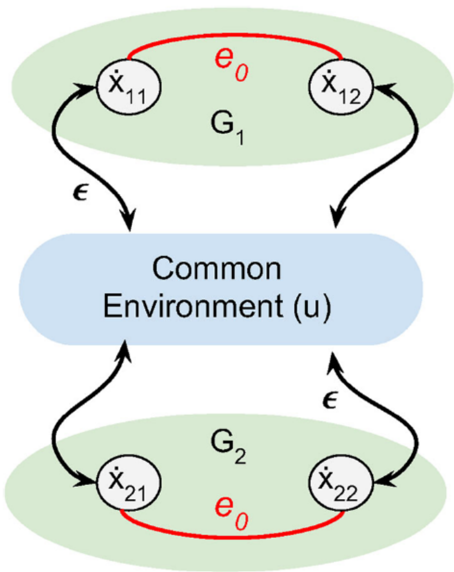

Figure 1.1: Schematic of the described network with $M, N = 2$.

$$\dot{X}_{i,j} = F(X_{i,j}) + e_o\beta(Q_i\bar{X}_i - X_{i,j}) + \epsilon\alpha_i u,$$
$$\dot{u} = -ku + \frac{\epsilon}{NM}\sum_{i=1}^{M}\alpha_i^T\sum_{j=1}^{N}X_{i,j} \tag{1.1}$$

where $i = 1, 2, ...M$ and $j = 1, 2, ...N$, i.e. there are M number of groups in this network with each group having N oscillators. The coupling strength e_o defines the mean-field coupling within the group and Q_i is the control parameter determining the weight of the mean-field. Coupling strength ϵ connects these independent groups of oscillators via the common environment. A schematic diagram is given in Fig.(1.1) to elucidate the overall structure of this network.

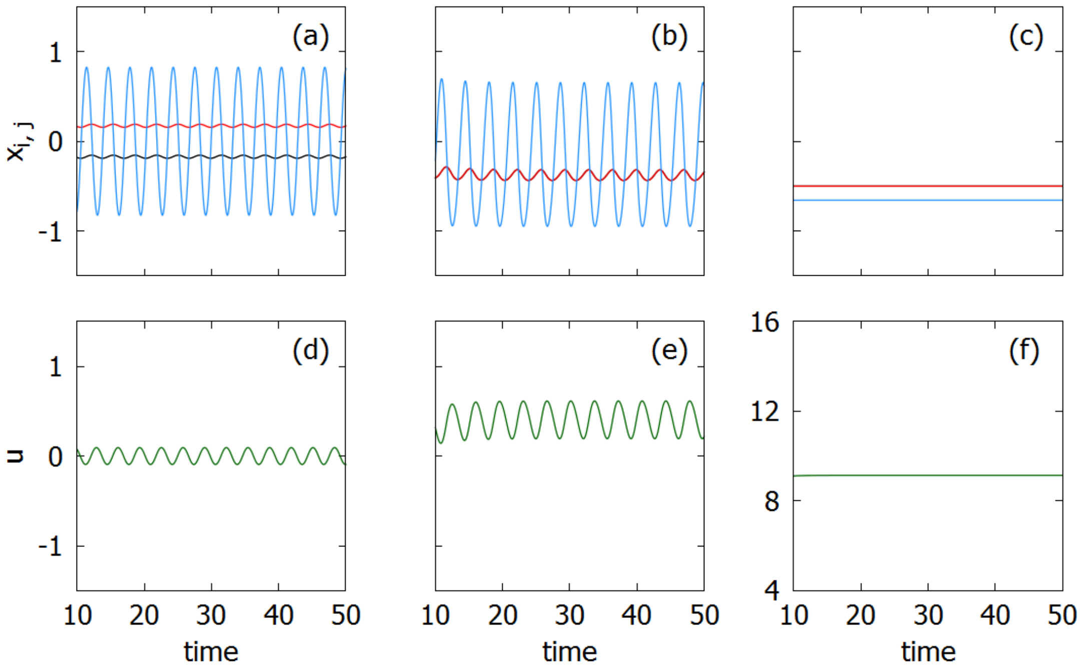

Figure 1.2: Network without blinking: time series with $\epsilon_o = 6$ (a) G_1 at $q_1 = 0.4$, (b) G_2 at $q_2 = 0.6$ and common environment for $\epsilon = 0.9$. Corresponding figures (d), (e) and (f) for 1 blinking connection in G_1(black) with blinking time period $t_{pd} = 2$.

Stuart-Landau (LS) limit cycle oscillators with intrinsic-frequency $\omega = 2$ have been employed to carry out the numerical results given by Eq.(1.2).

$$
\begin{aligned}
\dot{x}_{i,j} &= (1 - x_{i,j}^2 - y_{i,j}^2)x_{i,j} - \omega_i y_{i,j} + e_o(Q_i \bar{x}_i - x_{i,j}) \\
\dot{y}_{i,j} &= (1 - x_{i,j}^2 - y_{i,j}^2)y_{i,j} + \omega_i x_{i,j} + \epsilon u \\
\dot{u} &= -ku + \epsilon \bar{y}
\end{aligned}
\tag{1.2}
$$

Here, the choice of parameters e_o and Q_i allows us to put each of the individual group either in the oscillatory state or steady state (HSS/IHSS). To see the competition between groups of different dynamical states parameters has been chosen (e_o) such that G_1 will remain in oscillation death state (IHSS) with $Q_1 = 0.4$ and G_2 in the oscillatory state with $Q_2 = 0.9$. As a result of the indirect interaction, oscillations revive in the group G_1 which initially had fixed point dynamics shown in Fig.1.2(a),(b) for different values of ϵ. If ϵ is increased too much, revived oscillations again die out, but this time it drags the healthy group G_2 with itself towards the steady state (Fig.1.2(c)).

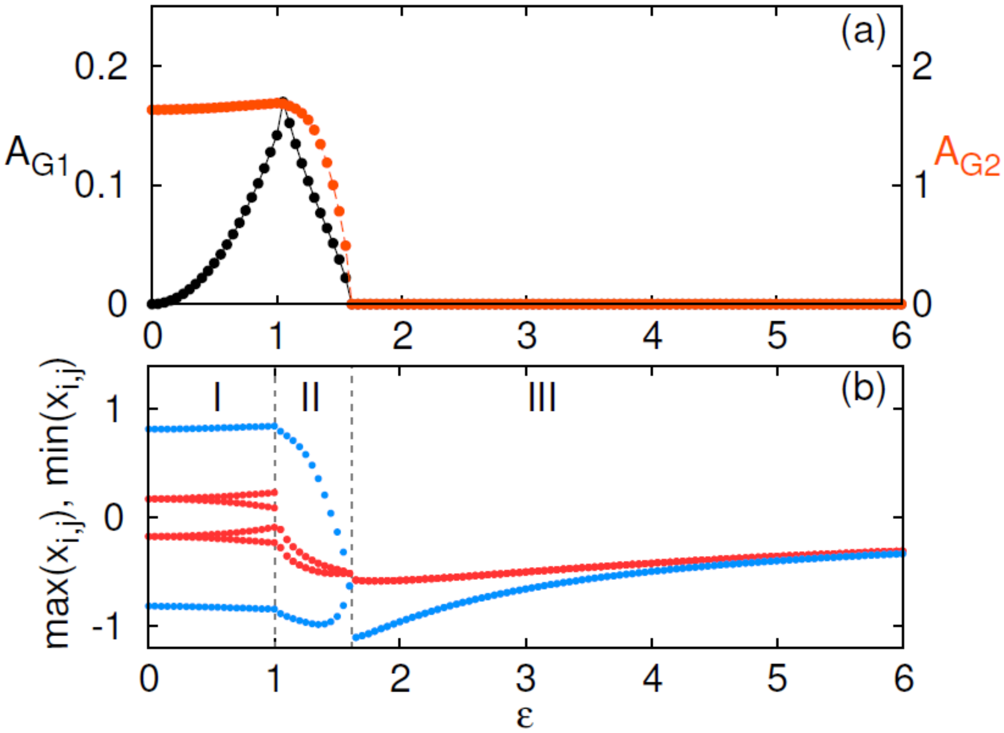

Figure 1.3: Network without blinking: Bifurcation diagram showing Amplitude (Eq.2.4) and local minima-maxima x-variable of revived oscillations changing with ϵ.

A bifurcation diagram has been given in Fig.1.3(b) to understand the role of ϵ more clearly which shows the local minima and maxima of the x variable of all the oscillators of both the groups. Here 2 transition points are clearly visible; one at $\epsilon = 1$ and another at $\epsilon \simeq 1.7$. First transition point makes both the revived oscillators (red) synchronized, and another one is the transition towards the steady state of the entire network. Fig.1.3(a) shows the amplitude of the revived oscillations (black) which keeps increasing till first transition point, then keeps decreasing till the second transition where all the oscillations die out.

Extending the results mentioned above, in this thesis we study the effect of *time-varying links* on the spatiotemporal patterns. Specifically, we consider one or more connections with the environment to be periodically *blinking*, i.e. the links to the environment of some of the oscillators are switched on and off periodically. We look for the effects of these blinking links on pattern formation in the entire network.

Chapter 2

Network with Blinking Connections with the Environment

First, we describe the network with blinking connections, mathematically given by the following generalized dynamical equation:

$$\begin{aligned}
\dot{x}_{i,j} &= f(x_{i,j}, y_{i,j}) + e_o(Q_i\bar{x}_i - x_{i,j}) \\
\dot{y}_{i,j} &= g(x_{i,j}, y_{i,j}) + \epsilon\Gamma_{i,j}(t_{pd})u \\
\dot{u} &= -ku + \frac{\epsilon}{MN}\langle\Gamma, y\rangle_F
\end{aligned} \tag{2.1}$$

where all the parameters are as defined in the previous section. $\Gamma(t_{pd})$ is the connectivity matrix with elements $\Gamma_{i,j}(t_{pd})$ which is either 1 or a square wave function oscillating between 0 and 1 with time period t_{pd}. $\langle\Gamma, y\rangle_F$ is the Frobenius inner product which is a component-wise inner product of two matrices (Γ and y) and returns the sum of all elements of the resulting matrix.

We consider two groups of Stuart-Landau(LS) oscillators connected via common environment given by Eq.(2.2) and can be schematically represented by Fig.(2.1).

$$\begin{aligned}
\dot{x}_{i,j} &= (1 - x_{i,j}^2 - y_{i,j}^2)x_{i,j} - \omega_i y_{i,j} + e_o(Q_i\bar{x}_i - x_{i,j}) \\
\dot{y}_{i,j} &= (1 - x_{i,j}^2 - y_{i,j}^2)y_{i,j} + \omega_i x_{i,j} + \epsilon\Gamma_{i,j}(t_{pd})u \\
\dot{u} &= -ku + \frac{\epsilon}{4}\langle\Gamma, y\rangle_F
\end{aligned} \tag{2.2}$$

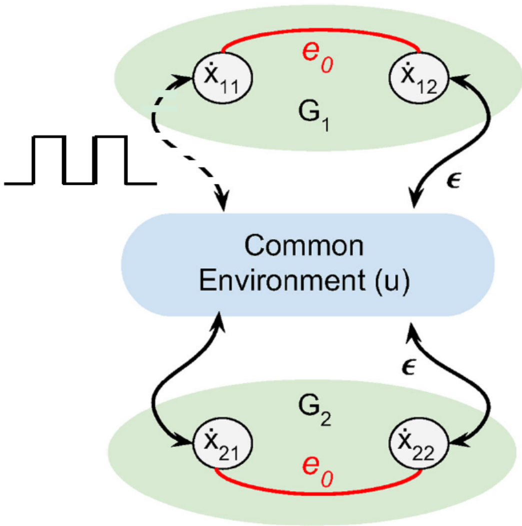

Figure 2.1: Schematic of the described network with $M, N = 2$.

Here matrix Γ can be given by Eq.2.3 where only its first element $\Gamma_{1,1}$ is blinking with time-period t_{pd} and rest are 1, i.e. one link switches on-off, and the rest are static connections permanently connecting the corresponding oscillators with the environment.

$$\Gamma(t_{pd}) = \begin{pmatrix} \Gamma_{1,1} & 1 \\ 1 & 1 \end{pmatrix}_{2\times 2} \tag{2.3}$$

We consider values of e_o and Q_i for which each group of oscillators occur in different dynamical states: oscillatory/active or dead/inactive (which could be an AD or OD). We consider the following combinations of Dead and Oscillatory groups with blinking connections:

G_1 (with one blinking connection)	G_2 (with all static links)
OD	OD
OD	Oscillatory
Oscillatory	OD
Oscillatory	Oscillatory

2.1 G_1 and G_2 in OD(IHSS)-State with one blinking connection

Here one oscillator from the group G_1 has a blinking connection with the environment, with the period of blinking $t_{pd} = 2$. We study the system with representative values of $\epsilon = 1.5$ and $e_o = 6$, where both groups (if independent) will yield Oscillation Death (OD) states at $Q_1 = 0.4$ and $Q_2 = 0.6$.

On comparing with the case of static connections between the groups and the environment (cf. Fig.2.2a,b and c), we observe that oscillations arise when one of the connections with the environment starts blinking (cf. Fig.2.2d). Further oscillations also emerge in the environment (cf. Fig.2.2f). This oscillatory influence reaches group G_2, which has only static links, via coupling through the common environment, and this group also starts to oscillate.

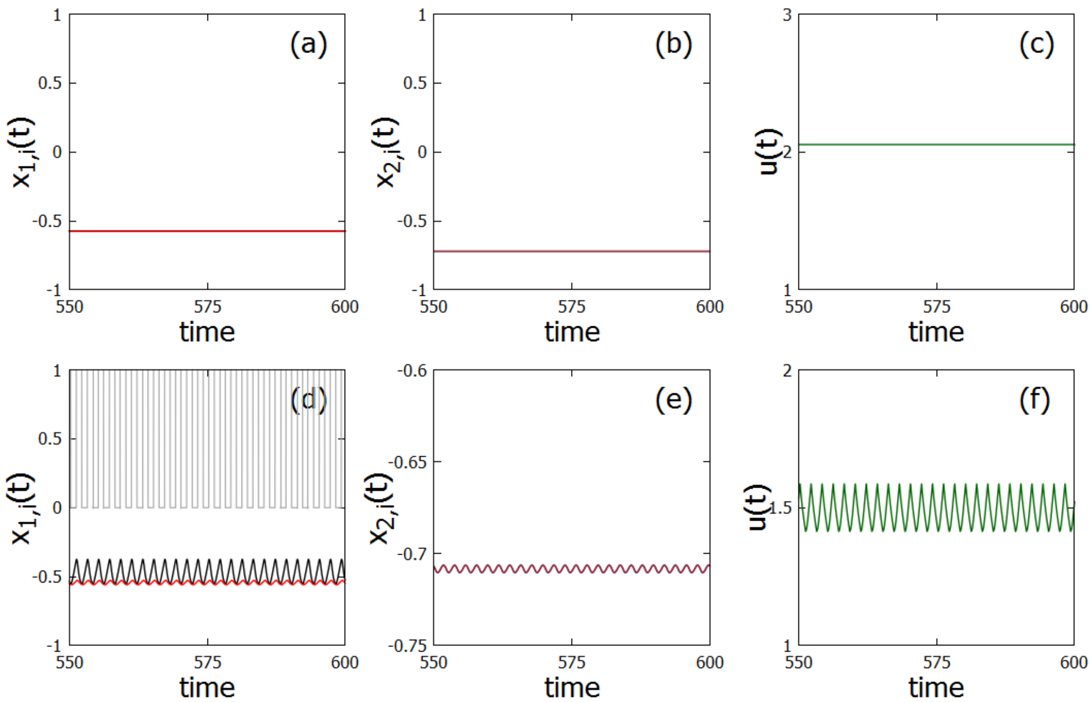

Figure 2.2: Time series of the network without blinking connection, for $\epsilon = 1.5$ and $\epsilon_o = 6$ in Eqn. 2.2. Here (a) group G_1 with $q_1 = 0.4$, (b) group G_2 with $q_2 = 0.6$, and (c) environment u. Time series of the network with one blinking connection: (d), (e) and (f) for the case where group G_1 (black) has one blinking connection, with blinking time period $t_{pd} = 2$.

To obtain a clearer picture of the changing dynamics of the oscillators, we plotted the bifurcation diagrams of the oscillator with blinking connections, $X_{1,1}$, in Fig. 2.3 Row-1(a). Specifically, the figure shows the local minima and maxima of the x-variable of oscillator $X_{1,1}$ from the group G_1. Up to coupling strength $\epsilon \simeq 2$, period-1 oscillations appeared in $X_{1,1}$. After that, the period of oscillations increased, and more significantly the effect of the blinking link is now apparent even in the oscillator $X_{1,2}$ which has a static connection with the environment (cf. Fig. 2.3 Row-1(b)).

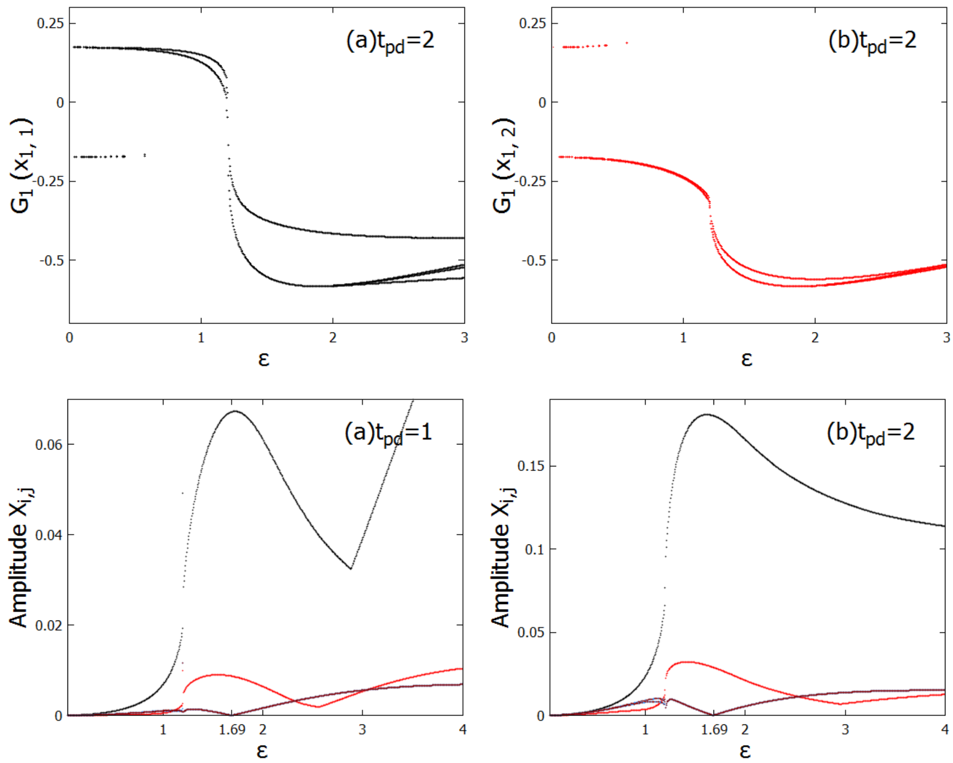

Figure 2.3: Row-1: Bifurcation diagram showing local minima and maxima of (a) $X_{1,1}$ oscillator with blinking connection and (b) $X_{1,2}$ oscillator with fixed connection with the common medium. Row-2: Amplitude of $X_{1,1}$(black):blinking, $X_{1,2}$(red):non-blinking and of G_2(brown) for (a) $t_{pd} = 1$ and (b) $t_{pd} = 2$. Other parameters are $e_o = 6, Q_1 = 0.4$ and $Q_2 = 0.6$ in Eqn. 2.2.

Then we study the *amplitude* of the x-variables of the oscillators, from both groups, given by equation2.4:

$$\text{Amplitude } X_{i,j} = |\text{ Global maxima - Global minima }| \tag{2.4}$$

The Amplitude $X_{i,j}$ of x-variable is plotted in Fig. 2.3 Row-2 for different ϵ

values. For blinking time period $t_{pd} = 1$ the amplitude of $X_{1,1}$ (i.e. the oscillator with blinking environmental connection) and also $X_{1,2}$ (i.e. the oscillator with static connection) reaches its maximum at $\epsilon \simeq 1.69$ (cf. Fig. 2.3 Row-2(a)). However, at this ϵ, the amplitudes of the oscillators in group G_2 becomes close to 0. If we look at Fig. 1.3(a) and (b), $\epsilon \simeq 1.69$ is the point of transition from the region of revived oscillations to the global steady state region. Further, this ϵ value remains constant under varying blinking time-periods (such as $t_{pd} = 2$ shown in Fig. 2.3 Row-2(b)).

2.2 G_1 in OD-State (IHSS) with one Blinking Connection and G_2 in Oscillatory State

Here the dead group G_1 ($q_1 = 0.4$) has one blinking connection and the oscillatory group G_2 ($q_2 = 0.9$) has static links, with $\epsilon_o = 6$. The time series of group G_1 for $\epsilon = 0.9$ is shown in Fig.2.4(d). The oscillator with blinking connection $X_{1,1}$ is shown in black colour, and it exhibits a modified waveform as a result of its blinking connection with the common medium. Due to the altered dynamics of $X_{1,1}$, the dynamics of $X_{1,2}$ is also affected, as it is connected directly to $X_{1,1}$ through mean-field coupling within the group G_1. This can be more clearly visualized in Fig.2.4(e) which displays the phase portraits, where a fuzzy band starts to appear in $X_{1,2}$ (red), representing irregularity in the period of oscillations.

A bifurcation diagram can be drawn showing local minima and maxima of the time-series of each of the oscillators $X_{i,j}$. Such a local minimum and maximum could be obtained from the time-series using the code given in **Appendix-**A. Bifurcation diagram shown in Fig. 2.5 for $0.1 \leq \epsilon \leq 3$ with different blinking time periods $t_{pd} = 1$ and 5. The case of no blinking connections is displayed as a reference. When $X_{1,1}$ has a blinking connection with time-period $t_{pd} = 1$, for higher ϵ values (up to $\epsilon < 1.7$), the period of oscillations increases. For $\epsilon > 1.7$ the entire network collapses into a steady state region (see Fig. 1.3).

2.3 G_1 in Oscillatory State with one Blinking Connection and G_2 in OD-State (IHSS)

As in the previous section, here we again consider a network containing 2-groups G_1 and G_2, where one is in the oscillatory state and another is in the steady state. Unlike in the section here, we consider 1-blinking connection with the oscillatory

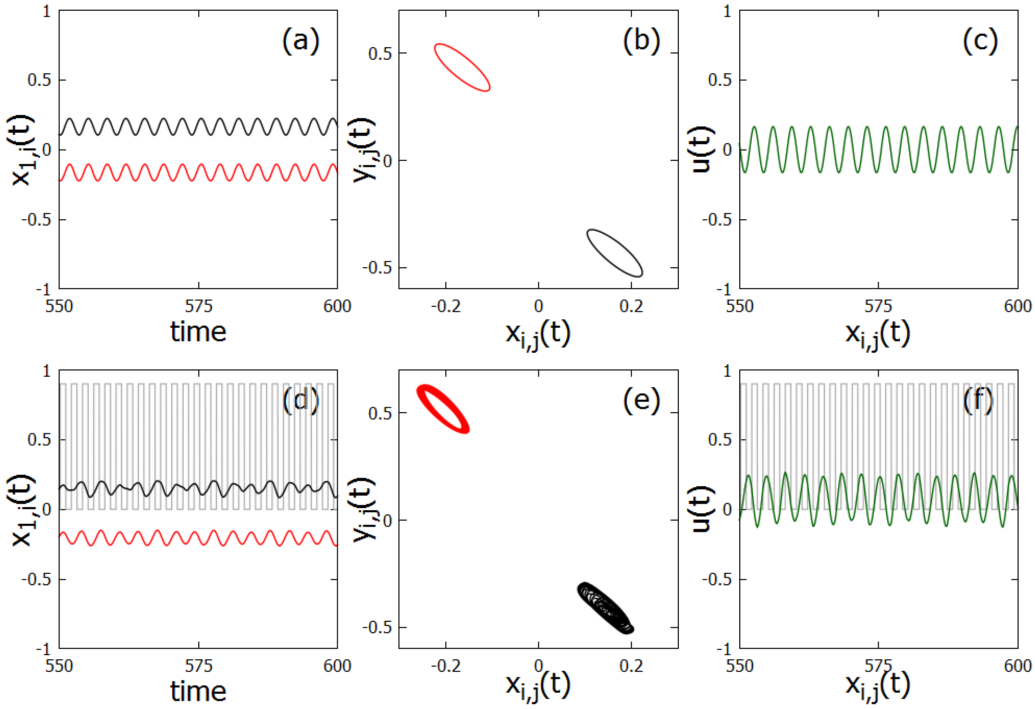

Figure 2.4: Time series of a network without blinking connections connections, with $\epsilon = 0.9$ and $\epsilon_o = 6$ in Eqn. 2.2. Here (a) group G_1 with $q_1 = 0.4$, (b) group G_2 with $q_2 = 0.9$, and (c) environment u. Corresponding figures (d), (e) and (f), for the case where group G_1 (black) has one blinking connection, with blinking time period $t_{pd} = 2$.

group G_1 with $Q_1 = 0.9$ to have one blinking connection, while group G_2 with $Q_2 = 0.4$ has only static connections. Parameter $e_o = 6$ in the entire network structure.

The revived oscillations of G_2 show increased time-period of oscillations due to the blinking connection of the common environment with G_1 (which is in the oscillatory regime). This is evident from Fig. 2.6 for blinking time-period $t_{pd} = 1$. As we increase the blinking time-period to 2, the pattern of oscillations changes drastically in group G_2. This difference in patterns can be seen easily by comparison with Fig. 2.6 (b) and (e). The common environment u, which is exponentially decaying in nature when uncoupled to the oscillator groups, develops regular multi-periodic oscillations due to the blinking connection of $X_{1,1}$ with it (see Fig. 2.6 (c) and (f)).

The bifurcation diagram showing local minima and maxima of X-variable is plotted in Fig. 2.7 for blinking time-period $t_{pd} = 1$ and 5. At $t_{pd} = 1$ the

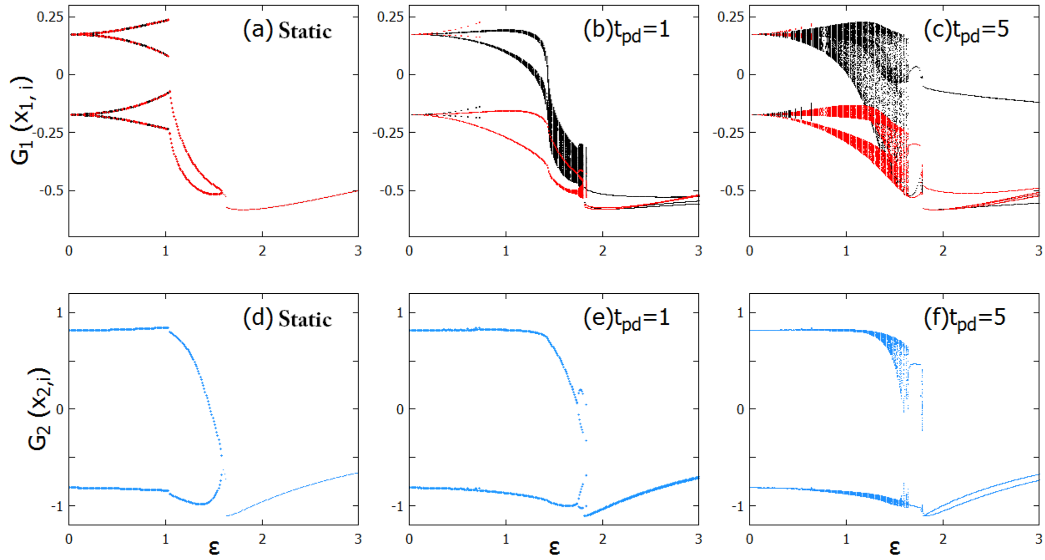

Figure 2.5: Bifurcation diagrams displaying the local minima and maxima of x-variables in the network for (a,b,c) group G_1, and (d,e,f) group G_2, for the static case (a,d), for $t_{pd} = 1$ (b,e) and $t_{pd} = 5$ (c,f).

bifurcation diagram of G_2 (with static connections) shows almost no fuzziness, nor any major modification in temporal patterns. However, the transition point to steady state, which was observed earlier to be at $\epsilon \simeq 1.7$, reduces to $\epsilon \simeq 1.3$ (cf. Fig. 2.7(b)).

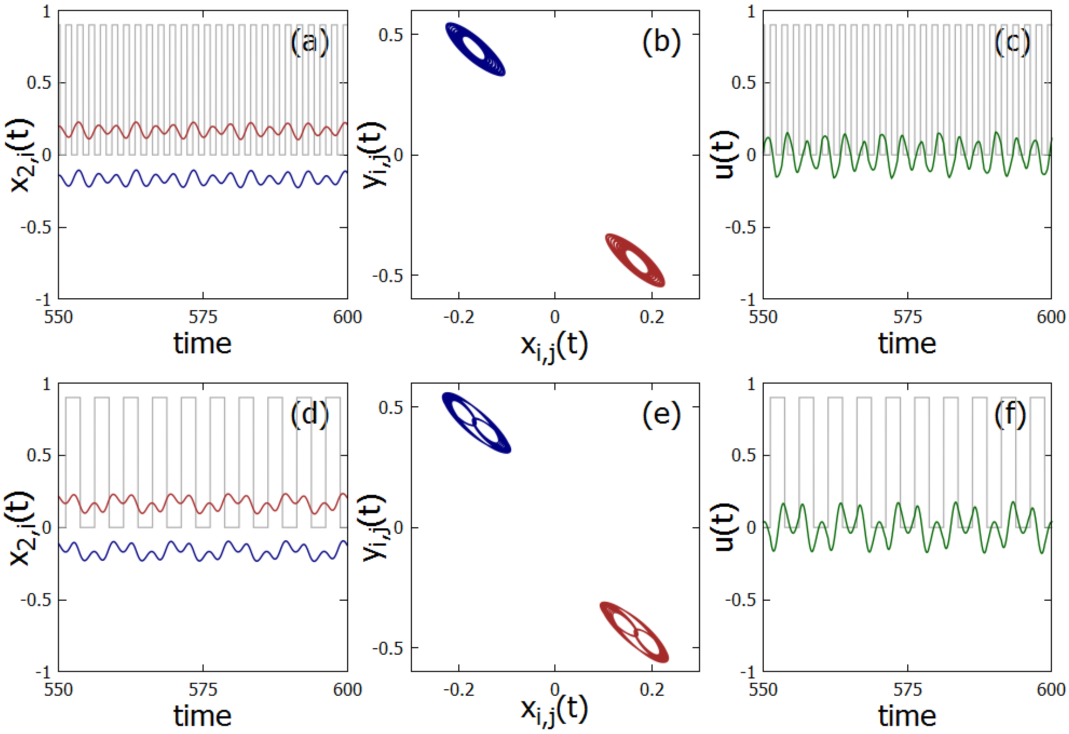

Figure 2.6: Network with one blinking connection in the Oscillatory Group: time series with $e_o = 6$, G_1 at $q_1 = 0.9$ with blinking time period $t_{pd} = 1$ in (a),(b) and (c) for $\epsilon = 0.9$. Similarly, time series with $e_o = 6$, G_2 at $q_2 = 0.4$ with blinking time period $t_{pd} = 2$ in (d),(e) and (f) for $\epsilon = 0.9$.

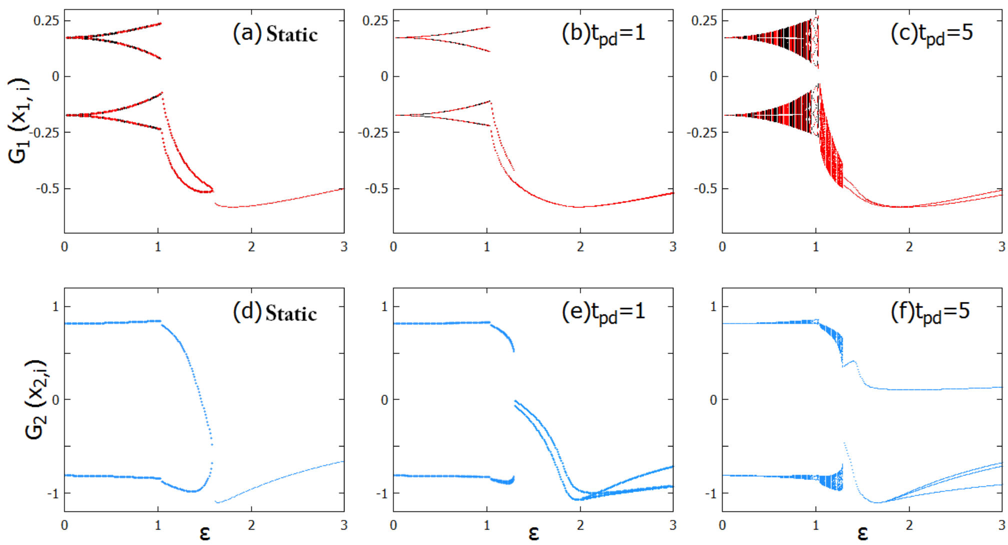

Figure 2.7: Bifurcation diagram of local minima and maxima of x-variables in the network for blinking time periods: (a) $G_1(q_1 = 0.9)$, (d) $G_2(q_2 = 0.4)$ for Static connections. Similarly (b) and (e) at $t_{pd} = 1$ and (c) and (f) at $t_{pd} = 5$ for 1-blinking connection in G_1.

Chapter 3

Network with Multiple Blinking Connections with the Environment

Following the lines of the previous chapter, we introduce blinking effects in both the connections of $X_{1,1}$ and $X_{1,2}$ of the group G_1 or/and G_2 with the common environment. In the subsequent sections we explore the following cases: (i) both groups are initially in the steady-state (OD) regime, and (ii) one of the groups is initially in the steady-state (OD) regime.

3.1 G_1 and G_2 in OD-state with multiple-blinking connections

To study the effect of multiple-blinking connections when both oscillator groups G_1 (with $Q_1 = 0.4$) and G_2 (with $Q_2 = 0.6$) are in Oscillation Death (OD) state, we first redefine the connectivity matrix Γ:

$$\Gamma(t_{pd}) = \begin{pmatrix} \Gamma_{1,1} & \Gamma_{1,2} \\ 1 & 1 \end{pmatrix}_{2\times 2} \tag{3.1}$$

which shows connections of $X_{1,1}$ and $X_{1,2}$ to be blinking with the environment while rest of the connections (i.e. of G_2) are static. In this section, we study two cases:
(a) when both the connections are blinking together, i.e. with zero phase difference and

(b) when they are blinking alternatively, i.e. with a phase difference of $\pi/2$ between them.

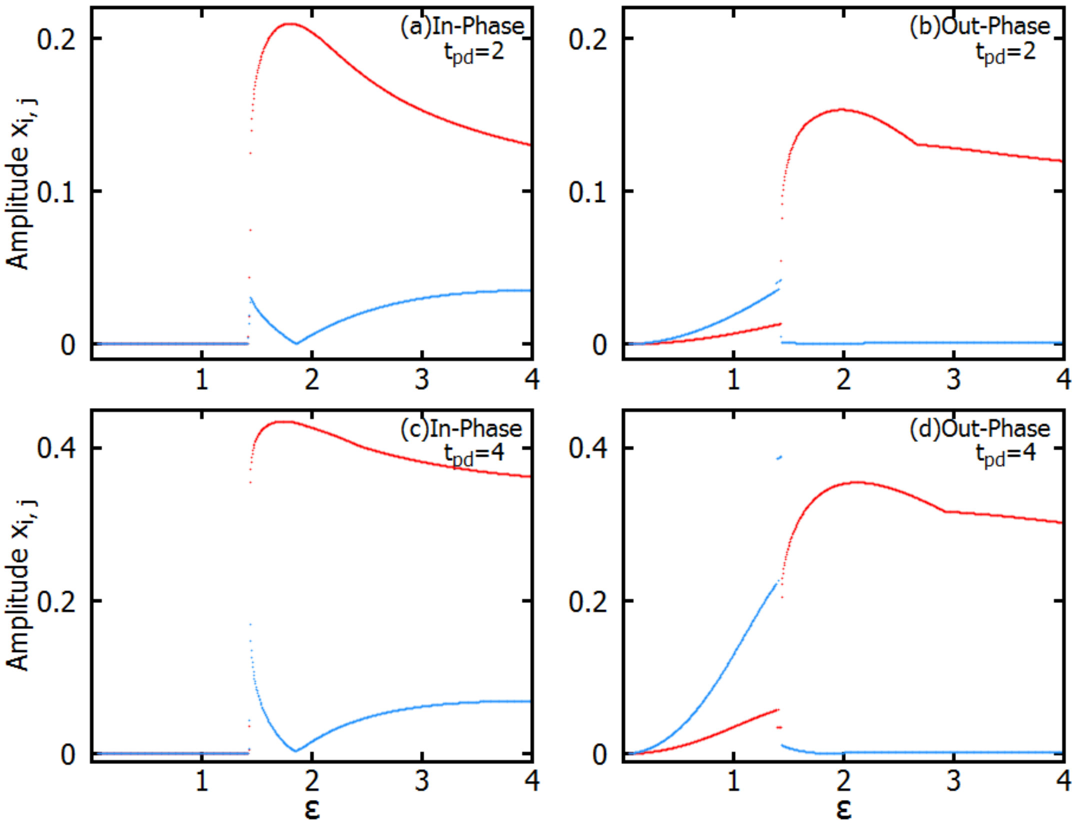

Figure 3.1: Amplitude of G_1 (red) where both connections are blinking. Here (a) and (c) are the cases with zero-phase difference between the blinking connections, while (b) and (d) have a phase difference of $\pi/2$ between the blinking connections. G_2-oscillators are shown with blue color. The blinking time-period $t_{pd} = 2$ in (a),(b) and $t_{pd} = 4$ in (c),(d).

We observed that when both the connections blink in synchronization, i.e. with the zero-phase difference between them, the entire network remains in steady-state dynamics up to $\epsilon \simeq 1.4$ (see Fig. 3.1 (a) and (c)). For $\epsilon > 1.4$ the oscillations revive, and the G_1 oscillators (both with blinking connections) have higher amplitude oscillations as compared to that of the oscillators in group G_2. However, when the connections blink alternately, i.e. with a $\pi/2$-phase difference, oscillations arise in both the groups with the time-period of oscillation equal to that of the blinking time-period t_{pd} given in Fig.3.1 (b) and (d).

Interestingly, for $0.1 < \epsilon < 1.4$ the amplitude of G_2 (which has all-static connections) becomes more than that of G_1 (with alternate-blinking connections).

It appears that the common environment has *amplified* the oscillations arising in group G_1 due to the blinking connections. On increasing the t_{pd} from 2 to 4, the amplitude of the oscillations also increased (ref. Fig. 3.1 (b) and (d)). Further, for $\epsilon > 1.4$ there is a sudden flip in the amplitude of both the groups. Now the amplitude of group G_1 becomes large, while in group G_2 the amplitude becomes almost 0.

Next, we introduced blinking connections in G_2 also, along with the blinking connections in G_1, while keeping both groups in the OD-state. So the connectivity matrix becomes:

$$\Gamma(t_{pd}) = \begin{pmatrix} \Gamma_{1,1} & \Gamma_{1,2} \\ \Gamma_{2,1} & \Gamma_{2,2} \end{pmatrix}_{2\times 2} \tag{3.2}$$

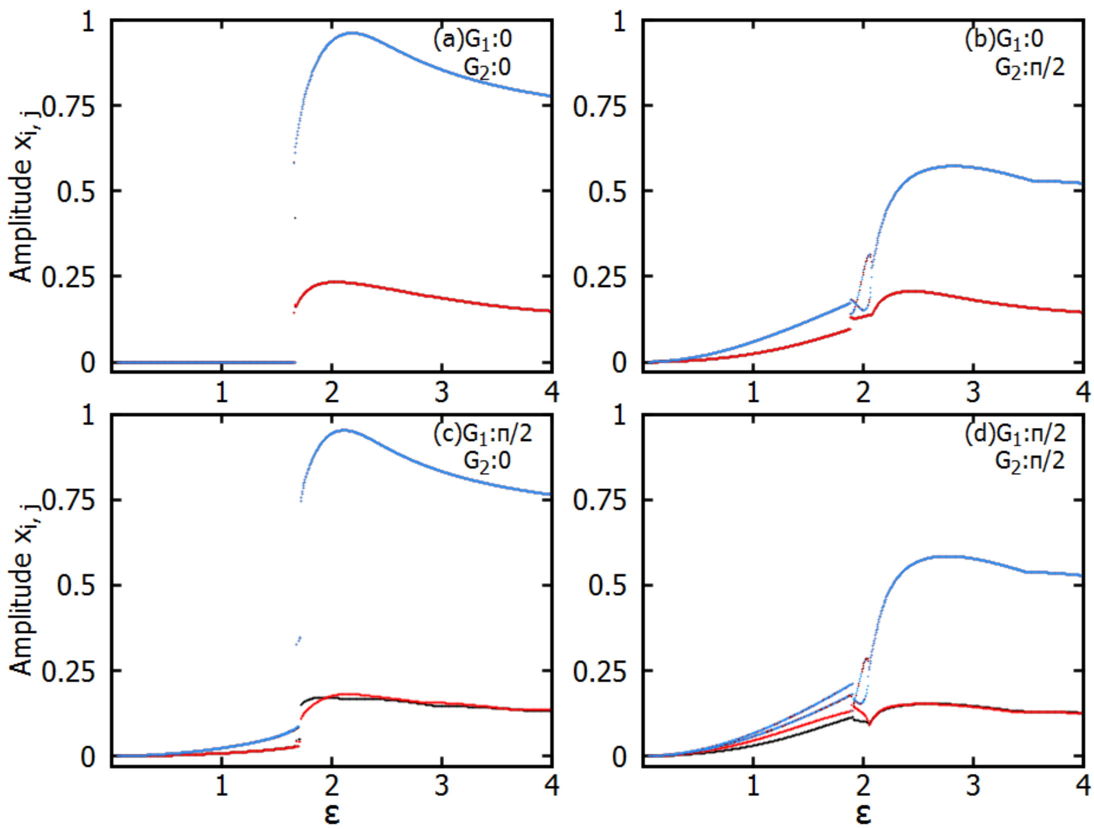

Figure 3.2: Amplitude of oscillations in group G_1 (red and black) and group G_2 (blue and purple), where both groups have blinking connections, with the blinking time-period for G_1 being $t_{pd1} = 2$, and for G_2 being $t_{pd2} = 4$. The phase difference of the blinking connections (0 or $\pi/2$ within the group) is mentioned on the top right corner of each box.

We introduced the blinking effect also in connections of G_2 for the case given in Fig.3.1 (a). So, here G_1 and G_2 both have in-sync (i.e. zero phase difference) blinking connections with the environment. The amplitude of oscillations in group G_2 still remains 0 for $0.1 < \epsilon < 1.7$. However, the transition point to revived oscillations now increases to $\epsilon \simeq 1.7$, as compared to 1.4 for the case of all-static G_2 connections. For $\epsilon > 1.7$ the amplitude of oscillations in G_2 becomes larger than that of G_1 (see Fig. 3.2 (a)). This is also observed when G_2 connections are made to blink alternately (see Fig. 3.2 (b)).

Fig. 3.1 (b) showed the case where G_1 had alternately blinking connections. Now, if we make the links in G_2 also blink on-off, the oscillators $X_{2,1}$ and $X_{2,2}$ which had zero amplitude for $\epsilon > 1.4$ show a revival of oscillations, with the amplitude of G_2 being more than that of G_1. This holds for the case of both 0 and $\pi/2$ phase difference between the blinking connections of G_2.

3.2 G_1 in OD-state with multiple-blinking connections and G_2 in Oscillatory state

Here we consider G_1 in the OD state with $Q_1 = 0.4$, with both its connections to the environment blinking on-off. Group G_2 has all static connections and is in the oscillatory state with $Q_2 = 0.9$. When both connections of G_1 blink together (in-phase), the oscillations still revive as one can expect from all-static connections[1] or for the case of one blinking connection given in Section 2.2. When both connections blink in-synchronization, an envelope develops in the revived oscillations and the waveforms of this envelope are in phase for both $X_{1,1}$ and $X_{1,2}$ (Fig. 3.3 (a)). On the other hand, as expected, when these connections blink alternately, the envelope of the waveforms of $X_{1,1}$ and $X_{1,2}$ is out-of-phase (Fig. 3.3 (b)).

The bifurcation diagram showing local minima and maxima of x-variables of $X_{1,1}$ and $X_{1,2}$ for in-phase blinking is shown in Fig. 3.3(c). A system where both connections are blinking displays a symmetric bifurcation diagram till $\epsilon \simeq 2$ (unlike the case of one blinking connection shown in Fig. 2.5). The spread in local minima and maxima becomes less when the connections blink out-of-phase.

Further, we consider the connections of G_2 with the environment to also switch on-off with the blinking time-period $t_{pd} = 1.6$, with group G_1 being in the OD-state (with $Q_1 = 0.4$) and group G_2 in an oscillatory state (with $Q_2 = 0.9$). Fig. 3.4 (a) shows the bifurcation diagram of x-variables of oscillators in group

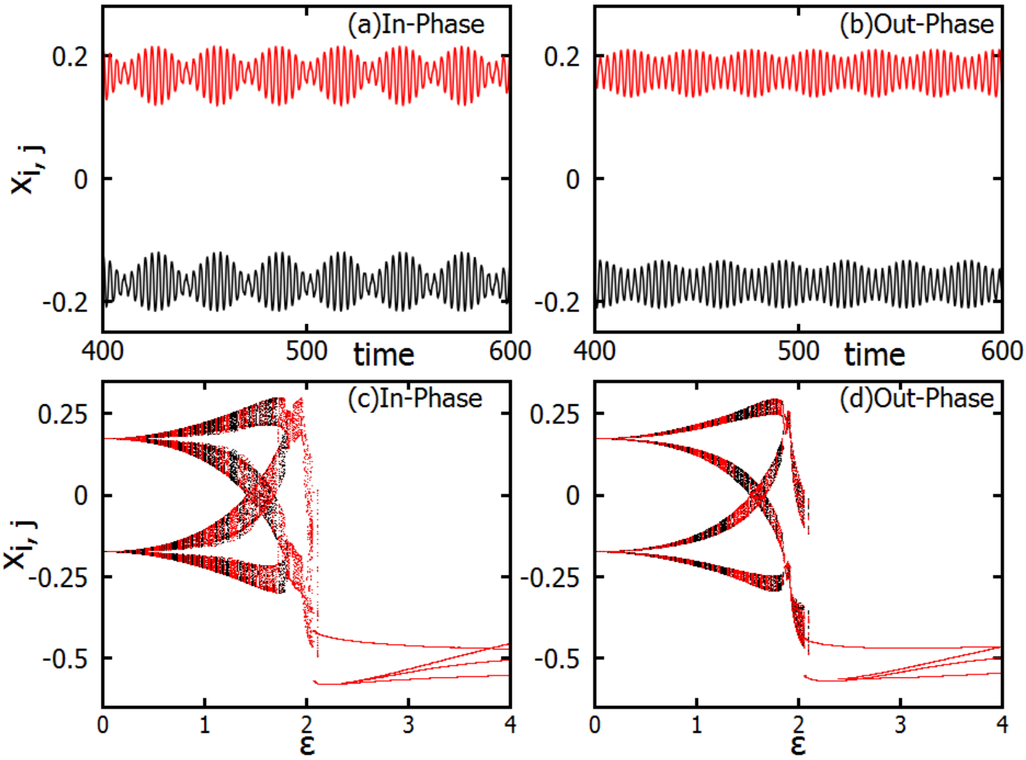

Figure 3.3: Time series for the case of $\epsilon = 0.9$, where both the oscillators of G_1 have blinking connections with (a) zero phase difference and (b) $\pi/2$-phase difference between the on-off connections. Bifurcation diagram of local minima and maxima of x-variables of oscillators in group G_1, for blinking time-period $t_{pd} = 1.6$, when there is (c) zero phase difference and (d) $\pi/2$ phase difference between the blinking connections.

G_1 when all connections are blinking on-off in-sync with each other within the group. If we compare it with the case of all-static connections (cf. Fig. 1.3b) the transition from bistable to mono-stable orbit is again clearly visible, though this occurs at higher coupling strengths $\epsilon \simeq 2$ (vis-a-vis $\epsilon \simeq 1$ in Fig. 1.3b). This transition disappeared when G_1 had only in-phase blinking connections (ref. Fig.3.3 (c)). When all the connections of G_1 and G_2 are alternately blinking within the group (Fig.3.4 (c)), the bifurcation diagram is similar to that of all-static connections (Fig.1.3 (b)), with an additional small fuzziness in the multiple local minima and maxima arising from the connections blinking on-off.

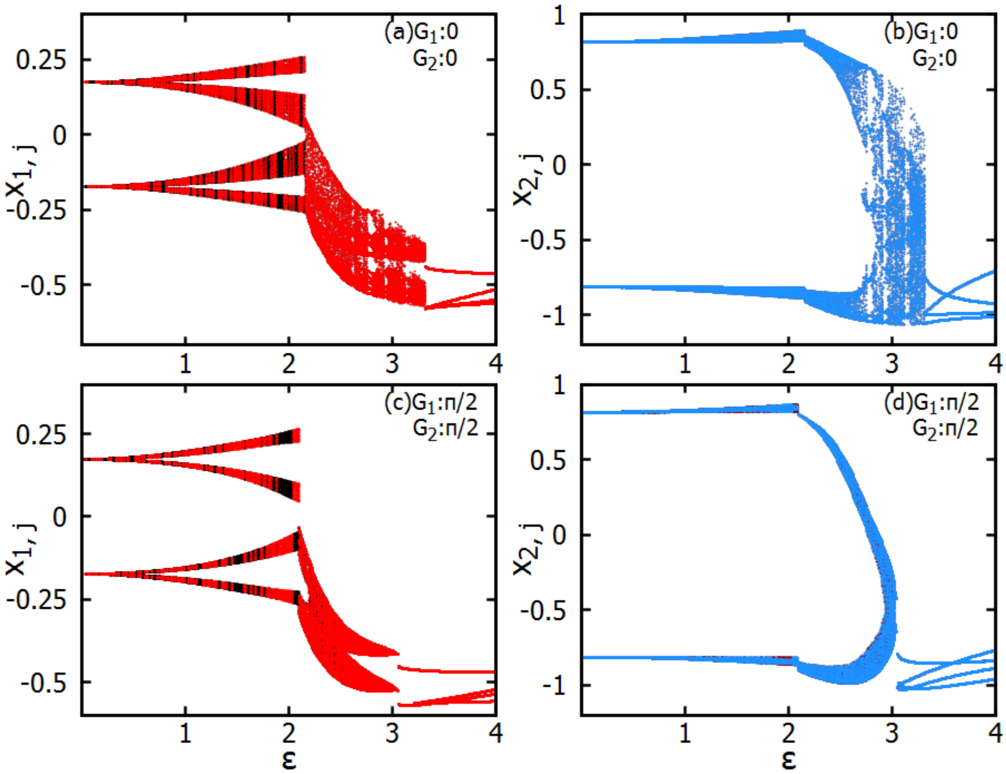

Figure 3.4: Bifurcation diagram of local minima and maxima of x-variables for blinking time-period $t_{pd} = 1.6$: when (a),(b) zero phase difference and (c),(d) $\pi/2$-phase difference between blinking connections in both G_1 (Q_1=0.4) and G_2 (Q_2=0.9).

Conclusion

The phenomenon of oscillation revival has been a trending topic of research nowadays in various fields [3, 4, 5, 6]. This thesis focuses on such oscillation revivals in networks of nonlinear systems mediated by a common environment. Specifically, we explore the dynamics of groups of Landau-Stuart (LS) limit-cycle oscillators, in similar or distinct dynamical states, connected indirectly via a damped common environment. Such an environment is shown to aid the revival of suppressed oscillations at sufficiently high coupling strengths [1]. Here we have extended this investigation further, and considered the dynamical effects of single and multiple blinking connections.

First, we started with a single blinking oscillator-environment connection in a network comprised of two groups, with two oscillators in each group. We explored different combinations of dead and oscillatory group/s. We found that when both the groups were initially in the steady-state (OD) regime, their oscillations revived when one of the connections blinked on-off. Theplitude of these oscillations increased with increasing time-period t_{pd} of blinking. When one of the groups was initially in the oscillatory regime, the revived oscillations displayed distortions in the waveforms of their time series due to the connection switching on-off. Further, the bifurcation diagram for local minima and maxima which showed only one minima and one maxima for static connections, now exhibited a continuum or band of minimas and maximas.

For multiple blinking connections we first investigated the scenario where one group had blinking connections, while the other group had all static connections. We then went on to study the case where all oscillator-environment connections were blinking. There were two distinct cases we considered here. First we consider the links to switch on-off together (i.e. in-phase blinking connections) and secondly, the links switch on-off alternately (i.e. out of phase blinking connections). When connections of one group were blinking in-sync, the oscillations do not revive till $\epsilon \simeq 1.4$. On the other hand the oscillations revive quickly if the connections switch on-off out-of-phase, i.e. the connections are alternately blinking.

In summary, we have investigated the dynamics of groups of limit cycle oscillators connected via a common external environment. Specifically we have explored the role of blinking oscillator-environment connections in the important phenomenon of oscillation revival.

Appendix A

C++ function for finding local minimas and maximas

The time-series of the oscillators can be filled into an array after leaving enough of the transients. Such an array of time-series (data[ ]) can easily be used for finding the local minima and maxima by accessing each element of that array.

```
float * gminmax(float data[])
{
        int length = 10000;
        float min[20000], max[20000];
        int c1=0, c2=0;
        float AvgMin=0, AvgMax=0, fmin, fmax;
        static float packed[2];

        for(int i=5; i<length-5; i++)
        {
                if(data[i]<= data[i-2] && data[i]<= data[i+2])
                {
                        min[c1] = data[i];
                        c1++;
                }

                if(data[i]>= data[i-2] && data[i]>= data[i+2])
                {
```

```
                        max[c2] = data[i];
                        c2++;
                }
        }

        for(int i=0; i<= c1; i++)
        {
                AvgMin += min[i];
        }
        for(int i=0; i<= c2; i++)
        {
                AvgMax += max[i];
        }

        fmin = AvgMin/(c1+1);
        fmax = AvgMax/(c2+1);

        packed[0] = fmin;
        packed[1] = fmax;

        return packed;
}
```

Appendix B

C++ function for finding Global minima and maxima

Similar to **Appendix A** the Global minimum and maximum can be found using an array of time-series (data[ ])

```
float * gminmax(float data[])
{
        int length = 10000;
        float min, max;
        min = data[0];
        max = data[0];
        static float packed[2];

        for(int i=10; i<length; i++)
        {
                if(data[i]<= min)
                {
                        min = data[i];
                }

                if(data[i]>= max)
                {
                        max = data[i];
                }
```

```
        }

        packed[0] = min;
        packed[1] = max;

        return packed;
}
```

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